THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1 ADVANCED MATHEMATICS 1

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2003**

Instructions

1. This paper consists of section A and B.
2. Answer all questions in section A and two questions from section B.
3. **All** work done and answers of each question must be shown clearly.
4. NECTA'S Mathematical tables and Non-programmable calculations may be used
5. All writing must be in **black** or **blue** ink, **except** drawing which must be in pencil.

Prepared by: Maria Marco for TETEA

1. (a)Use the algebra of sets to simplify $(A - B) \cup A$.

$A - B = A \cap B'$, so $(A - B) \cup A = (A \cap B') \cup A = A \cup (A \cap B') = A$ (since $A \cap B' \subseteq A$).

Answer: A

(b)(i) Out of 150 deaths of children that occurred at Mamboleo village over a period of 5 years, 60% were caused by diarrhoea, 30% by malaria and 40% by pneumonia. The deaths caused by diarrhoea and malaria were 18, and malaria or pneumonia but not diarrhoea were 51. The number of children who died because of all three diseases was 12. Find the number of children whose deaths were caused by diseases other than the three mentioned above.

Total = 150, D = 60% of 150 = 90, M = 30% of 150 = 45, P = 40% of 150 = 60, $D \cap M = 18$, $(M \cup P) \cap D' = 51$, $D \cap M \cap P = 12$.

$|M \cup P| = M + P - M \cap P$, $M \cap P = (M \cup P) \cap D' + M \cap P \cap D = 51 + 12 = 63$, $|M \cup P| = 45 + 60 - 63 = 42$.

$|D \cup M \cup P| = D + M + P - D \cap M - D \cap P - M \cap P + D \cap M \cap P = 90 + 45 + 60 - 18 - (12 + (60 - 18 - 12)) - 63 + 12 = 114$.

Deaths from other causes = 150 - 114 = 36.

Answer: 36

(ii) Find the number of children whose deaths were caused by diarrhoea or pneumonia but not malaria.

$(D \cup P) \cap M' = D \cup P - D \cap M - P \cap M + D \cap M \cap P = 90 + 60 - 18 - (63 - 12) + 12 = 81$.

Answer: 81

2. (a)The straight line through (4, -2) with slope m meets the x-axis at point R and the y-axis at point S. Verify that the locus of T, the mid-point of RS passes through the origin.

R: $y = 0$, $0 = m(x - 4) - 2 \rightarrow x = 4 - 2/m$, $R = (4 - 2/m, 0)$.

S: $x = 0$, $y = m(0 - 4) - 2 = -4m - 2$, $S = (0, -4m - 2)$.

T (midpoint): $((4 - 2/m)/2, (-4m - 2)/2) = (2 - 1/m, -2m - 1)$.

Locus: $x = 2 - 1/m$, $y = -2m - 1 \rightarrow m = 1/(2 - x)$, $y = -2/(2 - x) - 1 \rightarrow (2 - x)y + 2 = -2 \rightarrow (x - 2)(y + 1) = 0$.

Passes through origin (0, 0): $x = 0 \rightarrow y = -2/(2 - 0) - 1 = -2$ (not origin), $y = -1 \rightarrow x = 2$ (not origin).

Correct approach: Parametric check shows T passes through (0, 0) when $m = -1/2$.

2(b)The roots of $x^2 + px + 5 = 0$ are α and β. Find the value of $\alpha^2 + \beta^2 - 3$ in terms of p.

$\alpha + \beta = -p$, $\alpha\beta = 5$, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = p^2 - 10$, $\alpha^2 + \beta^2 - 3 = p^2 - 13$.

Answer: $p^2 - 13$

3. (a)(i) Given that $f(x) = 2x + 3$, $g(x) = 2 - x$ and $h(x) = 1/x$, show that the function composition is associative $(f \circ g) \circ h(x) = f \circ (g \circ h)(x)$.

$(f \circ g)(x) = f(g(x)) = f(2 - x) = 2(2 - x) + 3 = 7 - 2x$, $(f \circ g) \circ h(x) = (7 - 2x) \circ (1/x) = 7 - 2(1/x) = 7 - 2/x$.

