THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1 ADVANCED MATHEMATICS 1

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2022**

Instructions

1. This paper consists of **ten (10)** questions.
2. Answer all questions.
3. **All** work done and answers of each question must be shown clearly.
4. NECTA'S Mathematical tables and Non-programmable calculations may be used
5. All writing must be in **black** or **blue** ink, **except** drawing which must be in pencil.

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1. (a) By using a non-programmable calculator, find the value of the expression $\sqrt{(\cos^{-1}((e^{(15 + \tan^{-1}(\ln 0.25))} / (\sqrt{m})))} / 3.14$ correct to 3 decimal places.

Answer: 0.345 (with assumption $m = e^{(30)}$)

(b) If $h = 3$, $p = 500$, $g = 10$ and $m = 0.25$, use a non-programmable calculator to find the value of $T = (2hp) / \sqrt{(g(p - mg))}$ correct to nine significant figures.

Answer: 42.5288250

(c) By using the statistical functions of a non-programmable calculator and the following frequency distribution table, find the mean, variance and standard deviation correctly to three decimal places.

Values	250	230	210	190	170	150	130	110	90	70	50	30
Frequency	4	11	5	6	21	40	26	4	8	35	28	12

Answer: Mean = 120.200, Variance = 3455.960, Standard deviation = 58.787

2. (a) If $2 \cosh 2x + 10 \sinh 2x = 5$, obtain the value of x in logarithmic form.

$2 \cosh 2x + 10 \sinh 2x = 5$.

$\cosh 2x = (e^{(2x)} + e^{(-2x)})/2$, $\sinh 2x = (e^{(2x)} - e^{(-2x)})/2$.

Let $u = e^{(2x)}$, equation becomes: $2(u + 1/u)/2 + 10(u - 1/u)/2 = 5 \rightarrow u + 1/u + 5(u - 1/u) = 5 \rightarrow 6u - 4/u = 5$.

Multiply by u: $6u^2 - 5u - 4 = 0$.

Solve: $u = (5 \pm \sqrt{(25 + 96)}) / 12 = (5 \pm 11) / 12 \rightarrow u = 4/3$ or $u = -1/2$ (discard negative).

$e^{(2x)} = 4/3$, $2x = \ln(4/3)$, $x = (1/2) \ln(4/3)$.

Answer: $x = (1/2) \ln(4/3)$

(b) If $\theta = \ln(\tan \varphi)$, show that $\tanh \theta = -\cos 2\varphi$.

$\theta = \ln(\tan \varphi)$, so $e^{\theta} = \tan \varphi$.

$\tanh \theta = (e^{\theta} - e^{(-\theta)}) / (e^{\theta} + e^{(-\theta)}) = (\tan \varphi - 1/\tan \varphi) / (\tan \varphi + 1/\tan \varphi) = (\tan^2 \varphi - 1) / (\tan^2 \varphi + 1)$.

Use $\tan^2 \varphi = \sin^2 \varphi / \cos^2 \varphi$: $(\sin^2 \varphi - \cos^2 \varphi) / (\sin^2 \varphi + \cos^2 \varphi) = -\cos 2\varphi$ (since $\sin^2 \varphi - \cos^2 \varphi = -\cos 2\varphi$, $\sin^2 \varphi + \cos^2 \varphi = 1$).

Answer: $\tanh \theta = -\cos 2\varphi$, proved.

(c) By using the integration by parts technique, evaluate the integral $\int_0^1 x \sin 2x \, dx$ correct to 7 decimal places.

$u = x$, $dv = \sin 2x \, dx \rightarrow du = dx$, $v = \int \sin 2x \, dx = (-1/2) \cos 2x$.

$\int x \sin 2x \, dx = x (-1/2) \cos 2x - \int (-1/2) \cos 2x \, dx = (-1/2) x \cos 2x + (1/4) \sin 2x$.

Evaluate: $[(-1/2) x \cos 2x + (1/4) \sin 2x]_0^1 = [(-1/2)(1) \cos 2 + (1/4) \sin 2] - [0] = (-1/2) \cos 2 + (1/4) \sin 2$.

$\cos 2 \approx -0.4161$, $\sin 2 \approx 0.9093$.

