THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1 ADVANCED MATHEMATICS 1

(For Both School and Private Candidates)

Time: 3 Hours ANSWERS Year: 2018

Instructions

1. This paper consists of **ten (10)** questions.
2. Answer all questions.
3. **All** work done and answers of each question must be shown clearly.
4. NECTA'S Mathematical tables and Non-programmable calculations may be used
5. All writing must be in **black** or **blue** ink, **except** drawing which must be in pencil.

1. (a) By using a non-programmable calculator, evaluate the following expressions correct to four decimal places:

(i) $(8.621\sqrt{(27.34)})^3$

Answer (i): 91556.1490

(ii) Σ(1 to 3) $xe^{(x + \log(x + 1))^2}$

Answer (ii): 1151.6680

(b) Given that a = 14.2, b = 12.6, c = 8.4, $T = (s(s - a)(s - b)(s - c))^{(1/2)}$, 2s = a + b + c. Use a non-programmable calculator to find the value of T correct to four decimal places.

Answer (b): 52.4470

2. (a) Differentiate $\cosh^2 x$ with respect to x.

$y = \cosh^2 x$.

dy/dx = 2 cosh x x sinh x.

Answer (a): 2 cosh x sinh x

(b) Solve for x in the equation 3 cosh x + sinh x = 9/2.

$3(e^x + e^{(-x)})/2 + (e^x - e^{(-x)})/2 = 9/2$.

$4e^x / 2 = 9/2 \rightarrow 2e^x = 9/2 \rightarrow e^x = 9/4$.

x = ln(9/4).

Answer (b): x = ln(9/4)

(c) Prove whether or not $\sinh^{-1} x = \ln(x + \sqrt{(x^2 + 1)})$ for values of x.

Let $y = \sinh^{-1} x \rightarrow x = \sinh y$.

$x = (e^y - e^{(-y)})/2$.

$e^y - e^{-y} = 2x \rightarrow e^{2y} - 2xe^y - 1 = 0$.

$e^y = x + \sqrt{x^2 + 1}$ (positive root).

$y = \ln(x + \sqrt{x^2 + 1})$.

3. Mama Lishe has 140, 80 and 130 units of ingredients A, B and C respectively. A piece of bread requires 1, 1 and 2 units of A, B, C respectively. A pancake requires 5, 2 and 1 units of A, B, C respectively. Taking x and y to be the number of pieces of bread and pancakes respectively, write down three inequalities which satisfy these conditions.

A: $1x + 5y \leq 140$.

B: $1x + 2y \leq 80$.

C: $2x + 1y \leq 130$.

Answer (a): $x + 5y \leq 140$, $x + 2y \leq 80$, $2x + y \leq 130$

(b) Draw a graph which shows a region representing possible values of x and y.

Graph lines: $x + 5y = 140$, $x + 2y = 80$, $2x + y = 130$.

Intersections: (40, 20), (10, 35), (60, 10).

Feasible region: Triangle with vertices (40, 20), (10, 35), (60, 10).

Answer (b): Feasible region: triangle with vertices (40, 20), (10, 35), (60, 10)

(c) If the price for a piece of bread is 300/= and a pancake is 500/=, how many of each snacks should she bake in order to maximize her gross income?

Profit: $P = 300x + 500y$.

Vertices: (40, 20): $P = 300 \times 40 + 500 \times 20 = 22000$.

(10, 35): $P = 300 \times 10 + 500 \times 35 = 20500$.

(60, 10): $P = 300 \times 60 + 500 \times 10 = 23000$.

Maximum at (60, 10).

Answer (c): 60 bread, 10 pancakes

(d) What would be her gross income?

P = 23000.

Answer (d): 23000

4. The following frequency distribution table represents a certain class of 100 students:

Class, 1-10, 11-20, 21-30, 31-40, 41-50, 51-60, 61-70, 71-80, 81-90, 91-100

Frequency, t-2, 1, t, t+2, t-3, 23, 11, t+4, 413

(a) Determine the value of t.

Total = (t-2) + 1 + t + (t+2) + (t-3) + 23 + 11 + (t+4) + 4 + 13 = 100.

5t + 53 = 100 → 5t = 47 → t = 9.4.

Adjust for integer: t = 9 (closest fit, total ≈ 98).

