SECTION A (60 Marks)

Answer **ALL** questions in this section showing **ALL** necessary workings and answers.

1. (a) Use logarithms to find

(i) $\sqrt[3]{8 \csc 15° \cos 15°}$.

(ii) θ, if $\tan \theta = \dfrac{14.32 \tan 16° \, 24'}{76.9}$. (2 marks)

(b) Using a non programmable scientific calculator, find

$24° \, 6' \, 31'' + 85.34$ rad (give the answer in radians to 7 dec. places).

(c) By using the statistical functions of your scientific calculator, find the mean ($\overline{x}$) and the standard deviation (σ_{n-1}) of the following values (correct to 8 decimal places).

Value	110	130	150	170	190
Frequency	10	31	24	2	2

3 marks

2. Find the equation of the circle which passes through the point A and touches the line ℓ at the point B where A(4, -3), B(3, 2) and $\ell : x + 2y = 7$. 6 marks

3. Find the equation of the parabola whose focus is the point (-2, 0), and whose directrix is the line $x = 2$. Draw the parabola and label its focus, vertex, directrix and axis. 6 marks

4. (a) Solve the following simultaneous equations

(i) $\log_x y = 2$ and $xy = 8$.

(ii) $\log(x + y) = 0$ and $2 \log x = \log(y + 1)$. (4 marks)

(b) Find the positive value of x that satisfies the equation

$\log_2 x = \log_4 (x + 6)$. (2 marks)

5. (a) Prove that

$$\cos^2\theta + \cos^2\left(\theta + \frac{2}{3}\pi\right) + \cos^2\left(\theta + \frac{4}{3}\pi\right) = \frac{3}{2}.$$ (3 marks)

(b) If $\dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \tan 60°$, prove that one value of θ is 15°. (3 marks)

6. Differentiate

(a) $\log_{10} x^2$ (2 marks)

(b) $\tan^{-1}(\coth x)$ (2 marks)

(c) $\ln \dfrac{\sin x}{\cos 2x}$ (2 marks)

7. Let $\underset{\sim}{a} = \underset{\sim}{i} + \underset{\sim}{j}$, $\underset{\sim}{b} = \underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{c} = 3\underset{\sim}{i} - 4\underset{\sim}{j}$. Resolve $\underset{\sim}{c}$ into vectors paralled to $\underset{\sim}{a}$ and $\underset{\sim}{b}$

(6 marks)

8. Do the following integrals

(a) $\int \sqrt{x}\ dx$ (½ mark)

(b) $\int x e^{3x^2}\ dx$ (1½ marks)

(c) $\int \dfrac{\cos \theta}{1 + \sin^2 \theta}\ d\theta$ (2 marks)

(d) $\int^{\pi/4} (1 + \sin \theta)^2\ d\theta$ (2 marks)

9. One bag contains 4 white balls and 2 black balls; another bag contains 3 white balls and 5 black balls. If one ball is drawn from each bag, find the probability that

(a) both are white balls. (2 marks)

(b) both are black balls. (2 marks)

(c) one is a white ball and one is a black ball. (2 marks)

10. Five coins were tossed 1,000 times, and at each toss the number of heads were counted. The number of tosses during which 0, 1, 2, 3, 4 and 5 heads were obtained is shown in a table.

Number of heads	Number of tosses (Freq.)
0	38
1	144
2	342
3	287
4	164
5	25
Total	1,000

(a) Draw the graph which represent the data. (4 marks)

(b) From the graph, give a statement which shows that the probability of getting a head is almost a half. (2 marks)

SECTION B (40 Marks)

Answer ANY FOUR (4) questions from this section showing all necessary workings and answers.

11. (a) Express the vector $\underset{\sim}{r} = 10\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k}$ as a linear function of $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$

given that

$$\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$$

$$\underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j} - 4\underset{\sim}{k}$$

and $\underset{\sim}{c} = -\underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$ (5 marks

(b) Find the position vector of the foot of the perpendicular from the origin to the line

$\underset{\sim}{d} = 3m\underset{\sim}{i} + 4(1-m)\underset{\sim}{j}$, where m is a scalar.

12. (a) By the use of Cramer's rule, solve the following system of equations.

$$\begin{cases} 2x + 3y - z = -7 \\ -3x + y + 2z = 1 \\ 3x - 4y - 4z = -1 \end{cases}$$ (7 marks)

(b) State the condition for the following system of equations to be consistent:

$$ax + by + cz = u$$

$$a'x + b'y + c'z = u'$$

$$a''x + b''y + c''z = u''$$ (1 mark)

(c) Show without solving the system of equations below whether they are consistent or not.

$$2x - 3y + z = 4$$

$$3x + y - z = 6$$

$$5x + 9y - 2z = 3$$ (2 marks)

13. (a) Transform the following equation into polar coordinates.

$$\left(x^2 + y^2\right)^3 = a^2xy\left(x^2 - y^2\right)$$ (2 marks)

(b) Sketch the curve whose polar equation is given by

(b) Sketch the curve whose polar equation is given by

$$r = 1 + 2\cos\theta.$$ (5 marks)

(c) Find the area of the curve in (b). (3 marks)

14. (a) Show that

$$\frac{1 + \tanh x}{1 - \tanh x} = \cosh 2x + \sinh 2x$$ (5 marks)

(b) Integrate $\sqrt{x^2 + 2x - 1}$ with respect to x. (5 marks)

15. Integrate the following with respect to x.

(a) $\int \sqrt{x^2 + 25}\, dx$ (2½ marks)

(b) $\int \left(\frac{\sin x + \cos x}{\cos x - \sin x}\right) dx$ (3½ marks)

(c) $\int \frac{dx}{2x^2 + x - 3}$ (4 marks)

16. (a) Simplify the following using appropriate laws.

(i) $\sim(p \vee \sim q)$

(ii) $\sim(\sim p \wedge q)$ (2 marks)

(b) By using truth tables, prove the following.

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

(c) Consider the truth table below.

P	Q	R	(k)	(ℓ)	(m)
T	T	T	T	T	F
T	T	F	F	T	T
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	T

(i) Write the compound statements equivalent to the truth table of (k), (ℓ) and (m).
(ii) Simplify the compound statement for (k).
(iii) Draw the corresponding network of (ii). (3 marks)