$(-1/2)(-0.4161) + (1/4)(0.9093) = 0.20805 + 0.227325 = 0.435375 \approx 0.4353750$.

Answer: 0.4353750

3. (a) Mr. Safari wants 10, 12 and 12 units of chemicals A, B and C respectively for his garden. A liquid product contains 5 units of A, 2 units of B and 1 unit of C per jar and each jar is sold at 3,000/-. On the other hand, a dry product contains 1 unit of A, 2 units of B and 4 units of C per carton and each carton is sold at 2,000/-. If x and y are the number of jars of liquid products and cartons of dry products respectively, formulate a linear programming problem to minimize the cost.

Constraints:

$5x + y \geq 10$ (A), $2x + 2y \geq 12$ (B), $x + 4y \geq 12$ (C), $x \geq 0$, $y \geq 0$.

Cost: $C = 3000x + 2000y$.

Answer: Minimize $C = 3000x + 2000y$ subject to $5x + y \geq 10$, $2x + 2y \geq 12$, $x + 4y \geq 12$, $x \geq 0$, $y \geq 0$.

(b) A cement dealer has two depots; D_1 and D_2, holding 180 and 250 tons of cement respectively. The customers C_1 and C_2 have ordered 200 and 150 tons of cement respectively. The transport cost per ton from each depot to each customer are as shown in the following table:

From | C_1 | C_2

Depot D_1 | 1,000/= | 1,500/=

Depot D_2 | 2,000/= | 1,800/=

(i) How many tons of cement should be delivered to each customer in order to minimize the transport cost?

(ii) After meeting the orders, how many tons of cement will remain at D_2?

Let x = tons from D_1 to C_1, y = tons from D_1 to C_2.

D_1: $x + y \leq 180$.

D_2: $(200 - x) + (150 - y) \leq 250 \rightarrow 350 - x - y \leq 250 \rightarrow x + y \geq 100$.

C_1: $x + (200 - x) = 200$ (satisfied).

C_2: $y + (150 - y) = 150$ (satisfied).

Cost: $1000x + 1500y + 2000(200 - x) + 1800(150 - y) = -1000x - 300y + 670000$.

Minimize: $-1000x - 300y$ subject to $x + y \leq 180$, $x + y \geq 100$.

Vertices: (100, 0), (180, 0), (50, 50), (0, 100).

Cost at (100, 0): $1000(100) + 1500(0) + 2000(100) + 1800(150) = 570000$.

Minimum at (180, 0): $1000(180) + 1500(0) + 2000(20) + 1800(150) = 500000$.

(i) D_1 to C_1: 180 tons, D_1 to C_2: 0 tons, D_2 to C_1: 20 tons, D_2 to C_2: 150 tons.

(ii) D_2 remaining: $250 - (20 + 150) = 80$ tons.

Answer: (i) D_1 to C_1: 180 tons, D_1 to C_2: 0 tons, D_2 to C_1: 20 tons, D_2 to C_2: 150 tons; (ii) 80 tons

4. The masses of a sample of new potatoes were measured to the nearest gram and are summarized in the following table:

Mass (g)	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	2	14	21	73	42	13	9	4	2

(a) Determine the following measures of dispersion correct to three decimal places: first and third quartiles.

$\Sigma f = 180$.

Cumulative frequency: 2, 16, 37, 110, 152, 165, 174, 178, 180.

Q_1: $(180+1)/4 = 45.25 \rightarrow$ 40-49 class.

$Q_1 = 40 + 10(45.25-37)/73 = 40 + 1.130 = 41.130$.

Q_3: 3(180+1)/4 = 135.75 → 50-59 class.

$Q_3 = 50 + 10(135.75-110)/42 = 50 + 6.131 = 56.131$.

Answer: $Q_1 = 41.130$, $Q_3 = 56.131$

(b) semi-interquartile range.

Semi-interquartile range = $(Q_3 - Q_1) / 2 = (56.131 - 41.130) / 2 = 7.501$.

Answer: 7.501

(c) seventh decile.

D_7: $0.7 \times 180 = 126$ → 50-59 class.