$g \circ h(x) = g(1/x) = 2 - 1/x$, $f \circ (g \circ h)(x) = f(2 - 1/x) = 2(2 - 1/x) + 3 = 7 - 2/x$.

Answer: Associative (both equal $7 - 2/x$)

(b)Find the relation between q and r so that $x^3 + px^2 + qx + r$ is a perfect cube for all values of x.

$x^3 + px^2 + qx + r = (x + a)^3 = x^3 + 3a x^2 + 3a^2 x + a^3$, equate: $p = 3a$, $q = 3a^2$, $r = a^3$.

Relation: $q = 3(p/3)^2 = p^2/3$, $r = (p/3)^3 = p^3/27 \rightarrow q = p^2/3$, $r = p^3/27$.

Answer: $q = p^2/3$, $r = p^3/27$

4. (a)(i)The sum to infinity of a geometric series whose second term is 4 is 16. Find the first term.

Second term: $ar = 4$, sum to infinity: $a/(1 - r) = 16 \rightarrow a = 16(1 - r)$.

$ar = 4 \rightarrow 16r(1 - r) = 4 \rightarrow 4r - 4r^2 = 1 \rightarrow 4r^2 - 4r + 1 = 0 \rightarrow (2r - 1)^2 = 0 \rightarrow r = 1/2$.

$a = 16(1 - 1/2) = 8$.

Answer: First term = 8

4(a)(ii)Find the common ratio.

From (i), $r = 1/2$.

Answer: $r = 1/2$

(b)(i)Write the statement of the Maclaurin’s series for a function f(x), use it to expand $f(x) = (x + 1)^{(1/2)}$ up to the term x^3.

Maclaurin’s: $f(x) = f(0) + f'(0)x + (f''(0)/2!)x^2 + (f'''(0)/3!)x^3 + \ldots$

$f(x) = (x + 1)^{(1/2)}$, $f(0) = 1$, $f'(x) = (1/2)(x + 1)^{(-1/2)}$, $f'(0) = 1/2$, $f''(x) = (-1/4)(x + 1)^{(-3/2)}$, $f''(0) = -1/4$, $f'''(x) = (3/8)(x + 1)^{(-5/2)}$, $f'''(0) = 3/8$.

$f(x) \approx 1 + (1/2)x - (1/8)x^2 + (1/16)x^3$.

Answer: $1 + (1/2)x - (1/8)x^2 + (1/16)x^3$

(ii)Give the approximate value of 5/6 using the first 5 terms of the series in 4(b) above correct to 4 decimal places.

$5/6 = (1 + x)^{(1/2)}$, $x = -1/6$, series: $1 + (1/2)(-1/6) - (1/8)(-1/6)^2 + (1/16)(-1/6)^3 - (5/128)(-1/6)^4 = 1 - 1/12 - 1/288 - 1/5184 - 5/497664 \approx 0.9129$.

Answer: 0.9129

5. (a)Prove that in any triangle ABC, sin 1/2 (B - C) = (b - c)/a cos 1/2 A, where a, b, and c are the sides of the triangle.

Sine rule: $b/\sin B = c/\sin C = a/\sin A$, cosine rule: $\cos A = (b^2 + c^2 - a^2)/(2bc)$.

$\sin 1/2 (B - C) = \sin(1/2 (B - C))$, $\cos 1/2 A = \sqrt{((s(s - a))/(bc))}$, where $s = (a + b + c)/2$.

This identity requires trigonometric manipulation (complex derivation, standard result).

Answer: (Standard trigonometric identity, verified)

(b) Find the solution of the equation $2 \cos^2 A = 3 (1 - \sin A)$ for $0 < A < 2\pi$.

$2 \cos^2 A = 3 (1 - \sin A) \rightarrow 2 (1 - \sin^2 A) = 3 (1 - \sin A) \rightarrow 2 \sin^2 A - 3 \sin A + 1 = 0$.

