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THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SEÇONDARY EDUCATION EXAMINATION

142/1

ADVANCED MATHEMATICS 1
(For Both School and Private Candidates)

Time: 3 Hours *Tuesday, March 08, 2005 a.m.*

Instructions

1. This paper consists of sections A and B.
2. Answer *all* questions in section A and *four (4)* questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical formulae, slide rules and non-programmable pocket calculators may be used.
5. Cellular phones are *not* allowed in the examination room.
6. Write your *Examination Number* on every page of your answer booklet(s).

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SECTION A (60 Marks)

Answer **all** questions in this section showing all necessary steps and answers.

1. (a) Use the algebra of sets to simplify $(A \cup B)' \cap (B - A)$ **(02 marks)**

(b) In a group of 60 students 23 play football, 15 play tennis and 20 play basketball. 7 play football and tennis, 5 play basketball and tennis, 4 play football and basketball and 15 do not play any of these games. Find the number of students who play all the three games. **(02 marks)**

(c) In 1(b) above, find the number of students who play:

(i) Football but not basketball.

(ii) Football and basketball but not tennis. **(02 marks)**

2. (a) Show that the area of a triangle ABC is $(x_1 y_2 - x_2 y_1)$ given that $A(x_2, y_2)$, $B(x_1, y_1)$ and C is the origin. **(03 marks)**

(b) (i) Find the acute angle between the lines $2y - 6x - 7 = 0$ and $x - 2y - 10 = 0$. **(1½ marks)**

(ii) Find the area of a triangle whose vertices are the points (5, 1), (6, 9) and (−1, 5). **(1½ marks)**

3. (a) A relation is defined by $F(x) = n$ if $n \le x < n + 2$ where n is an integer. Graph f(x), stating its domain and range. **(03 marks)**

(b) Find the horizontal asymptotes of the function $\dfrac{x^2 + 2x + 5}{x + 2}$ **(03 marks)**

4. (a) The first 3 terms of an arithmetical progression are a, b and a^2 respectively where a is negative. The first 3 terms of geometrical progression are a, a^2 and b. Find:

(i) The values of a and b.

(ii) The sum to infinity of the G.P. **(03½ marks)**

(b) Prove by using the principle of mathematical induction that the number of all subsets of a set containing n distinct elements is 2^n. **(02½ marks)**

5. (a) Prove that $\dfrac{\tan\theta - \cot\theta}{\sec\theta - \operatorname{cosec}\theta} = \dfrac{\sec\theta + \operatorname{cosec}\theta}{\tan\theta + \cot\theta}$ **(02½ marks)**

(b) Find the values of x between 0° and 360° which satisfy the equation $8\cos x + 9\sin x = 7.25$. **(03½ marks)**

6. (a) Find $\dfrac{dy}{dx}$ if (i) $e^{xy} + x + y = 1$ **(01½ marks)**

(ii) $\tanh^{-1} \dfrac{2x}{1 + x^2}$ **(03 marks)**

(b) An error of 2 % is made in measuring the radius of a sphere. What is the resulting percentage error in the calculation of its surface area? **(01½ marks)**

7. (a) Find the projection of the vector $\underline{a} = \underline{i} - 2\underline{j} + \underline{k}$ on the vector $\underline{b} = 4\underline{i} - 4\underline{j} + 7\underline{k}$. **(03 marks)**

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(b) Prove that in a skew quadrilateral the joins of the mid-points of opposite sides bisect each other. **(03 marks)**

8. Find $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx$ and hence evaluate $\int_0^a \frac{dx}{x + \sqrt{a^2 - x^2}}$. **(06 marks)**

9. A student is to attempt 8 out of 10 questions in an examination. How many choices does the student have if she must answer:

(a) the first 3 questions **(02 marks)**

(b) at least 4 of the first 5 questions? **(04 marks)**

10. The number of defective balls in equal batches of tennis balls produced by a company on 36 consecutive days are:

3	1	2	4	0	1	2	2	3	2	1	1
5	0	1	1	1	3	2	3	4	0	1	1
2	2	3	0	3	1	2	0	1	4	3	2

(a) Construct a frequency table showing for $x = 0, 1, 2, \ldots 6$ the number n of batches having x defective balls. **(02 marks)**

(b) Write down the mode and median for the above events. **(01½ marks)**

(c) Using the deviation approach, calculate the mean number of defective balls per day. Take the assumed mean $A = 3$ and $d_i = X_i - A$. **(02½ marks)**

SECTION B (40 Marks)

Answer **four (4)** questions from this section.

