THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1 **ADVANCED MATHEMATICS 1**

(For Both School and Private Candidates)

TIME: 3 Hours 3 May 2000 A.M.

Instructions

1. This paper consists of sections A and B.
2. Answer ALL questions in section A and any FOUR (4) questions from section B.
3. Mathematical Tables, Mathematical formulae, slide rules and electronic pocket calculators may be used.
4. All answers must be written in the answer booklet provided.
5. All work done in answering each question must be shown clearly.
6. Write your examination number on every page of your answer booklet provided.

This paper consists of 5 printed pages

SECTION A (60 marks)

Answer all questions in this section, showing all necessary steps and answers.

1. (a) If A and B are non-empty sets, use the laws of algebra of sets to simplify the following expressions:

   (i) $[(A\cup B)'\cup(A-B)]'$

   (ii) $(A-B)\cap(B-A)$

   (04 marks)

   (b) Of the 26 animals in a zoo, 5 animals eat all types of food in the zoo i.e. grass, meat and bones. 6 animals eat grass and meat only, 2 animals eat grass and bones only, and 4 animals eat meat and bones only. The number of animals eating one type of food only is divided equally between the three types of food.

   (i) Illustrate the above information by a labelled Venn diagram.

   (ii) Find the number of animals eating grass.

   (02 marks)

2. (a) Find the equations of the tangents from the origin to the circle $x^2+y^2-5x-5y+10=0$

   (02 marks)

   (b) Find the shortest distance from the point P(-1, 2) to the circle
   $x^2+y^2-6x-8y+21=0$

   (02 marks)

   (c) A and B are the points (3, 4) and (-1, 2). Find the cartesian equation of the locus of the point P(x, y) which moves so that:
   AP = 2BP.

   (02 marks)

3. The function $f:\{x:x\in R,\ x\neq -1\}\rightarrow R$ is defined by $f(x)=\dfrac{3x-2}{x+1}$

   (a) What is the range of f?

   (b) Find $f^{-1}(x)$

   (c) Determine the domain and range of $f^{-1}(x)$

   (06 marks)

4. Prove by method of induction that

   (a) $1^3+2^3+3^3+\cdots+n^3=\dfrac{1}{4}n^2(n+1)^2$

   (03 marks)

   (b) 9^n-1 is divisible by 8

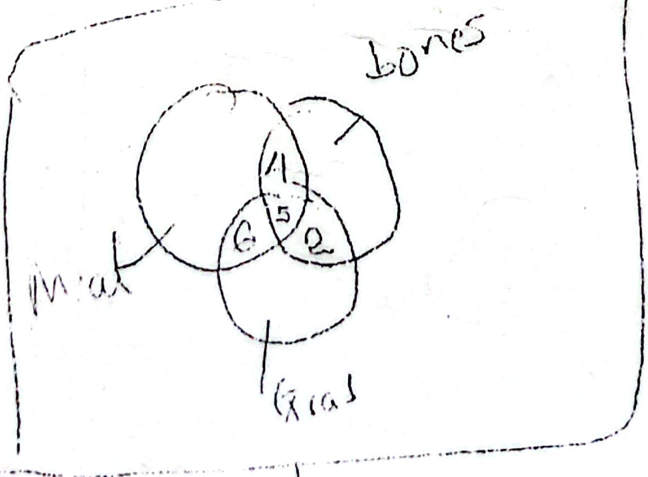
   (03 marks)

5. (a) Evaluate

   (i) $\tan\left[\sin^{-1}(3/4)\right]$

   (ii) $\cos(\sin^{-1}x)$

   (02 marks)



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(b) Show that

$$(\cot x + \operatorname{cosec}\theta)^2 = \frac{(1+\cos\theta)}{(1-\cos\theta)}$$

(c) If $8\sin^2\theta + 2\cos\theta - 5 = 0$, show that $\cos\theta = {}^3/_4$ or $-{}^1/_2$. Hence find the possible values of $\tan\theta$.

(02 marks)

6. The lengths of the sides of a rectangular sheet of metal are 8cm and 3cm. A square of side x is cut from each corner of the sheet and the remaining piece is folded to make an open box.

(a) Show that the volume V of the box is given by $V = 4x^3 - 22x^2 + 24x \text{ cm}^3$.

(b) Find the value of x for which the volume of the box is a maximum and calculate this maximum volume.

(06 marks)

7. By means of vectors and not otherwise prove that

(a) $|\underline{c}|^2 = |\underline{a}|^2 + |\underline{b}|^2 - 2|\underline{a}||\underline{b}|\cos\theta$.
where $\underline{a}$, $\underline{b}$, and $\underline{c}$ are the vectors for three sides of the triangle, ABC.

(b) the position vector of the midpoint M of $\underline{AB}$ is $\frac{1}{2}(\underline{OA} + \underline{OB})$

(06 marks)

8. (a) Evaluate (i) $\int_0^{1/2} \sinh 2x \, dx$

(ii) $\int_0^2 \frac{dx}{\sqrt{4x^2+9}}$

(04 marks)

(b) $\int \frac{4x+3}{2x+1} dx$.

(02 marks)

9. A biased die is thrown thirty times and the number of sixes seen is eight. If the die is thrown a further twelve times, find:

(a) the probability that a six will occur exactly twice

(b) the expected number of sixes

(c) the variance of the number of sixes.