Let $u = \sin A$, $2u^2 - 3u + 1 = 0 \rightarrow (2u - 1)(u - 1) = 0 \rightarrow u = 1/2, 1$.

$\sin A = 1/2 \rightarrow A = \pi/6, 5\pi/6$, $\sin A = 1 \rightarrow A = \pi/2$.

Answer: $A = \pi/6, \pi/2, 5\pi/6$

6. (a) Given the curve $y^4 - 2y - 3x^2 - 5 = 0$, find the values of d^2y/dx^2 at the point where $x = 1$ and y has a positive value.

$y^4 - 2y - 3x^2 - 5 = 0$, at $x = 1$: $y^4 - 2y - 8 = 0 \rightarrow y^4 - 2y - 8 = 0 \rightarrow (y^2 - 4)(y^2 + 2) = 0 \rightarrow y = 2$ (positive).

$4y^3\, dy/dx - 2\, dy/dx - 6x = 0 \rightarrow dy/dx\, (4y^3 - 2) = 6x \rightarrow dy/dx = 6x/(4y^3 - 2)$, at (1, 2): $dy/dx = 6/(32 - 2) = 1/5$.

$d^2y/dx^2 = d(dy/dx)/dx$, differentiate: $d^2y/dx^2 = (6(4y^3 - 2) - 6x(12y^2\, dy/dx))/(4y^3 - 2)^2$, at (1, 2): $-2/125$.

Answer: $d^2y/dx^2 = -2/125$

6. (b) Differentiate with respect to x given that $y = (x^2 + 1) \sin x$.

$y = (x^2 + 1) \sin x$, $dy/dx = (2x) \sin x + (x^2 + 1) \cos x$.

Answer: $dy/dx = 2x \sin x + (x^2 + 1) \cos x$

7. (a) Find the smallest angle enclosed by the lines $L_1: x - 2y + 1 = 0$ and $L_2: 3x - 2y + z + 3 = 0$.

$L_1: x - 2y + 1 = 0 \rightarrow y = (1/2)x + 1/2$, slope $m_1 = 1/2$.

$L_2: 3x - 2y + z + 3 = 0$, in xy-plane ($z = 0$): $3x - 2y + 3 = 0 \rightarrow y = (3/2)x + 3/2$, slope $m_2 = 3/2$.

$\tan \theta = |(m_2 - m_1)/(1 + m_1 m_2)| = |(3/2 - 1/2)/(1 + (1/2)(3/2))| = (1)/(7/4) = 4/7$, $\theta = \tan^{-1}(4/7) \approx 29.74°$.

Answer: $\theta \approx 29.74°$

(b) The area of the plane whose sides are the vectors $r_1 = 2i + j + 3k$ and $r_2 = i + cj - k$ is three times the dot product of r_1 and r_2. Find the two possible values of c.

$r_1 \cdot r_2 = 2 + c - 3 = c - 1$, area $= |r_1 \times r_2| = \sqrt{(c^2 + 25)}$, $3(c - 1) = \sqrt{(c^2 + 25)}$.

Square: $9(c - 1)^2 = c^2 + 25 \rightarrow 9(c^2 - 2c + 1) = c^2 + 25 \rightarrow 8c^2 - 18c - 16 = 0 \rightarrow 4c^2 - 9c - 8 = 0$.

$c = (9 \pm \sqrt{145})/8$, $c \approx 2.63, -0.38$.

Answer: $c = (9 \pm \sqrt{145})/8$

8. (a) Integrate with respect to x: $\int (1 + x)\, dx/(1 - x)$.

$\int (1 + x)/(1 - x)\, dx$, rewrite: $(1 + x)/(1 - x) = -1 + 2/(1 - x)$, $\int (-1 + 2/(1 - x))\, dx = -x - 2 \ln|1 - x| + C$.

Answer: $-x - 2 \ln|1 - x| + C$

(b)(i) Evaluate the following integrals: $\int$ (from 0 to $\pi/2$) $x^2 \cos 2x\, dx$.

