THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2

ADVANCED MATHEMATICS 2
(For School and Private Candidates)

Time: 3 Hours **2006 February, 13 Monday, p.m.**

INSTRUCTIONS

1. This paper consists of **sixteen (16)** questions in sections A and B.
2. Answer **all** questions in section A and **four (4)** questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables, mathematical formulae and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

ACS This paper consists of 6 printed pages.

SECTION A (60 marks)

Answer all questions in this section.

1. (a) By using mathematical tables, compute:

(i) $\sqrt{0.031}$ correct to 3 decimal places.

(ii) $\dfrac{\tan 36.75^{\circ}}{\sin 60^{\circ}48'}$

(3 marks)

(b) By using a scientific calculator, find:

(i) $e^{-2/3} + 12\cosh^{-1} 1.5$ correct to 6 significant figures.

(ii) $\ln \frac{2}{3} - 8^{\frac{1}{6}}$ correct to 6 significant figures.

(iii) The mean and standard deviation for the scores; 64, 44, 56, 75 and 66.

(3 marks)

2. (a) Let A_{pq} denote $p \wedge q$ and let Np donate ~p. Write the following propositions using A and N instead of $\wedge$ and ~:

(i) $\sim p \wedge q$

(ii) $\sim(p \wedge \sim q)$

(iii) $(\sim p \wedge q)$

(3 marks)

(b) Use algebra of propositions to show that $(p \wedge q) \vee [\sim r \wedge (q \wedge p] \equiv p \wedge q$.

(3 marks)

3. Determine the image of the curve $(x-4)^2 + (y+1)^2 = 9$ under the following transformations:

(a) Translation by the arrow $(-4, 1)$.

(b) Reflection on the line $y = x$.

(c) Dilation by factor 2 in y direction.

(d) Magnification by factor c.

(e) What important property can be seen about the shape of the locus in 3. (a) above after the transformation?

(6 marks)

4. (a) Solve the equation:

$\cos 2x + \cos x + 1 = 0$ for $0 \leq x \leq = 360°$.

(3 marks)

(b) Show that

$$\frac{\sin 2A + \cos 2A + 1}{\sin 2A + \cos 2A - 1} = \frac{\tan(45° + A)}{\tan A}$$

(3 marks)

5. (a) For what values of x is $\sin\theta = \frac{x-1}{x+1}$?

(3 marks)

(b) Find the sum of the following series and then simplify it.

$1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + n(n+1)$.

(3 marks)

6. The normal to the parabola $y^2 = 4x$ at P(1, 2) meets the x axis at G, and M is the midpoint of PG. A line through M parallel to the y axis meets the x axis at N and the parabola at Q. Prove that QN = PG.

(6 marks)

7. (a) If $\tan\theta = \sinh\theta$, prove that $\theta = \ln(\sec\theta + \tan\theta)$.

(3 marks)

(b) Show that if $a\cosh x + b\sinh x = c$ has equal roots, then $c^2 = a^2 - b^2$.

(3 marks)

8. (a) For the random variable x, show that $Var(x) = E^2(x) - [E(x)]^2$.

(3 marks)

(b) The random variable has a probability density function P(X = x) for x = 1, 2, 3 as shown in the table below:

x	1	2	3
P(x)	0.1	0.6	0.3

Find:

(i) $E(5x + 3)$.

(ii) $E(x^2)$.

(iii) $Var(5x + 3)$.

(3 marks)

$P(1,2)$

$\sum n(n+1)$

$\sum n^2 + \sum n$

$\frac{n}{6}(n+1)(2n+1) + \frac{n}{2}(n+1)$

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9. A and B play 12 games of chess of which 6 games are won by A, 4 games are won by B and 2 end in a draw. They agree to play a tournament consisting of 3 games. Find the probability that:

(a) A and B win alternately.

(b) 2 games end in a draw.

(c) B wins at least one game.

(6 marks)

10. (a) Express $\dfrac{1}{1+\cos\theta+i\sin\theta}$ in the form $x+iy$.

(2½ marks)

(b) Given that $x+iy=(-3-2i)^n$ where $x, y, \in \mathbb{R}$, $n \in Z$. Prove that $x^2+y^2=13^n$.