(06 marks)

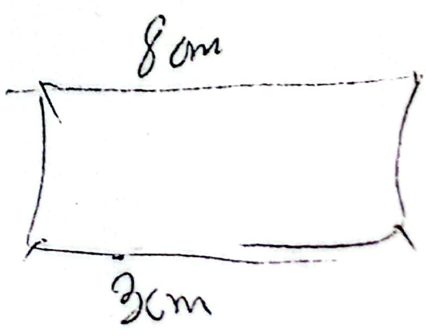
10. A random sample of 120 broad bean seeds was collected. Each seed was weighed to the nearest 0.01gm and the results are summerized below.

Weight (gm)	No. of Beans
1.10 – 1.29	7
1.30 – 1.49	24
1.50 – 1.69	33
1.70 – 1.89	32
1.90 – 2.09	14
2.10 – 2.29	8
2.30 – 2.49	1
2.50 – 2.69	1

1.195

0.19

0.095



Calculate

(a) the mean

(b) the standard deviation of weights correct to three decimal places.

(06 marks)

SECTION B (40 Marks)

Answer any FOUR questions from this section showing all necessary steps and answers.

11. A mining company owns two small copper mines, each of which produces three grades of copper ore, high grade ore, medium grade ore and low grade ore. Mine A produces 1 tonne of high grade ore, 3 tonnes of medium grade ore and 5 tonnes of low grade ore each day. Mine B produces 2 tonnes of each of the three grades of ore each day. The company needs 80 tonnes of high grade ore, 160 tonnes of medium grade ore and 200 tonnes of low grade ore. How many days should each mine be operated if it costs 20,000/= per day to work each mine? (10 marks)

12. (a) Use De Moivres theorem to prove that:

$$(\cos m\theta + i\sin m\theta)(\cos n\theta + i\sin\theta) = \cos(m+n)\theta + i\sin(m+n)\theta$$

Hence or otherwise simplify

$$\frac{\cos 3\theta + i\sin 3\theta}{\cos 5\theta - i\sin 5\theta}$$

(04 marks)

(b) If z is a complex number, find the locus in cartesian coordinates represented by the equation $|z - 3| = 2$. (02 marks)

(c) Find the roots of the quadratic equation $z^2 - 3(1 + i)z + 5i = 0$ expressing your answers in the form $c + id$ where c and d are real numbers (04 marks)

13. (a) An experiment is being performed which produces oxygen at a continuous rate. The rate of oxygen produced is measured each minute and the results tabulated.

Minutes	0	1	2	3	4	5	6	7	8	9	10
Oxygen cm^3/min	0	14	18	22	30	42	41	36	29	20	12

Use the trapezoidal rule to estimate the total amount of oxygen produced in 10 minutes.

(b) Use Simpson's rule with five ordinates to find an approximate value of $\int_0^\pi \sqrt{\sin\theta}\, d\theta$

(04 marks)

(c) The period, T of a simple pendulum is calculated from the formula $T = 2\pi\sqrt{\ell/g}$ where ℓ is the length of the pendulum and g is the constant gravitational acceleration. Find the percentage change in the period caused by lengthening the pendulum by 2%. (02 marks)

14. (a) If $(x^2 + 1)\frac{dy}{dx} = \frac{1}{2}x(4 - y^2)$, express y as a function of x if y = 1 when x = 0. (03 marks)

(b) Solve the equation

$X + 2X \geq 80$

$3Y + 2Y \geq 160$

$5Z + 2Z = 200$

$X + Y + Z =$

$X + 3Y + 5Z =$

$Z + 2Z =$

$A + B =$

$(y + x^3y^2)\,dx + x\,dy = 0.$

(03 marks)

(c) Find the general solution of

$$\frac{d^2y}{dx^2} - \frac{5dy}{dx} + 4y = e^x$$

(04 marks)

15. (a) The random variable β denotes the number of weeks of life of a certain type of bulb. If the probability density function of β is given by

$$f(\beta) = \begin{cases} \dfrac{20{,}000}{\beta^3}, & \beta > 1000 \\ 0, & \text{elsewhere} \end{cases}$$

find the expected life of this type of bulb. (05 marks)

(b) From a certain garden, the mean length of 500 mandolo leaves was 151mm and the standard deviation was 15mm. Assuming that the lengths are normally distributed find how many leaves measure:

(i) between 120 and 155mm.

(ii) more than 185mm

(05 marks)

16. (a) A particle moves so that after t sec. its displacement S is given by

$S = (3t^2 + 1)\,\underline{i} + (t^4 - 5t)\,\underline{j}$. Find the magnitude and direction of the velocity and acceleration after 3 sec.

(05 marks)

(b) A particle of mass m moves so that its position vector at time t is

$\underline{r} = \underline{a}\cos\omega t + \underline{b}\sin\omega t$. where $\underline{a}$ and $\underline{b}$ are constant vectors and ω is constant.

(i) Show that the force acting on the particle is $-m\omega^2\underline{r}$

(ii) If $\underline{a}$ is perpendicular to $\underline{b}$, find the kinetic energy of the particle. (05 marks)

$|3i + i|\ |i - 8i|$

$S = ut + \frac{1}{2}at^2$

$S = 0 \times 3 + \frac{1}{2}$

$|4i - 4j|$

$V = U + at$

$S = 0 + a3$

$S = 3 \times (3)^2 + 1)(3^4 - 5 \cdot 3)$

$S = (27 + 1)(81 - 15)$

$= 28 + 66$

$9\varphi = 0 + 3a$

$3a$