

**THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION
EXAMINATION**

142/2

**ADVANCED MATHEMATICS 2
(For Both School and Private Candidates)**

Time: 3 Hours

Thursday, 10th February 2011 a.m.

INSTRUCTIONS

1. This paper consists of **sixteen (16)** questions in sections A and B.
2. Answer **all** questions in section A and **four (4)** questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables, mathematical formulae and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

This paper consists of 5 printed pages.

3,245,159.557

SECTION A (60 marks)Answer **all** questions in this section.

1. (a) By using mathematical tables evaluate:

$$\frac{[12 \tan^{-1}(3.42)]e^{8.22}}{(0.001182)^{1/2}}$$

95.55 x 10⁶

- (b) By using a non-programmable scientific calculator compute the following expression to eight significant figures:

$$\frac{(7 \ln 13.42)(42.8425)^{4/5}}{(0.03482)^{3/2} [1 \sin^{-1}(0.8543)]}$$

0.03482

(6 marks)

2. (a) Check the validity of the following argument
- $\sim p \rightarrow (q \leftrightarrow \sim r)$
- ,
- $\sim r \rightarrow \sim p$
- ,
- $q \rightarrow \sim r$
- and
- $\sim r$
- .

- (b) What does the proposition
- $[(p \wedge q) \wedge r] \rightarrow (p \wedge q)$
- represent?

(6 marks)

3. (a) What is the length of the tangent to the circle
- $4x^2 + 4y^2 - 8x - 2y + 15 = 0$
- from the point (2, 3)?

- (b) Show that the circles
- $x^2 + y^2 - 16x - 12y + 75 = 0$
- and
- $5x^2 + 5y^2 - 32x - 24y + 75 = 0$
- touch each other.

(6 marks)

4. (a) Solve the equation
- $\frac{1}{2} \tan^{-1} x = \tan^{-1} \left(\frac{1-x}{1+x} \right)$
- .

- (b) In the triangle ABC,
- $AB = x - y$
- ,
- $BC = x$
- ,
- $CA = x + y$
- . Show that

$$\cos B = \frac{x-4y}{2(x-y)}$$

- (c) Prove that
- $\frac{\cos 3\theta}{\cos \theta} - \frac{\cos 6\theta}{\cos 2\theta} = 2(\cos 2\theta - \cos 4\theta)$
- .

(6 marks)

5.

- (a) If α , β and γ are the roots of the equation $x^3 - 5x^2 + 5x - 2 = 0$, find the equation whose roots are $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$.
- (b) Prove that the equation $(k-2)x^2 + 2x - k = 0$ has real roots for all values of k .
- (c) When the expression $x^5 + 2x^2 + ax + b$ is divided by $x^2 - 4$, the remainder is $3x + 1$. Find the values of a and b .

(6 marks)

6.

- (a) Find the centre and equations of the asymptotes of the hyperbola $xy - x - 2y = 6$.
- (b) Find the equation for the set of points $P(x, y)$ such that they are equidistant from the origin O and the line $x = 4$.

(6 marks)

7.

- (a) If $y = (\cosh^{-1} x)^2$, show that $(x^2 - 1) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = 2$.
- (b) Show that $y = C_1 e^{2x} + C_2 e^{-2x}$ is the same as $y = b_1 \cosh 2x + b_2 \sinh 2x$.

(6 marks)

8.

The table below shows the marks obtained by students in a certain test.

Marks	Number of Students
0 - 10	10
10 - 20	7
20 - 30	3
30 - 40	5
40 - 50	15
50 - 60	10
60 - 70	15
70 - 80	10
80 - 90	10
90 - 100	5

- (a) Using the assumed mean $A = 55$, calculate the mean mark.
- (b) Calculate the standard deviation of the distribution.

(6 marks)

9. There are four defective oranges in a package of 10. If two oranges are selected at random one after another without replacement, what is the probability of each of the following?
- One defective and one good oranges will be selected.
 - At least one defective orange will be selected.
- (6 marks)
10. (a) Show that, when n is a positive integer:
- $$(\sqrt{3} - i)^n + (\sqrt{3} + i)^n = 2^{n+1} \cos\left(\frac{n\pi}{6}\right).$$
- (b) If $z = x + iy$, find real values of x and y which satisfies $z\bar{z} - 6i = 12 - 2iz$ where $\bar{z}$ is the conjugate of z .
- (6 marks)

SECTION B (40 marks)

Answer any **four (4)** questions from this section. Extra questions will **not** be marked.

11. (a) Use knowledge on vectors to prove the sine rule for plane triangles.
- (b) A force given by $\underline{F} = 3\underline{i} + 2\underline{j} - 4\underline{k}$ is applied at the point $(1, -1, 2)$. Find the moment of $\underline{F}$ about the point $(2, -1, 3)$.
- (c) Show that $\underline{a} = (2\underline{i} - 2\underline{j} + \underline{k})/3$, $\underline{b} = (\underline{i} + 2\underline{j} + 2\underline{k})/3$ and $\underline{c} = (2\underline{i} + \underline{j} - 2\underline{k})/3$ are mutually orthogonal unit vectors.
- (10 marks)
12. (a) The roots of the equation $3x^2 + 4x - 5 = 0$ are α and β . Find:
- $\frac{1}{\alpha} + \frac{1}{\beta}$,
 - $\alpha^2 + \beta^2$,
 - an equation whose roots are α^3 and β^3 .
- (b) If $f(x) = ax^2 + bx + c$ leaves remainders 1, 25 and 1 on division by $x - 1$, $x + 1$ and $x - 2$ respectively, show that $f(x)$ is a perfect square.
- (10 marks)

13. (a) Determine the coordinates of the point at which the normal to the curve $xy = 8$ at the point $(4, 2)$, cuts the tangent to the curve $16x^2 - y^2 = 64$ at the point $\left(\frac{5}{2}, 6\right)$.
- (b) AB is a chord of the rectangular hyperbola $xy = c^2$ and M is its midpoint. If AB has a constant length, find the locus of M. (10 marks)

14. (a) Differentiate the following hyperbolic functions with respect to x :

(i) $f(x) = \cosh^{-1}(3 - 2x)$,
 (ii) $f(x) = \sinh^{-1}(\tan x)$.

(b) Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$.

(c) Show that $\tanh^{-1}(x) = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$.

(10 marks)

15. (a) Simplify each of the following propositions by using laws of algebra of propositions:

(i) $q \rightarrow (\sim p \rightarrow \sim q)$,
 (ii) $\sim (p \vee q) \vee (\sim p \wedge q)$.

- (b) Use the truth table to test the validity of the following argument:
 "You like Geography and Advanced Mathematics or you do not like Geography and Economics". (10 marks)

16. (a) A curve is given by the parametric equations $x = a(\theta + \sin \theta)$ and $y = a(1 + \cos \theta)$. Find the length of the curve between the points $\theta = 0$ and $\theta = \pi/2$.

- (b) Integrate the following:

$\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}} dx$.

- (c) By using binomial expansion of $(1 + 2x)^{1/2}$, evaluate $(1.02)^{1/2}$ correct to five decimal places. (10 marks)