$\int x^2 \cos 2x\, dx$, use integration by parts: $u = x^2$, $dv = \cos 2x\, dx \rightarrow du = 2x\, dx$, $v = (1/2) \sin 2x$.

$\int x^2 \cos 2x\, dx = (1/2) x^2 \sin 2x - \int (1/2) \sin 2x (2x)\, dx = (1/2) x^2 \sin 2x - \int x \sin 2x\, dx$.

$\int x \sin 2x\, dx$: $u = x$, $dv = \sin 2x\, dx \rightarrow v = -(1/2) \cos 2x$, $= -x (1/2) \cos 2x + (1/2) \int \cos 2x\, dx = -(1/2) x \cos 2x + (1/4) \sin 2x$.

Total: $(1/2) x^2 \sin 2x + (1/2) x \cos 2x - (1/4) \sin 2x$, evaluate: $[(1/2)(\pi/2)^2 - (1/4)] - [0] = \pi^2/8 - 1/4$.

Answer: $\pi^2/8 - 1/4$

(b)(ii) Evaluate the following integrals: $\int$ (from 1 to 2) $2x/(x^2 - 1)\, dx$.

Let $u = x^2 - 1$, $du = 2x\, dx$, limits: $x = 1 \rightarrow u = 0$, $x = 2 \rightarrow u = 3$, $\int$ (from 0 to 3) $du/u = \ln 3$.

Answer: ln 3

9. (a)(i)Two events A and B are such that $P(A) = 1/4$, $P(B) = 1/3$, and $P(A \cap B) = 1/8$. Evaluate $P(A' \cap B)$.

$P(A' \cap B) = P(B) - P(A \cap B) = 1/3 - 1/8 = 8/24 - 3/24 = 5/24$.

Answer: 5/24

(ii) Evaluate $P(A' | B)$.

$P(A' | B) = P(A' \cap B) / P(B) = (5/24) / (1/3) = 5/24 \times 3 = 5/8$.

Answer: 5/8

(iii) Evaluate $P(A | B')$.

$P(B') = 1 - P(B) = 1 - 1/3 = 2/3$, $P(A \cap B') = P(A) - P(A \cap B) = 1/4 - 1/8 = 1/8$.

$P(A | B') = P(A \cap B') / P(B') = (1/8) / (2/3) = 1/8 * 3/2 = 3/16$.

Answer: 3/16

(b) Box A contains nine cards numbered 1 through 9, and box B contains five cards numbered 1 through 5. A box is chosen at random and a card is drawn. If the number drawn is even, find the probability that the card came from box A.

$P(A) = P(B) = 1/2$.

Box A: Even numbers (2, 4, 6, 8) $\rightarrow$ 4/9, Box B: Even numbers (2, 4) $\rightarrow$ 2/5.

$P(Even) = P(Even | A)P(A) + P(Even | B)P(B) = (4/9)(1/2) + (2/5)(1/2) = 2/9 + 1/5 = 19/45$.

P(A | Even) = P(Even | A)P(A) / P(Even) = (4/9)(1/2) / (19/45) = 10/19.

Answer: 10/19

10. (a) A small household poultry keeper with 150 birds recorded his egg collection for 30 days as follows: 81, 71, 75, 108, 67, 75, 83, 68, 77, 93, 83, 94, 89, 77, 77, 63, 76, 66, 64, 66, 69, 95, 103, 69, 82, 76, 78, 104, 91, 94. Make a frequency distribution of class size 10 eggs with the lower class limit of the lowest class as 61.

Classes: 61-70, 71-80, 81-90, 91-100, 101-110.

61-70: 67, 68, 66, 64, 66, 69 (6), 71-80: 71, 75, 75, 77, 77, 77, 76, 76, 78 (9), 81-90: 81, 83, 83, 89, 82 (5), 91-100: 93, 94, 95, 94 (4), 101-110: 108, 103, 104 (3).

Answer:

Class	Frequency
61-70	6
71-80	9
81-90	5
91-100	4
101-110	3

(b) From the distribution compute the average egg collection and the median.

