SECTION A (60 Marks)

Answer ALL questions in this section showing ALL necessary workings and answers.

1. (a) Use logarithms to find

(i) \( \sqrt[3]{8 \csc 15^\circ \cos 15^\circ} \).

(ii) \( \theta \), if \( \tan \theta = \frac{14.32 \tan 16^\circ 24'}{76.9} \). (2 marks)

(b) Using a non programmable scientific calculator, find

\[ 24^\circ 6' 31'' + 85.34 \text{ rad} \] (give the answer in radians to 7 dec. places).

(c) By using the statistical functions of your scientific calculator, find the mean \((\bar{x})\) and the standard deviation \((\sigma_{x-1})\) of the following values (correct to 8 decimal places).

<table>
<thead>
<tr>
<th>Value</th>
<th>110</th>
<th>130</th>
<th>150</th>
<th>170</th>
<th>190</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>10</td>
<td>31</td>
<td>24</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

3 marks

2. Find the equation of the circle which passes through the point \( A \) and touches the line \( l \) at the point \( B \) where \( A(4, -3), \ B(3, 2) \) and \( l: x + 2y = 7 \). 6 marks

3. Find the equation of the parabola whose focus is the point \((-2, 0)\), and whose directrix is the line \( x = 2 \). Draw the parabola and label its focus, vertex, directrix and axis. 6 marks

4. (a) Solve the following simultaneous equations

(i) \( \log_4 y = 2 \) and \( xy = 8 \).

(ii) \( \log (x + y) = 0 \) and \( 2 \log x = \log (y + 1) \). (4 marks)

(b) Find the positive value of \( x \) that satisfies the equation

\[ \log_4 x = \log_4 (x + 6) \]. (2 marks)

5. (a) Prove that

\[ \cos^3 \theta + \cos^3 (\theta + \frac{2\pi}{3}) + \cos^3 (\theta + \frac{4\pi}{3}) = \frac{3}{2} \]. (3 marks)

(b) If \( \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} = \tan 60^\circ \), prove that one value of \( \theta \) is \( 15^\circ \). (3 marks)
6. Differentiate
   (a) \( \log_{10} x^2 \) \hspace{1cm} (2 marks)
   (b) \( \tan^{-1} (\coth x) \) \hspace{1cm} (2 marks)
   (c) \( \ln \frac{\sin x}{\cos 2x} \) \hspace{1cm} (2 marks)

7. Let \( a = i + j \), \( b = i - j \) and \( c = 3i - 4j \). Resolve \( c \) into vectors parallel to \( a \) and \( b \) \hspace{1cm} (6 marks)

8. Do the following integrals
   (a) \( \int x \cdot x \, dx \) \hspace{1cm} (½ mark)
   (b) \( \int x e^{3x^2} \, dx \) \hspace{1cm} (1½ marks)
   (c) \( \int \frac{\cos \theta}{1 + \sin^2 \theta} \, d\theta \) \hspace{1cm} (2 marks)
   (d) \( \int \sqrt{4(1 + \sin \theta)^2} \, d\theta \) \hspace{1cm} (2 marks)

9. One bag contains 4 white balls and 2 black balls; another bag contains 3 white balls and 5 black balls. If one ball is drawn from each bag, find the probability that
   (a) both are white balls. \hspace{1cm} (2 marks)
   (b) both are black balls. \hspace{1cm} (2 marks)
   (c) one is a white ball and one is a black ball. \hspace{1cm} (2 marks)

10. Five coins were tossed 1,000 times, and at each toss the number of heads were counted. The number of tosses during which 0, 1, 2, 3, 4 and 5 heads were obtained is shown in a table.

<table>
<thead>
<tr>
<th>Number of heads</th>
<th>Number of tosses (Freq.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>38</td>
</tr>
<tr>
<td>1</td>
<td>144</td>
</tr>
<tr>
<td>2</td>
<td>342</td>
</tr>
<tr>
<td>3</td>
<td>287</td>
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<tr>
<td>4</td>
<td>164</td>
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<tr>
<td>5</td>
<td>25</td>
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<tr>
<td>Total</td>
<td>1,000</td>
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</table>

(a) Draw the graph which represent the data. \hspace{1cm} (4 marks)
(b) From the graph, give a statement which shows that the probability of getting a head is almost a half. (2 marks)

SECTION B (40 Marks)

Answer ANY FOUR (4) questions from this section showing all necessary workings and answers.

11. (a) Express the vector \( \mathbf{r} = 10 \mathbf{i} - 3 \mathbf{j} - \mathbf{k} \) as a linear function of \( \mathbf{a}, \mathbf{b} \) and \( \mathbf{c} \) given that
   \[
   \mathbf{a} = 2 \mathbf{i} - \mathbf{j} + 3 \mathbf{k}
   \]
   \[
   \mathbf{b} = 3 \mathbf{i} + 2 \mathbf{j} - 4 \mathbf{k}
   \]
   and \( \mathbf{c} = -\mathbf{i} + 3 \mathbf{j} - 2 \mathbf{k} \) (5 marks)

(b) Find the position vector of the foot of the perpendicular from the origin to the line
   \( \mathbf{d} = 3m \mathbf{i} + 4(1 - m) \mathbf{j} \), where \( m \) is a scalar. (7 marks)

12. (a) By the use of Cramer’s rule, solve the following system of equations.
   \[
   \begin{align*}
   2x + 3y - z &= -7 \\
   -3x + y + 2z &= 1 \\
   3x - 4y - 4z &= -1
   \end{align*}
   \] (7 marks)

(b) State the condition for the following system of equations to be consistent:
   \[
   ax + by + cz = u
   \]
   \[
   a'x + b'y + c'z = u'
   \]
   \[
   a''x + b''y + c''z = u''
   \] (1 mark)

(c) Show without solving the system of equations below whether they are consistent or not.
   \[
   \begin{align*}
   2x - 3y + z &= 4 \\
   3x + y - z &= 6 \\
   5x + 9y - 2z &= 3
   \end{align*}
   \] (2 marks)

13. (a) Transform the following equation into polar coordinates.
   \[
   \left( x^2 + y^2 \right)^3 = a^2 x y \left( x^2 - y^2 \right)
   \] (2 marks)

(b) Sketch the curve whose polar equation is given by
(b) Sketch the curve whose polar equation is given by
\[ r = 1 + 2 \cos \theta. \] (5 marks)

(c) Find the area of the curve in (b). (3 marks)

14. (a) Show that
\[ \frac{1 + \tanh x}{1 - \tanh x} = \cosh 2x + \sinh 2x \] (5 marks)

(b) Integrate \( \sqrt{x^2 + 2x - 1} \) with respect to \( x \). (5 marks)

15. Integrate the following with respect to \( x \).

(a) \[ \int \sqrt{x^2 + 25} \, dx \] (2½ marks)

(b) \[ \int \left( \frac{\sin x + \cos x}{\cos x - \sin x} \right) \, dx \] (3½ marks)

(c) \[ \int \frac{dx}{2x^2 + x - 3} \] (4 marks)

16. (a) Simplify the following using appropriate laws.

(i) \( \sim (p \lor q) \) (2 marks)

(ii) \( \sim (\sim p \land q) \)

(b) By using truth tables, prove the following.
\[ p \land (q \lor r) = (p \land q) \lor (p \land r) \]

(c) Consider the truth table below.

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
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<th>(k)</th>
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(i) Write the compound statements equivalent to the truth table of (k), (l) and (m).
(ii) Simplify the compound statement for (k).
(iii) Draw the corresponding network of (ii). (3 marks)