$D_7 = 50 + 10(126-110)/42 = 50 + 3.810 = 53.810$.

Answer: 53.810

(d) 80th percentile.

P_{80}: $0.8 \times 180 = 144$ → 50-59 class.

$P_{80} = 50 + 10(144-110)/42 = 50 + 8.095 = 58.095$.

Answer: 58.095

(e) variance and standard deviation.

Midpoints: 14.5, 24.5, 34.5, 44.5, 54.5, 64.5, 74.5, 84.5, 94.5.

Mean: $\Sigma(fx) = (14.5\times2) + (24.5\times14) + (34.5\times21) + (44.5\times73) + (54.5\times42) + (64.5\times13) + (74.5\times9) + (84.5\times4) + (94.5\times2) = 29 + 343 + 724.5 + 3248.5 + 2289 + 838.5 + 670.5 + 338 + 189 = 8670$.

Mean = 8670 / 180 = 48.167.

Variance: $\Sigma(fx^2) = (14.5^2\times2) + (24.5^2\times14) + (34.5^2\times21) + (44.5^2\times73) + (54.5^2\times42) + (64.5^2\times13) + (74.5^2\times9) + (84.5^2\times4) + (94.5^2\times2) = 429250 / 180 - (48.167)^2 = 2384.722 - 2319.806 = 64.916$.

Standard deviation = $\sqrt{64.916} \approx 8.057$.

Answer: Variance = 64.916, Standard deviation = 8.057

5. (a) If sets A and B are defined by $A = \{x \in \mathbb{R} : -1 \leq x \leq 2\}$ and $B = \{x \in \mathbb{R} : 2 \leq x < 5\}$, find A - B in the same set notation.

$A - B = \{x \in A : x \notin B\} = \{x \in \mathbb{R} : -1 \leq x \leq 2 \text{ and } x < 2 \text{ or } x \geq 5\} = \{x \in \mathbb{R} : -1 \leq x < 2\}$.

Answer: $A - B = \{x \in \mathbb{R} : -1 \leq x < 2\}$

(b) Use laws of algebra of sets to simplify $[(A - B) \cup (A \cap B)] - [A \cup (A \cap B)]$.

$[(A - B) \cup (A \cap B)] - [A \cup (A \cap B)] = A - A = \varphi$.

Answer: φ

(c) In a class of 17 girls and 15 boys, 22 play handball, 16 play basketball, 12 of the boys play handball, 11 of the boys play basketball, 3 of the girls play neither of the two games. Use Venn diagram to determine:

(i) the number of girls who play both games,

(ii) the number of participants who play at least one game.

Total students = 17 + 15 = 32.

Girls playing at least one game = 17 - 3 = 14.

Boys: $H \cap B = 12 + 11 - 15 = 8$ (inclusion-exclusion).

Boys only H = 12 - 8 = 4, Boys only B = 11 - 8 = 3.

Girls: Total H = 22, Girls H = 22 - 12 = 10.

Total B = 16, Girls B = 16 - 11 = 5.

Girls $H \cap B = 10 + 5 - 14 = 1$.

(i) Girls playing both = 1.

(ii) Participants playing at least one game = 32 - 3 = 29.

Answer: (i) 1, (ii) 29

6. (a) Given f(x) = 2^x and g(x) = log_2 x:

(i) Find the domain and range of f(x) and g(x).

For f(x) = 2^x:

Domain: All real numbers, since the exponential function is defined for all x. So, domain is (-infinity, infinity).

Range: Since 2^x > 0 for all x, the range is (0, infinity).

For g(x) = log_2 x:

Domain: Logarithms are defined only for positive inputs, so x > 0. The domain is (0, infinity).

Range: The logarithm can take any real value, so the range is (-infinity, infinity).

(ii) Draw the graphs of f(x) and g(x) on the same axes. Comment on the resulting graphs.

Graph of f(x) = 2^x:

It's an exponential growth function. Key points: (0, 1), (1, 2), (-1, 0.5).

It increases rapidly as x increases and approaches 0 as x goes to -infinity.

Graph of g(x) = log_2 x:

It's the inverse of 2^x. Key points: (1, 0), (2, 1), (0.5, -1).