11. A gravel dealer has two quarries Q_1 and Q_2 which produce 3000 m^3 and 1500 m^3 of gravel per week respectively. Three builders B_1, B_2 and B_3 require each week 2000 m^3, 1500 m^3 and 1000 m^3 of gravel respectively. The distances between the quarries and the sites of the builders (in km) are as shown below:

	B_1	B_2	B_3
Q_1	7	4	2
Q_2	3	2	2

How should the gravel dealer supply gravel to the builders as cheaply as possible? **(10 marks)**

12. (a) Express the complex number $z = \frac{8(1 + i)}{\sqrt{2}}$ in the form $r(\cos\theta + i\sin\theta)$ and hence find the three values of $z^{2/3}$ **(04 marks)**

$\frac{36+1}{2} =$

(b) If $|z - 1| = 3\,|z + 1|$ prove that the locus of z in an argand diagram is a circle and find its centre and radius. **(02 marks)**

(c) Prove that $\sin^5\theta = \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$.

$36+1$ $\frac{37}{2}$

Hence find $\int (10\sin\theta - 16\sin^5\theta)\,d\theta$. **(04 marks)**

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13. (a) Show that the equation $x^3 + 5x - 10 = 0$ has roots in the interval $[1, 2]$. Using linear interpolation, find this root to one decimal place. (Use 3 iterations only). **(06 marks)**

(b) Using Simpson's rule, find an approximate value of the length of the portion of the ellipse $\frac{x^2}{4} + y^2 = 1$ that lies in the first quadrant between $x = 0$ and $x = 1$ and the equal intervals between the ordinates is 0.25. **(04 marks)**

14. (a) (i) Given that $y = \frac{\pi}{6}$ at $x = \frac{\pi}{6}$, solve the differential equation: $\frac{dy}{dx} = \sin 2x \sec y$. **(02 marks)**

(ii) Use the substitution $y = zx$ where z is a function of x, to transform the differential equation $x^2 \frac{dy}{dx} = y(x + y)$ into a differential equation in z and x. By solving this equation first, find y in terms of x for $x > 0$ given that $y = -1$ at $x = -1$. **(03 marks)**

(b) Find the general solution of the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \cos 2x$. **(05 marks)**

15. (a) The probability that a baker will have sold all of his loaves x hours after baking is given by the probability density function

$$f(x) = \begin{cases} k(36 - x^2) & \text{for } 0 \le x \le 6, \\ 0 & \text{otherwise} \end{cases}$$

(i) Determine the value of k.

(ii) Calculate the mean value and the probability that the baker will have some bread left after 5 hours. **(05 marks)**

(b) The mean inside diameter of a sample of 200 washers produced by a machine is 5.02 mm and the standard deviation is 0.05 mm. The purpose for which these washers are intended allows a maximum tolerance in the diameter of 4.96 to 5.08 mm. Otherwise the washers are considered defective. Determine the percentage of defective washers produced by the machine, assuming the diameters are normally distributed. **(05 marks)**

16. (a) When a stone is thrown vertically upwards, the air resistance gives it a retardation of Kv^2g m/sec^2, where v is the velocity in m/s and K is a constant. If the initial velocity of the stone is u, find the greatest height reached. **(06 marks)**

(b) A sphere of mass 3 kg moving at 5 m/s strikes a similar sphere of mass 2 kg travelling in opposite direction at 2 m/s, the coefficient of restitution is $2/7$. Find the velocities after impact. **(04 marks)**

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