THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1

ADVANCED MATHEMATICS 1

(For Both School and Private Candidates)

Time: 3Hours *Tuesday 06 May 2003 a.m.*

Instructions

1. This paper consists of sections A and B.
2. Answer ALL questions in section A and FOUR (4) from section B.
3. ALL work done in answering each question must be shown clearly.
4. Mathematical tables, mathematical formulae, slide rules and **nonprogrammable** pocket calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your Examination Number on every page of your answer booklet(s).

SECTION A (60 marks)

Answer ALL questions in this section showing all necessary steps and answers.

1. (a) Use the algebra of sets to simplify $(A - B) \cup A$. **(03 marks)**

(b) Out of 150 deaths of children that occurred at Mamboleo village over a period of 5 years, 60 % were caused by diarrhoea, 30 % by malaria and 40 % by pneumonia. The deaths caused by diarrhoea and malaria were 18 by diarrhoea and pneumonia were 30 and by malaria or pneumonia but not diarrhoea were 51 . The number of children who died because of all diseases was 12.

Find the number of children whose deaths were caused by

(i) diseases other than the three mentioned above **(01½ marks)**

(ii) diarrhoea or pneumonia but not malaria. **(01½ marks)**

2. (a) The straight line through (4, -2) with slope m meets the x-axis at point R and the y-axis at point S. Verify that the locus of T, the mid-point of RS passes through the origin. **(04 marks)**

(b) The roots of $x^2 + px - 5 = 0$ are α and β, find the value of $\alpha^2 + \beta^2 - 3$ in terms of p. **(02 marks)**

3. (a) Given that $f(x) = 2x + 3$, $g(x) = 2 - x$ and $h(x) = \frac{1}{x} - 3$, show that

(i) the function composition is associative

(ii) $(f \circ g \circ h)^{-1}(x) = h \circ^{-1} g \circ^{-1} f^{-1}$ **(05 marks)**

(b) Find the relation between q and r so that $x^3 + 3px^2 + qx + r$ is a perfect cube for all values of x. **(01 mark)**

4. (a) The sum to infinity of a geometric series whose second term is 4 is 16. Find

(i) the first term

(ii) the common ratio. **(02 marks)**

(b) (i) Write the statement of the Maclaurin's series for a function f(x) and use it to expand $f(x) = (x + 1)^{-1}$ up to the term x^4. **(02 marks)**

(ii) Give the approximate value of $5/6$ using the first 5 terms of the series in 4. (b) (i) above correct to 4 decimal places. **(02 marks)**

5. (a) Prove that in any triangle ABC, $\sin \frac{1}{2}(B - C) = \frac{b - c}{a} \cos \frac{1}{2}A$, where a ,b and c are the sides of the triangle. **(04 marks)**

(b) Find the solution of the equation $2 \cos^2 A = 3(1 - \sin A)$ for $0 < A < 2\pi$. **(02 marks)**

6. (a) Given the curve $y^2 + 2xy - 3x^2 - 5 = 0$. Find the values of $\frac{d^2y}{dx^2}$ at the point where x = 1 and y has a positive value. **(04 marks)**

(b) Differentiate with respect to x given that $y = (x^2 + 1)^{\sin x}$ **(02 marks)**

7. (a) Find the smallest angle enclosed by the lines L_1: $x-2=\frac{y+1}{2}=z-1$

L_2: $\frac{x-3}{3} = \frac{2-y}{2} = z+3$ (02½ marks)

(b) The area of the plane whose sides are the vectors $r_1 = 2i - j + 3k$ and $r_2 = i + cj - k$ is three times the dot product of r_1 and r_2. Find the two possible values of c. (03½ marks)

8. (a) Integrate with respect to x: $\int \frac{dx}{(1+x)\,\ell n\,(1+x)}$ (01½ marks)

(b) Evaluate the following integrals:

(i) $\int_0^{\pi/4} x^2 \cos 2x \, dx$ (02½ marks)

(ii) $\int_1^2 \frac{2x+1}{\sqrt{x-1}} dx$ (02 marks)