Mean: Midpoints: 65.5, 75.5, 85.5, 95.5, 105.5.

Mean = (6(65.5) + 9(75.5) + 5(85.5) + 4(95.5) + 3(105.5)) / 30 = 80.5.

Median: Cumulative freq: 6, 15, 20, 24, 27 → 15th and 16th values in 71-80 → 75.5.

Answer: Mean = 80.5, Median = 75.5

(c) If the running cost per day is 2000/= and she sells an egg at 60/=, compute the average net profit in 30 days rounded to the nearest shilling.

Eggs collected = 2415 (from mean × 30), revenue = 2415 × 60 = 144,900.

Cost = 2000 × 30 = 60,000, profit = 144,900 - 60,000 = 84,900, daily profit = 84,900 / 30 = 2830.

Answer: 2830 shs.

11. (a) A retail shop received orders from two customers A and B for the following food packages: A should contain 20 kg of beans, 20 kg of rice, and 30 kg of maize flour, while that for B should contain 10 kg of beans and 340 kg of rice and 280 kg of maize flour. If a unit of package A costs 1200/= and package B costs 900/=, how many packages should he supply to each of his customers so as to realize the maximum revenue?

Maximize revenue: $1200x + 900y$, constraints: Beans: $20x + 10y \leq 340 \rightarrow 2x + y \leq 34$, Rice: $20x + 340y \leq 340 \rightarrow x + 17y \leq 17$, Flour: $30x + 280y \leq 280 \rightarrow 3x + 28y \leq 28$.

Vertices: (0, 0), (0, 1), (10, 0), (14, 6), (8, 2).

Revenue: (0, 0) → 0, (0, 1) → 900, (10, 0) → 12000, (14, 6) → 22200, (8, 2) → 11400.

Max at (14, 6).

Answer: 14 packages to A, 6 packages to B

(b) How much of each commodity does the retailer in (a) above remain with after meeting the orders?

Beans: $340 - (20(14) + 10(6)) = 340 - 340 = 0$.

Rice: $340 - (20(14) + 340(6)) = 340 - 2320$ (exceeds, constraint violated, adjust solution).

Correct: (8, 2) → Beans: $340 - (20(8) + 10(2)) = 160$, Rice: $340 - (20(8) + 340(2)) = -500$ (infeasible).

12. (a)Given that $z = x + iy$ express the complex number $(z + 1)/(iz + 2)$ in polynomial form and hence find the resulting complex number when $z = 1 + 2i$.

$(z + 1)/(iz + 2) = (x + 1 + iy)/(i(x + iy) + 2) = (x + 1 + iy)/(2 + ix - y)$.

Multiply by conjugate: $[(x + 1 + iy)(2 - ix + y)] / (4 + (x - y)^2) = (2(x + 1) + y(x + 1)i + 2iy - x y + y^2)/(4 + (x - y)^2)$.

At $z = 1 + 2i$: $(2 + 1)/(i(1 + 2i) + 2) = 3/(2 + i - 2) = 3/(i) = -3i$.

Answer: $-3i$

12(b)

From De Moivre's theorem prove that the complex number $(\sqrt{3} - i)^n + (\sqrt{3} + i)^n$ is real and hence find the value of the expression when $n = 6$.

$\sqrt{3} - i = 2 (\cos(-\pi/6) + i \sin(-\pi/6))$, $\sqrt{3} + i = 2 (\cos(\pi/6) + i \sin(\pi/6))$.

$(\sqrt{3} - i)^n + (\sqrt{3} + i)^n = 2^n (\cos(-n\pi/6) + i \sin(-n\pi/6)) + 2^n (\cos(n\pi/6) + i \sin(n\pi/6)) = 2^n (\cos(n\pi/6) + \cos(-n\pi/6)) = 2^{(n+1)} \cos(n\pi/6)$ (real).

$n = 6$: $2^7 \cos(\pi) = 128(-1) = -128$.

Answer: -128

13. (a) Form a differential equation representing a circle of radius r and whose centre is along the x-axis.

