Find

(i) The value of the constant k,

(ii) $E(X)$,

(iii) $P\left(\dfrac{1}{2} \le x \le 1\tfrac{1}{2}\right)$.

(d) If X follows binomial distribution with mean 4 and variance 2, find $P\left(|x - 4| \le 2\right)$ and write your answer in four significant figures.

7. (a) Form the differential equation by eliminating arbitrary constants, in the equation
$Ax^2 + By^2 = 1$.

(b) Solve $\left(1 + y^2\right)dx = \left(\tan^{-1} y - x\right)dy$.

(c) The rate of decrease of the temperature of a body is proportional to the difference between the temperature of the body and that of the surrounding air. If water at temperature $100^{\circ}C$ cools in 20 minutes to $78^{\circ}C$ in a room of temperature $25^{\circ}C$, find the temperature of water after 30 minutes correctly to two decimal places.

(d) Find the general equation for the equation $\dfrac{d^2 y}{dx^2} - 7\dfrac{dy}{dx} + 6y = 2\sin x$ given that $y = 1$,

$\dfrac{dy}{dx} = 0$ when $x = 0$.

8. (a) Show that the point $B(5, -5)$ lies on the parabola $y^2 = 5x$ and find the equation of the normal to the parabola at the point B in the form $y = mx + c$.

(b) If $y = mx + c$ is a tangent to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, find c in terms of a, b, m.

(c) (i) Find the rectangular equation of $r = 12(1 + \sin\theta)$.

(ii) Sketch the graph of $r = \sin 2\theta$ for $0 \le \theta \le \pi$.