

Basic Applied Maths Paper 1 - May 1991

Answers

1. a) $\log(x^4 + 3x^3) - \log(x+3) + \log z - \log 6 = 2 \log x$
 $\log\left(\frac{x^4 + 3x^3}{x+3}\right) + \log \frac{1}{3} = \log x^2$

$$x^3 + \frac{1}{3} = x^2$$

b) $y^2 = 3^2 \cdot 2^4 = 9 \cdot 16 = 144$

$$y = \pm 12$$

2. a) Given $N(K \cup L) = N(K) + N(L) - N(K \cap L)$

Let A, B, C be 3 non-empty sets and $D = B \cup C$.

Then $A \cup B \cup C = A \cup D$ and.

$$N(A \cup D) = N(A) + N(D) - N(A \cap D)$$

But $N(D) = N(B) + N(C) - N(B \cap C)$

So, $N(A \cup D) = N(A) + N(B) + N(C) - N(B \cap C) - N(A \cap D)$

$$= N(A) + N(B) + N(C) - N(B \cap C) - N(A \cap B) - N(A \cap C) + N(A \cap B \cap C)$$

b) $N(D \cup S \cup T) = 28 + 24 + 23 - 18 - 12 - 14 + 9 = 40$

3. a) $a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$

$$\Rightarrow y_1 = -\frac{a_1}{b_1}x - \frac{c_1}{b_1} \quad \Rightarrow y_2 = -\frac{a_2}{b_2}x - \frac{c_2}{b_2}$$

if $y_1 = y_2 \Rightarrow m_1 = -\frac{1}{m_2} \Rightarrow -\frac{a_1}{b_1} = -\frac{1}{-\frac{a_2}{b_2}}$

$$\Rightarrow -\frac{a_1}{b_1} = \frac{b_2}{a_2}$$

$$\Rightarrow -a_1 a_2 = b_1 b_2$$

$$\Rightarrow a_1 a_2 + b_1 b_2 = 0 \text{ grad.}$$

b) $y = 2x + 3$

$$m = 2$$

$$m_{\perp} = -\frac{1}{2}$$

$$y - 1 = -\frac{1}{2}(x - 3)$$

$$y = -\frac{1}{2}x + \frac{5}{2}$$

4. a) $C(n, 2) = 10$

$$\binom{n}{2} = \frac{n!}{(n-2)!2!} = 10$$

$$\frac{n \cdot (n-1)}{2} = 10$$

$$n^2 - n - 20 = 0 \quad n = 5$$

b)

	paint	no paint	
heads	10	24	34
no head	10	6	16
	20	30	50

i) $P(\text{no paint \& no head}) = \frac{6}{50}$

ii) $P(\text{no head or paint}) = P(\text{no head}) + P(\text{paint}) - P(\text{no head + paint})$
 $= \frac{16}{50} + \frac{20}{50} - \frac{10}{50} = \frac{26}{50}$

5. $\left[\begin{array}{cc|cc} 2 & -4 & 10 & 0 \\ 6 & 9 & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & -2 & 1/2 & 0 \\ 0 & 21 & -3 & 1 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 3/14 & 2/21 \\ 0 & 1 & -1/7 & 1/21 \end{array} \right] \quad C^{-1} = \left[\begin{array}{cc|cc} 3/14 & 2/21 \\ -1/7 & 1/21 \end{array} \right]$

$$\left[\begin{array}{cc|cc} 24 & 10 \\ 0 & 3 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 20 & 1 & -4/3 \\ 0 & 1 & -3 & 1 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1/2 & -2/3 \\ 0 & 1 & 0 & 1/3 \end{array} \right] \quad D^{-1} = \left[\begin{array}{cc|cc} 1/2 & -2/3 \\ 0 & 1/3 \end{array} \right]$$

$$CD = \begin{bmatrix} 2 & -4 \\ 6 & 9 \end{bmatrix} \begin{bmatrix} 24 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 12 & 51 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 4 & -4 & 10 & 0 \\ 12 & 51 & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & -1 & 1/4 & 0 \\ 0 & 63 & -3 & 1 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 17/84 & 1/63 \\ 0 & 1 & -1/21 & 1/63 \end{array} \right]$$

$$(CD)^{-1} = \left[\begin{array}{cc|cc} 17/84 & 1/63 \\ -1/21 & 1/63 \end{array} \right]$$

$$D^{-1}C^{-1} = \left[\begin{array}{cc|cc} 1/2 & -2/3 \\ 0 & 1/3 \end{array} \right] \left[\begin{array}{cc|cc} 3/14 & 2/21 \\ -1/7 & 1/21 \end{array} \right] = \left[\begin{array}{cc|cc} 17/84 & 1/63 \\ -1/21 & 1/63 \end{array} \right]$$

6. a) $\vec{v} = 2i + 3j - k$

$$\vec{u} = -2i + 2j + 2k$$

if $\vec{v} \perp \vec{u}$ then $\vec{v} \cdot \vec{u} = 0$

$$\vec{v} \cdot \vec{u} = 2(-2) + (3)(2) + (-1)(2) = 0$$

b) $\vec{v} = 2a + b = 2(i + 2j - 2k) + (3i + 2j + 6k)$
 $= 5i + 6j + 2k$

$|\vec{v}| = (25 + 36 + 4)^{1/2} = (65)^{1/2} \approx 8.06$

c) $\vec{F} = i - 2j + 5k$

$(-1, 1, 0)$ to $(2, 4, 1)$ $\vec{r} = (2+1)i + (4-1)j + (1-0)k$
 $= 3i + 3j + k$

$W = \vec{F} \cdot \vec{r} = 3(4) + 3(-2) + 1(5) = 2 \text{ J}$

7.