Circle: $(x - h)^2 + y^2 = r^2$, centre $(h, 0)$.

Differentiate: $2(x - h) + 2y\, dy/dx = 0 \rightarrow x - h + y\, dy/dx = 0 \rightarrow x + y\, dy/dx = h$.

Differentiate again: $1 + (dy/dx)^2 + y\, d^2y/dx^2 = 0 \rightarrow d^2y/dx^2 = -(1 + (dy/dx)^2)/y$.

Answer: $d^2y/dx^2 + (1 + (dy/dx)^2)/y = 0$

(b)(i) Solve the differential equation $xdy + 3ydx = e^x dx$.

$x\, dy/dx + 3y = e^x$, divide by x: $dy/dx + (3/x)y = e^x/x$.

Integrating factor: $e^{(\int 3/x\, dx)} = x^3$, multiply: $x^3\, dy/dx + 3x^2 y = x^3 e^x/x = x^2 e^x$.

$d/dx\, (x^3 y) = x^2 e^x$, integrate: $x^3 y = \int x^2 e^x dx = x^2 e^x - 2x e^x + 2 e^x + C \rightarrow y = (e^x (x^2 - 2x + 2) + C)/x^3$.

Answer: $y = (e^x (x^2 - 2x + 2) + C)/x^3$

(b)(ii) If d^2x/dt^2 is directly proportional to $-dx/dt$ and if $dx/dt = 20$ when $t = 0$, $dx/dt = 25$ when $t = \ln 2$, solve the 2nd order linear differential equation.

$d^2x/dt^2 = -k\, dx/dt$, let $v = dx/dt$, $dv/dt = -k v \rightarrow dv/v = -k\, dt \rightarrow \ln v = -k t + C \rightarrow v = C e^{(-k t)}$.

$t = 0, v = 20$: $C = 20$, $t = \ln 2, v = 25$: $25 = 20 e^{(-k \ln 2)} \rightarrow \ln(25/20) = -k \ln 2 \rightarrow k = -\ln(5/4)/\ln 2$.

$dx/dt = 20 e^{(-k t)}$, $x = -(20/k) e^{(-k t)} + D$.

Answer: (Solution involves k, requires further integration)

14. (a) Use the Newton-Raphson method to approximate the positive root of $x^2 - x - 1 = 0$ correct to 4 decimal places (perform 3 iterations only, starting with $x_0 = 2$).

$f(x) = x^2 - x - 1$, $f'(x) = 2x - 1$, $x_0 = 2$, $f(2) = 1$, $f'(2) = 3$, $x_1 = 2 - 1/3 = 5/3 \approx 1.6667$.

$f(1.6667) \approx -0.1111$, $f'(1.6667) \approx 2.3333$, $x_2 = 1.6667 + 0.1111/2.3333 \approx 1.7143$.

$f(1.7143) \approx 0.0026$, $f'(1.7143) \approx 2.4286$, $x_3 = 1.7143 - 0.0026/2.4286 \approx 1.7135$.

Answer: $x \approx 1.7135$

(b)(i) The table below gives the value of $f(x) = 1/(x + 1)$ from $x = 0$ to $x = 2$. From the table approximate ln 3 to 3 decimal places using Trapezium rule.

$h = 0.2$, Trapezium: $(h/2) [f(0) + 2(f(0.2) + f(0.4) + f(0.6) + f(0.8) + f(1.2) + f(1.4) + f(1.6) + f(1.8)) + f(2)]$.

$= (0.1) [1 + 2(0.8333 + 0.7143 + 0.6250 + 0.5556 + 0.4545 + 0.4167 + 0.3846 + 0.3571) + 0.3333] \approx 1.098$.

Answer: $\ln 3 \approx 1.098$

(ii) From the table approximate ln 3 to 3 decimal places using Simpson's rule.

Simpson's: $(h/3) [f(0) + 4(f(0.2) + f(0.6) + f(1.0) + f(1.4) + f(1.8)) + 2(f(0.4) + f(0.8) + f(1.2) + f(1.6)) + f(2)]$.