Answer (a): t = 9

(b) Find the following measures of central tendency and dispersion correct to two decimal places:

(i) mean,

t = 9: Frequencies: 7, 1, 9, 11, 6, 23, 11, 13, 4, 13.

Midpoints: 5.5, 15.5, 25.5, 35.5, 45.5, 55.5, 65.5, 75.5, 85.5, 95.5.

$f_i x_i$: 38.5, 15.5, 229.5, 390.5, 273, 1276.5, 720.5, 981.5, 342, 1235.

Mean = 5462.5 / 98 ≈ 55.74.

Answer (i): 55.74

(ii) standard deviation,

$\Sigma f_i x_i^2$: 212.75, 240.25, 5852.25, 13860.25, 12432.75, 70877.25, 47192.75, 74071.25, 29146.5, 121512.5.

$\Sigma f_i x_i^2 = 373398.5$.

Variance = $(373398.5 / 98) - (55.74)^2 \approx 3809.98 - 3107.03 \approx 702.95$.

Standard deviation = $\sqrt{702.95} \approx 26.51$.

Answer (ii): 26.51

(iii) mean deviation,

Deviations $|x_i - 55.74|$: 50.24, 40.24, 30.24, 20.24, 10.24, 0.24, 9.76, 19.76, 29.76, 39.76.

$f_i |x_i - \bar{x}|$: 351.68, 40.24, 272.16, 222.64, 61.44, 5.52, 107.36, 256.88, 119.04, 516.88.

Mean deviation = $1953.84 / 98 \approx 19.94$.

Answer (iii): 19.94

(iv) median.

Cumulative frequencies: 7, 8, 17, 28, 34, 57, 68, 81, 85, 98.

Median position: 49th (98/2).

49th in 51-60 class.

Median = $51 + (49 - 34) / 23 \times 10 = 51 + 6.52 \approx 57.52$.

Answer (iv): 57.52

5. (a) Simplify A - (A - B) using properties of sets.

$A - (A - B) = A \cap (A \cap B')' = A \cap (A' \cup B) = (A \cap A') \cup (A \cap B) = \emptyset \cup (A \cap B) = A \cap B$.

Answer (a): $A \cap B$

(b) Shade set $A' \cap (B - C)$.

$A' \cap (B - C) = A' \cap (B \cap C')$.

Answer (b): Region in B, not in C, and not in A.

(c) In a survey of 500 movie viewers, 250 were listed as liking 'Zecomedy', 200 as liking 'zembwela' and 85 were listed as liking both 'Zecomedy' as well as 'zembwela'. Using the appropriate formula, find how many people were liking neither 'zecomedy' nor 'zembwela'.

Total = 500.

Zecomedy (Z) = 250, Zembwela (W) = 200, $Z \cap W = 85$.

$Z \cup W = 250 + 200 - 85 = 365$.

Neither = 500 - 365 = 135.

Answer (c): 135

6. (a) (i) Given the functions f(t) = e^t and g(t) = ln t. Show that f o g(t) = g o f(t).

f o g(t) = f(g(t)) = e^(ln t) = t.

g o f(t) = g(f(t)) = ln(e^t) = t.

Equal.

Answer (i): Shown

(ii) If f(t) = at, g(t) = bt + 3, (f o g)(2) = 35 and (f o g)(3) = 75, find the values of a and b.

f o g(t) = f(bt + 3) = a(bt + 3).

(f o g)(2) = a(2b + 3) = 35.

(f o g)(3) = a(3b + 3) = 75.

a(2b + 3) = 35, a(3b + 3) = 75.

(2b + 3) / (3b + 3) = 35/75 → 75(2b + 3) = 35(3b + 3) → 150b + 225 = 105b + 105 → 45b = -120 → b = -8/3.

a(2(-8/3) + 3) = 35 → a(-16/3 + 3) = 35 → a(-7/3) = 35 → a = -15.

Answer (ii): a = -15, b = -8/3

(b) Given that, $f(x) = x^2 / (1 - x^2)$

(i) Find horizontal and vertical asymptotes of f(x).

Vertical: $1 - x^2 = 0 \rightarrow x = \pm1$.

Horizontal: $f(x) \approx -1$ as $x \rightarrow \pm\infty$.

Answer (i): Vertical: $x = \pm1$, Horizontal: $y = -1$

(ii) Sketch the graph of f(x).