$2x + 3y + z = 3$
 $x - z = 3$
 $x + y + z = 4$

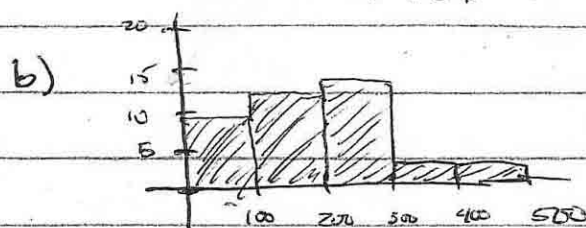
$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 3 \\ 1 & 0 & -1 & 3 \\ 1 & 1 & 1 & 4 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 0 & 3 & 3 & -3 \\ 1 & 0 & -1 & 3 \\ 2 & 1 & 0 & 7 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 0 & 1 & 1 & -1 \\ 1 & 1 & 0 & 2 \\ 2 & 1 & 0 & 7 \end{array} \right]$$

$\rightarrow \left[\begin{array}{ccc|c} 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -3 \\ 1 & 0 & 0 & 5 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{array} \right] \quad \begin{array}{l} x = 5 \\ y = -3 \\ z = 2 \end{array}$

8.

interval	x_i	f_i
0-99	50	9
100-199	150	11
200-299	250	12
300-399	350	2
400-499	450	2
		36

a) $\bar{x} = \frac{\sum x_i f_i}{\sum f_i} = \frac{50 \cdot 9 + 150 \cdot 11 + 250 \cdot 12 + 350 \cdot 2 + 450 \cdot 2}{36}$
 $= 186$



9. a) $f(x) = \frac{1}{x^2 - 1}$ $D = \mathbb{R} - \{1, -1\}$
 $R = \mathbb{R}$

b) $f(x) = \frac{1}{x^2 + 1}$ $D = \mathbb{R}$
 $R = \mathbb{R} \geq 0$

2. a) i) $\int (x^2 + 2)(3x - 1) dx = \int (3x^3 - x^2 + 6x - 2) dx = \frac{3x^4}{4} - \frac{x^3}{3} + 3x^2 - 2x + c$

b) i) $\int (4x + \frac{3}{x}) dx = 4 \frac{x^2}{2} + 3 \ln|x| + c$ a) ii) $\int (2x+3)^2 dx = \frac{1}{6} (2x+3)^3 + c$

ii) $\int_1^4 (x^2 + 3) dx = \left[\frac{x^3}{3} + 3x \right]_1^4 = \frac{4^3}{3} + 12 - \frac{1}{3} - 3 = 30$

11. a) i) $D(x^2 \sin x) = 2x \sin x + x^2 \cos x$

ii) $D(4xe^{3x}) = 4e^{3x} + 12xe^{3x}$

iii) $D\left(\frac{x^2-x}{(x+1)^2}\right) = \frac{(2x-1)(x+1)^2 - (x^2-x)2(x+1)}{(x+1)^4} = \frac{(2x-1)(x+1) - (x^2-x) \cdot 2}{(x+1)^3}$

b) $\int_1^{\sqrt{3}} \frac{1}{\sqrt{4-2x}} dx$ let $u = 4-2x$ $du = -2dx$

$= -\frac{1}{2} \int_2^{4-2\sqrt{3}} u^{-1/2} du = -\frac{1}{2} u^{1/2} \Big|_2^{4-2\sqrt{3}} = -\frac{1}{2} (4-2x)^{1/2} \Big|_1^{\sqrt{3}}$

$= -\frac{1}{2} \left[(4-2\sqrt{3})^{1/2} - (4-2)^{1/2} \right]$

$= -\frac{1}{2} \left[(4-2\sqrt{3})^{1/2} - \sqrt{2} \right]$

12. a) $X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}$

b) $f(x) = x^2 - 2$ $X_{n+1} = X_n - \frac{X_n^2 - 2}{2X_n}$

$f'(x) = 2x$

$= \frac{2X_n^2 - X_n^2 + 2}{2X_n} = \frac{X_n^2 + 2}{2X_n}$

c) $X_0 = 1.5$ $X_1 = \frac{(1.5)^2 + 2}{2(1.5)} = 1.42$

$X_2 = \frac{(1.42)^2 + 2}{2(1.42)} \approx 1.41$

d) $X_0 = 2$

$X_{n+1} = X_n - \frac{X_n^3 - 9}{3X_n^2}$

$f(x) = x^3 - 9 = 0$

$f'(x) = 3x^2$

$X_1 = 2 - \frac{2^3 - 9}{3 \cdot 4} = 2.083$

$X_2 = 2.083 - \frac{(2.083)^3 - 9}{3(2.083)^2} = 2.080$

13. a) i) $V = \int a dt = \int -4 dt = -4t + c$

$v(t) = -4t - 2$

ii) $S = \int v dt = \int (-4t - 2) dt = -2t^2 - 2t + d$

$s(0) = 0 \Rightarrow d = 0$ $s(t) = -2t^2 - 2t$