THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2

ADVANCED MATHEMATICS 2

(For Both School and Private Candidates)

Time: 3 Hours **Thursday, 09th May 2019 p.m.**

Instructions

1. This paper consists sections A and B with a total of **eight (8)** questions.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones and any unauthorized materials are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

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SECTION A (60 Marks)

Answer **all** questions in this section.

1. (a) Express the complex number $\left(\frac{1+i}{1-i}\right)^8 + \left(\frac{\sqrt{3}}{1-i}\right)^4$ in the form $a+ib$.

(b) Show that $[r(\cos\theta + i\sin\theta)]^n = r^n e^{in\theta}$.

(c) If the point P represents the complex number $z = x+iy$ on the Argand diagram, describe the locus of P if $|z-i| = 3|z+i|$.

2. (a) Let P be "She is tall" and Q be "She is beautiful". Write the verbal representation of the following statements:

(i) $P \wedge Q$.

(ii) $P \wedge \sim Q$.

(iii) $\sim P \wedge \sim Q$

(iv) $\sim (P \vee \sim Q)$

(b) Using the laws of algebra of propositions simplify $[P \wedge (P \vee Q)] \vee [Q \wedge (P \vee Q)]$.

(c) (i) Find a simplified sentence having the following truth table:

p	q	r	
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

(ii) Draw a simple electric network that corresponds to the compound statement obtained in c (i).

3. (a) If $\underline{a} = 2\underline{i} + 3\underline{j} + 4\underline{k}$ and $\underline{b} = 2\underline{i} + \underline{j} + 2\underline{k}$, find the following:

(i) The projection of $\underline{a}$ onto $\underline{b}$.

(ii) The angle between vectors $\underline{a}$ and $\underline{b}$.

(iii) The unit vector of $\underline{a} \times \underline{b}$.

(b) The point K has position vector $3\underline{i} + 2\underline{j} - 5\underline{k}$ and a point L has position vector $\underline{i} + 3\underline{j} + 2\underline{k}$. Find the position vector of a point M which divides $\overrightarrow{KL}$ in the ratio of $4:3$.

(c) A displacement vector is given by $\underline{r} = a\underline{i}\cos nt + b\underline{j}\sin nt$ where a and b are arbitrary constants. Find the corresponding velocity and acceleration when $t = 0$.

(a) (i) Express $\frac{1}{r(r+1)}$ in partial fractions.

(ii) From (a) (i) deduce the formula for $\sum_{r=1}^{n}\frac{1}{r(r+1)}$.

(b) A teacher bought pens, pencils and note books for her students. She bought 3 pens, 6 pencils and 3 note books in the first week; 1 pen, 2 pencils and 2 note books in the second week; as well as 4 pens 1 pencil and 4 note books in the third week. If she spent 3,000, 1,100 and 2,600 shillings in the first, second and third week respectively, use the inverse matrix method to find the price of each item.

(c) Use synthetic division to find the quotient and the remainder when $2x^4+3x^3-2x+5$ is divided by $x+5$.

SECTION B (40 Marks)

Answer **two (2)** questions from this section.

(a) Use factor formulae to show that $\sin 5\alpha+\sin 2\alpha-\sin\alpha=\sin 2\alpha(2co3\alpha+1)$.

(b) Simplify the expression $\frac{1+\sin\phi}{5+3\tan\phi-4\cos\phi}$ using small angles approximation up to the term containing ϕ^2.

(c) Prove that $\cos\beta(\tan\beta+3)(3\tan\beta+1)=3\sec\beta=10\sin\beta$.

(d) Find the greatest and least value of the function $\frac{1}{4\sin x-3\cos x+6}$.

5. (a) (i) Show that ${}^{n}C_{r+1}+{}^{n}C_{r}={}^{n+1}C_{r+1}$.

(ii) A machine produces a total of 10,000 nails a day which on average 5% are defective. Find the probability that out of 500 nails chosen at random 10 will be defective.

(b) (i) Find the probability that in four tosses of a fair die a 2 appears at most once.

(ii) The mean weight of 400 female pupils at a certain school is 65 kg and the standard deviation is 5 kg. Assuming that the weights are normally distributed, find how many pupils weigh between 50 and 67 kg.

(c) A random variable X has the probability density function

$$f(x)=\begin{cases} px, & \text{for } 0\le x<2 \\ p(4-x), & \text{for } 2\le x\le 4 \\ 0, & \text{elsewhere} \end{cases}$$

(i) Find the value of the constant p.

(ii) Sketch the graph of $f(x)$.

(iii) Evaluate $P\left(\frac{1}{2}\le X\le\frac{5}{2}\right)$.

7. (a) Form a differential equation whose general solution is given by $x = e^{2t}(A + Bt)$ where A and B are constants.

(b) (i) Show that $y = 2 - \cos x$ is a particular integral of the differential equation $\frac{d^2y}{dx^2} + 4y = 8 - 3\cos x$ and find the general solution.

(ii) Find the particular solution of the differential equation $\frac{d^2y}{dx^2} + 4y = 8 - 3\cos x$ such that when $x = 0$, $y = 1.5$ and $\frac{dy}{dx} = 0$.

(c) A rumour is spreading through a large city at a rate which is proportional to the product of the fractions of those who heard it and of those who have not heard it, so that x is the fraction of those who heard it after time t.

(i) If initially a fraction c has heard the rumour, show that $x = \frac{c}{c + (1-c)e^{-kt}}$.

(ii) If 10% have heard the rumour at noon and another 10% by 3:00 pm, find x as a function of t. What further population would you expect to have heard it by 6:00 pm?

8. (a) Show that the equation of a tangent to parabola $y^2 = 4ax$ at point (x_1, y_1) is $yy_1 = 2a(x + x_1)$.

(b) Find the perpendicular distance of a point $(10,10)$ from the tangent to the curve $4x^2 + 9y^2 = 25$ at $(-18,1)$.

(c) Show that the equation $16x^2 + 25y^2 - 64x + 150y - 111 = 0$ is an equation of ellipse.

(d) (i) Show that $y = mx + c$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ when $c^2 = a^2m^2 - b^2$.

(ii) Determine the equation of a tangent line to hyperbola $5x^2 - 4y^2 = 1$ if the slope of the tangent line is -2.

(e) (i) Transform the equation $x^2 + y^2 + 4x = 2\sqrt{x^2 + y^2}$ into a polar equation.

(ii) Draw the graph of the polar equation obtained in (i) above in the interval $0 \le \theta \le 2\pi$.