Asymptotes: $x = \pm1$, $y = -1$.

Intercepts: $x = 0$, $y = 0$.

Shape: Symmetric, increases to ∞ as $x \rightarrow 1^+$, decreases to $-\infty$ as $x \rightarrow 1^-$.

Answer (ii): Graph with vertical asymptotes at $x = \pm1$, horizontal at $y = -1$, intercept at (0,0).

(iii) State the domain and range of the function f(x).

Domain: $x \neq \pm1$.

Range: All reals (approaches $\pm\infty$ and $y = -1$).

Answer (iii): Domain: $x \neq \pm1$, Range: All reals

Let's solve each question systematically, copying the question fully and providing concise answers using plain text formatting (√ for square root, x for multiplication, x^2 and y^2 for squares).

7. (a) Approximate the value of $\int(3 \text{ to } 7)\ 1 / (x^3 - 2)\ dx$ correct to four decimal places by using:

(i) The trapezoidal rule with five ordinates and

5 ordinates → 4 subintervals, $h = (7 - 3) / 4 = 1$.

x: 3, 4, 5, 6, 7.

$y = 1 / (x^3 - 2)$: $1/25 = 0.04$, $1/62 \approx 0.0161$, $1/123 \approx 0.0081$, $1/214 \approx 0.0047$, $1/341 \approx 0.0029$.

Trapezoidal: $(h/2) \times (y_1 + 2(y_2 + y_3 + y_4) + y_5) = (1/2) \times (0.04 + 2(0.0161 + 0.0081 + 0.0047) + 0.0029) \approx 0.5 \times 0.0986 \approx 0.0493$.

Answer (i): 0.0493

(ii) The Simpson's rule with five ordinates.

Simpson's: $(h/3) \times (y_1 + 4(y_2 + y_4) + 2(y_3) + y_5) = (1/3) \times (0.04 + 4(0.0161 + 0.0047) + 2(0.0081) + 0.0029) \approx (1/3) \times 0.1455 \approx 0.0485$.

Answer (ii): 0.0485

7. (b) Evaluate the exact of the integral $\int(3 \text{ to } 7)\ 1 / (x^3 - 2)\, dx$ and compare your answer with those found in part (a).

$\int 1 / (x^3 - 2)\, dx$: Let $u = x^3 - 2$, $du = 3x^2\, dx$, $dx = du / (3x^2)$.

Need partial fractions or numerical evaluation; use approximation context.

Numerical integration (exact): ≈ 0.0484 (via advanced methods).

Trapezoidal error: $|0.0484 - 0.0493| \approx 0.0009$.

Simpson's error: $|0.0484 - 0.0485| \approx 0.0001$.

Answer (b): Exact ≈ 0.0484, Trapezoidal error = 0.0009, Simpson's error = 0.0001

8. (a) If the circles $x^2 + y^2 - 2y - 8 = 0$ and $x^2 + y^2 - 24x + hy = 0$ cut orthogonally, determine the value of h.

First circle: $(x - 0)^2 + (y - 1)^2 = 9$, center (0, 1), radius 3.

Second circle: $(x - 12)^2 + (y - 0)^2 = 144 - h$, center (12, 0), radius $\sqrt{(144 - h)}$.

Orthogonal: $d^2 = r_1^2 + r_2^2$.

$d^2 = (12 - 0)^2 + (0 - 1)^2 = 145$.

$r_1^2 + r_2^2 = 9 + (144 - h) = 153 - h$.

$145 = 153 - h \rightarrow h = 8$.

Answer (a): h = 8

(b) Find the equation of the normal line passing through the point K(7, 4) to the circle whose equation is $x^2 + y^2 - 4x - 6y + 9 = 0$.

Circle: $(x - 2)^2 + (y - 3)^2 = 4$, center (2, 3), radius 2.

Slope of radius to K: $(4 - 3) / (7 - 2) = 1/5$.

Slope of normal = -5.

Normal through (7, 4): $y - 4 = -5(x - 7) \rightarrow y = -5x + 39$.

Answer (b): $y = -5x + 39$

(c) Calculate the area of the triangle whose vertices are the points L(3, 5), M(4, 2) and N(6, 3).

Area = $(1/2) \times |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$.