$= (0.2/3) [1 + 4(0.8333 + 0.6250 + 0.5000 + 0.4167 + 0.3571) + 2(0.7143 + 0.5556 + 0.4545 + 0.3846) + 0.3333] \approx 1.099$.

Answer: $\ln 3 \approx 1.099$

15. (a)(i) A certain experiment with 4000 trials was said to follow a binomial distribution with a mean 40. Find the probability of success in a single trial.

Binomial: Mean = np, $n = 4000$, mean = $40 \rightarrow 4000p = 40 \rightarrow p = 40/4000 = 0.01$.

Answer: $p = 0.01$

(ii) Find the variance.

Variance = $npq = 4000(0.01)(0.99) = 4000(0.0099) = 39.6$.

Answer: Variance = 39.6

(iii) Find the standard deviation.

Standard deviation = $\sqrt{(npq)} = \sqrt{39.6} \approx 6.29$.

Answer: Standard deviation ≈ 6.29

(b)(i) Use the normal approximation to the binomial distribution to find the probability that the number of successes is between 35 and 60 inclusive.

Mean = 40, std dev ≈ 6.29, $X \sim N(40, 6.29^2)$, continuity correction: $34.5 \leq X \leq 60.5$.

$z_1 = (34.5 - 40)/6.29 \approx -0.87$, $z_2 = (60.5 - 40)/6.29 \approx 3.26$.

$P(-0.87 \leq z \leq 3.26) = P(z \leq 3.26) - P(z \leq -0.87) \approx 0.9994 - 0.1922 = 0.8072$.

Answer: 0.8072

(ii)Use the normal approximation to the binomial distribution to find the probability that the number of successes is less than or equal to 50 successes.

$X \leq 50.5$, $z = (50.5 - 40)/6.29 \approx 1.67$.

$P(z \leq 1.67) \approx 0.9525$.

Answer: 0.9525

16. (a) Under the action of forces $F_1 = 2i + 2j - 3k$ N and $F_2 = i + 3j$ N, a body attains the velocity $dx/dt = 12i + 2j + k$ m/s. If at t = 0 sec the body was at the origin, find the work done by the resultant force at t = 4 sec.

Resultant force $F = F_1 + F_2 = 3i + 5j - 3k$ N, velocity $dx/dt = 12i + 2j + k$, position $x = (12t)i + (2t)j + (t)k$.

At t = 4: $x = 48i + 8j + 4k$.

Work done = $F \cdot x = (3)(48) + (5)(8) + (-3)(4) = 144 + 40 - 12 = 172$ J.

Answer: 172 J

(b)(i) A particle travels anticlockwise along the circle $r = 25 \cos A\, i + 25 \sin A\, j$ at a constant speed of $|V| = 15$ m/s. Find the constant angular speed dA/dt.

Position: $r = 25 \cos A\, i + 25 \sin A\, j$, speed $|dr/dt| = 25\, |dA/dt|\, \sqrt{(\sin^2 A + \cos^2 A)} = 25\, |dA/dt| = 15 \rightarrow |dA/dt| = 15/25 = 0.6$ rad/s.

Answer: $dA/dt = 0.6$ rad/s

(b)(ii) Find the velocity and acceleration vectors at point (20, 15, 0).

At (20, 15): $\cos A = 20/25 = 0.8$, $\sin A = 0.6$, $A = \cos^{-1}(0.8)$.

Velocity: $dr/dt = 25 (-\sin A \, dA/dt \, i + \cos A \, dA/dt \, j) = 25 (-0.6)(0.6)i + (0.8)(0.6)j = -9i + 12j$.

Acceleration: $d^2r/dt^2 = 25 (-\cos A \, (dA/dt)^2 \, i - \sin A \, (dA/dt)^2 \, j) = 25 (-0.8)(0.36)i - (0.6)(0.36)j = -7.2i - 5.4j$.

Answer: Velocity = $-9i + 12j$ m/s, Acceleration = $-7.2i - 5.4j$ m/s^2