(3½ marks)

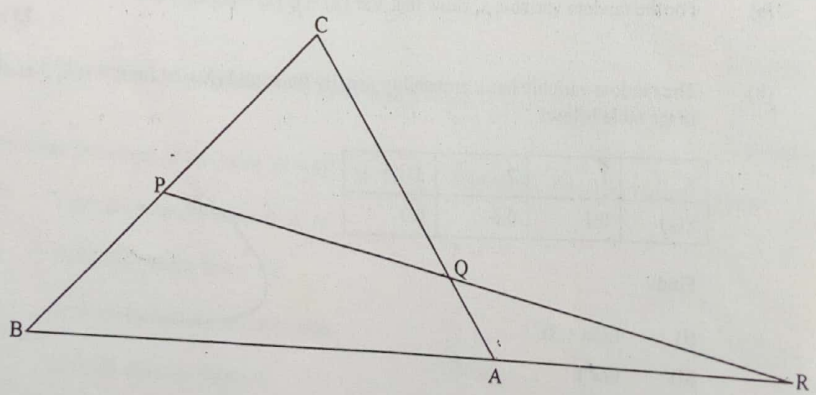
SECTION B (40 marks)

Answer **four (4)** questions from this section.

11. The vertices A, B and C of a triangle have position vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ respectively relative to the origin O. Point P is on BC such that BP : PC = 3 : 1. Point Q is on AC such that CQ : QA = 2 : 3. Point R on BA produced is such that BR : AR = 2 : 1. The position vectors of P, Q and R are $\underset{\sim}{p}$, $\underset{\sim}{q}$ and $\underset{\sim}{r}$ respectively.

(a) Show that $\underset{\sim}{q}$ can be expressed in terms of $\underset{\sim}{p}$ and $\underset{\sim}{r}$, hence show that P, Q and R are collinear.

(b) State the ratio of the line segments PQ and QR.



(10 marks)

12. (a) $(2x - y)^{20}$ is expanded as a series in descending powers of x. Find the:

(i) position and the value of the term in x^3.

(ii) position and the value of the term free from x.

(3 marks)

(b) Express $\frac{2}{n(n+2)}$ in partial fractions and deduce that

$$\frac{1}{1\times 3}+\frac{1}{2\times 4}+\frac{1}{3\times 5}---+\frac{1}{n(n+2)} = \frac{3}{4}-\frac{2n+3}{2(n+1)(n+2)}.$$

Hence find $\sum_{n=1}^{100}\frac{1}{n(n+2)}$.

(7 marks)

13. (a) Sketch the graph of $r = \sin 3\theta$ for $0° \le \theta \le 180°$.

(6 marks)

(b) Write down the equations for the asymptotes to the hyperbola $x^2 - y^2 = 1$ and hence sketch the hyperbola in the cartesian plane.

(2 marks)

(c) Transform the equation $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ to its corresponding polar equation.

(2 marks)

14. (a) Given that $y = \tan^{-1} x$, show that when $|x| < 1$ $\tanh^{-1} x = \frac{1}{2}\ell n\frac{1+x}{1-x}$.

(5 marks)

(b) A particle is moving along a curve so that its velocity t seconds later is given by

$$v = \frac{1}{\sqrt{t^2+4t-5}}.$$

Starting with $v = \frac{ds}{dt}$, find the expression for its displacement ~~S at~~ t seconds given that $S = 0$ when $t = 1$.

(5 marks)

15. (a) Using the laws of algebra of sets:

(i) Simplify $\sim(\sim(p \vee q) \vee (\sim p \wedge q))$

(ii) Verify that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

(4½ marks)

(b) Construct an electrical network for the proposition $(p \vee q) \wedge [(r \vee s) \wedge w]$.

(1½ marks)

(c) Draw the simplest network diagram corresponding to the statement of the following truth table.

p	q	r	Statement
T	T	T	F
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	T

(4 marks)

16. (a) Find

(i) $\int x^3 e^{2x} dx$.

(ii) $\int \frac{dx}{x^2\sqrt{4+x^2}}$.

(4 marks)

(b) Evaluate $\int_{1/2}^{1} \frac{\sin x}{x} dx$ accurate to 6 decimal places.

(6 marks)