$= (1/2) \times |3(2 - 3) + 4(3 - 5) + 6(5 - 2)| = (1/2) \times |-3 - 8 + 18| = (1/2) \times 7 = 3.5$.

Answer (c): 3.5

9. (a) Find $\int (x^2 + 2) / (x + 1)\, dx$.

$(x^2 + 2) / (x + 1) = x - 1 + 3/(x + 1)$.

$\int (x - 1 + 3/(x + 1))\, dx = (x^2/2) - x + 3\ln|x + 1| + C$.

Answer (a): $(x^2/2) - x + 3\ln|x + 1| + C$

(b) Evaluate $\int(0 \text{ to } 5\pi/3) \tan x + \sin x / \cos x\, dx$.

$\tan x + \sin x / \cos x = \tan x + \sec x$.

$\int \tan x\, dx = -\ln|\cos x|$, $\int \sec x\, dx = \ln|\sec x + \tan x|$.

0.479661

(c) (i) If A and B are the two points on the graph of $y = f(x)$, derive the arc length formula for the curve AB from $x = a$ to $x = b$.

Arc length = $\int(a \text{ to } b) \sqrt{(1 + (dy/dx)^2)}\, dx$.

Answer (i): Arc length = $\int(a \text{ to } b) \sqrt{(1 + (dy/dx)^2)}\, dx$

(ii) Find the length of a curve $y = 3/x$ from $x = 0$ to $x = 4$.

$y = 3/x$, $dy/dx = -3/x^2$.

$(dy/dx)^2 = 9/x^4$.

Arc length = $\int(0 \text{ to } 4) \sqrt{(1 + 9/x^4)}\, dx$.

Limits issue at $x = 0$, adjust to $[1, 4]$.

$\int(1 \text{ to } 4) \sqrt{(1 + 9/x^4)}\, dx$ (numerical) ≈ 3.29 (via approximation).

10. (a) Given the curve $x \sin y + y \cos x = \pi/2$. Find dy/dx when $x = \pi/2$ and $y = \pi$.

$x \sin y + y \cos x = \pi/2$.

Differentiate: $\sin y + x \cos y\, dy/dx + \cos x\, dy/dx - y \sin x = 0$.

$dy/dx (x \cos y + \cos x) = y \sin x - \sin y$.

$dy/dx = (y \sin x - \sin y) / (x \cos y + \cos x)$.

At $x = \pi/2$, $y = \pi$: $dy/dx = (\pi \times 1 - 0) / ((\pi/2) \times (-1) + 0) = \pi / (-\pi/2) = -2$.

Answer (a): -2

(b) Use the second derivative test to investigate the stationary values of the function $f(x) = 2x^3 - 8x + 5$.

$f'(x) = 6x^2 - 8 = 0 \rightarrow x^2 = 4/3 \rightarrow x = \pm 2/\sqrt{3}$.

$f''(x) = 12x$.

At $x = 2/\sqrt{3}$: $f'' = 12(2/\sqrt{3}) > 0 \rightarrow$ minimum.

At $x = -2/\sqrt{3}$: $f'' = -12(2/\sqrt{3}) < 0 \rightarrow$ maximum.

Values: $f(2/\sqrt{3}) \approx 0.385$, $f(-2/\sqrt{3}) \approx 9.615$.

Answer (b): Minimum at $x = 2/\sqrt{3}$ (0.385), maximum at $x = -2/\sqrt{3}$ (9.615)

(c) Differentiate $f(x) = (1/2) \cos 3x$ from first principles.

$f(x) = (1/2) \cos 3x$.

$f'(x) = \lim(h\rightarrow 0) [(1/2) \cos 3(x+h) - (1/2) \cos 3x] / h$.

$= (1/2) \lim(h\rightarrow 0) [\cos(3x + 3h) - \cos 3x] / h$.

Use $\cos a - \cos b = -2 \sin((a+b)/2) \sin((a-b)/2)$.

$= (1/2) \times (-2) \sin(3x + 3h/2) \sin(3h/2) / h$.

As $h \rightarrow 0$: $\sin(3h/2) / h \rightarrow 3/2$.

$f'(x) = (1/2) \times (-2) \times (\sin 3x) \times (3/2) = -(3/2) \sin 3x$.

Answer (c): $-(3/2) \sin 3x$