9. (a) Two events A and B are such that P(A) = ¼, P(B) = ½ and $P(A \wedge B) = \frac{1}{8}$. Evaluate

(i) $P(A' \cap B)$

(ii) $P(A'|B)$

(iii) $P(A|B')$ (03½ marks)

(b) Box A contains nine cards numbered 1 through 9, and box B contains five cards numbered 1 through 5. A box is chosen at random and a card is drawn. If the number drawn is even, find the probability that the card came from box A. (02½ marks)

10. A small household poultry keeper with 150 birds recorded his egg collection for 30 days as follows:

81	71	75	108	67	75
83	68	77	93	83	94
89	77	77	63	76	66
64	66	69	95	103	69
82	76	78	104	91	94

(a) Make a frequency distribution of class size 10 eggs with the lower class limit of the lowest class as 61. (02 marks)

(b) (i) From the distribution compute the average egg collection and the variance. (03 marks)

(ii) If the running cost per day is 2000/= and she sells an egg at 60/=, compute the average net profit in 30 days rounded to the nearest shilling. (01 mark)

SECTION B (40 marks)

Answer FOUR questions from this section showing all necessary steps and answers.

11. (a) A retail shop received orders from two customers A and B for the following food packages: The package for A should contain 20 kg of beans, 20 kg of rice and 20 kg of maize flour, while that for B should contain 10 kg of beans and 30 kg of rice. The shop has only340 kg of beans, 540 kg of rice and 280 kg of maize flour. If a unit of package A costs 1,200/= and package B costs 900/=, how many packages should he supply to each of his customers so as to realise the maximum sales? (08 marks)

(b) How much of each commodity does the retailer in (a) above remain with after meeting the orders? (02 marks)

12. (a) Given that $z = x + iy$ express the complex number $\frac{z+i}{iz+2}$ in polynomial form and hence find the resulting complex number when $z = 1 + 2i$. (04 marks)

(b) From De Moivre's theorem prove that the complex number $(\sqrt{3} + i)^n + (\sqrt{3} - i)^n$ is always real and hence find the value of the expression when $n = 6$. (06 marks)

13. (a) Form a differential equation representing a circle of radius r and whose centre is along the x-axis. (02 marks)

(b) (i) Solve the differential equation $xdy + 3ydx = e^x dx$ (04 marks)

(ii) If $\frac{d^2x}{dt^2}$ is directly proportional to $\frac{-dx}{dt}$, and if $x = 20$, $\frac{dx}{dt} = 25$ when $t = 0$, solve the resulting 2nd order linear differential equation. (04 marks)

14. (a) Use the Newton-Raphson method to approximate the positive root of $x^2 - x - 1 = 0$ correct to 4 decimal places (perform 3 iterations only, starting with $x_0 = 2$). (05 marks)

(b) The table below gives the value of $f(x) = \frac{1}{x+1}$ from $x = 0$ to $x = 2$.

x	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
f(x)	1.0000	0.8333	0.7143	0.6250	0.5556	0.5000	0.4545	0.4166	0.3846	0.3571	0.3333

From the table approximate $\ln 3$ to 4 decimal places using

(i) Trapezium rule (ii) Simpson's rule. (05 marks)

15. A certain experiment with 4000 trials was said to follow a binomial distribution with a mean 40.

(a) Find

(i) the probability of success in a single trial

(ii) the variance

(iii) the standard deviation. (04 marks)

(b) Use the normal approximation to the binomial distribution to find the probability that the number of success is

(i) between 35 and 60 inclusive

(ii) less than or equal to 50 successes. (06 marks)

16 (a) Under the action of forces $F = 2i + 2j - 3k$ Nand $F_2 = i + 3j$ N, a body attains the velocity $^{dr}/_{dt} = i + 2tj + k$ m/s.
If at $t = 0$ the body was at the origin find the work done by the resultant force at $t = 4$ sec. (04½ marks)

(b) A particle travels anticlockwise along the circle $r = 25 \cos Ai + 25 \sin A j$ at a constant speed of $|V| = 15$ m/s.

Find

(i) the constant angular speed $\frac{dA}{dt}$

(ii) the velocity and acceleration vectors at point (20, 15). (05 marks)