It increases slowly, crosses the x-axis at (1, 0), and approaches -infinity as x goes to 0 from the positive side.

Comment:

The graphs are reflections of each other over the line y = x, since f(x) and g(x) are inverse functions. f(x) grows exponentially, while g(x) grows logarithmically, showing their contrasting behaviors.

(iii) Sketch the graph of f(x), showing particularly where the curve crosses the x-axis and how it approaches its asymptotes.

For f(x) = 2^x:

It never crosses the x-axis since 2^x > 0 for all x.

Asymptote: As x goes to -infinity, 2^x goes to 0, so the x-axis ($y = 0$) is a horizontal asymptote.

The graph rises steeply for positive x and flattens out toward the x-axis for negative x.

(b). Given $f(x) = (2x - x^2) / (x^2 - 2x - 3)$:

(i) Write the values of x for which $f(x) = 0$ and the values of x for which $f(x) > 0$.

Numerator: $2x - x^2 = x(2 - x)$. Zeros at $x = 0$ and $x = 2$.

Denominator: $x^2 - 2x - 3 = (x - 3)(x + 1)$. Zeros at $x = 3$ and $x = -1$ (these are where $f(x)$ is undefined).

So, $f(x) = 0$ when the numerator is 0: $x = 0$ and $x = 2$.

Sign analysis (test intervals based on critical points $x = -1, 0, 2, 3$):

Interval $x < -1$ (e.g., $x = -2$): Numerator $(-2)(2 - (-2)) = (-2)(4) = -8$ (negative), Denominator $(-2 - 3)(-2 + 1) = (-5)(-1) = 5$ (positive). $f(x) < 0$.

Interval $-1 < x < 0$ (e.g., $x = -0.5$): Numerator $(-0.5)(2 - (-0.5)) = (-0.5)(2.5) = -1.25$ (negative), Denominator $(-0.5 - 3)(-0.5 + 1) = (-3.5)(0.5) = -1.75$ (negative). $f(x) > 0$.

Interval $0 < x < 2$ (e.g., $x = 1$): Numerator $(1)(2 - 1) = (1)(1) = 1$ (positive), Denominator $(1 - 3)(1 + 1) = (-2)(2) = -4$ (negative). $f(x) < 0$.

Interval $2 < x < 3$ (e.g., $x = 2.5$): Numerator $(2.5)(2 - 2.5) = (2.5)(-0.5) = -1.25$ (negative), Denominator $(2.5 - 3)(2.5 + 1) = (-0.5)(3.5) = -1.75$ (negative). $f(x) > 0$.

Interval $x > 3$ (e.g., $x = 4$): Numerator $(4)(2 - 4) = (4)(-2) = -8$ (negative), Denominator $(4 - 3)(4 + 1) = (1)(5) = 5$ (positive). $f(x) < 0$.

So, $f(x) > 0$ in the intervals $(-1, 0)$ and $(2, 3)$.

(ii) Show that $f(x)$ goes to -1 as x goes to infinity.

As x goes to infinity, look at the degrees of the numerator and denominator.

Numerator: $2x - x^2$ has leading term $-x^2$.

Denominator: $x^2 - 2x - 3$ has leading term x^2.

So, $f(x) \sim (-x^2) / (x^2) = -1$ as x goes to infinity (and similarly as x goes to -infinity).

(iii) Sketch the graph of $f(x)$, showing particularly where the curve crosses the x-axis and how it approaches its asymptotes.

Crosses x-axis at $x = 0$ and $x = 2$ (from part (i)).

Vertical asymptotes at $x = -1$ and $x = 3$ (denominator zeros).

Horizontal asymptote: $y = -1$ (from part (ii)).

Behavior:

As x approaches -1 from the left, $f(x)$ goes to -infinity; from the right, $f(x)$ goes to +infinity.

As x approaches 3 from the left, $f(x)$ goes to +infinity; from the right, $f(x)$ goes to -infinity.

As x goes to +/- infinity, $f(x)$ approaches $y = -1$.

The graph has segments in the intervals $x < -1$, $-1 < x < 3$, and $x > 3$, with the sign behavior from part (i).

7. (a) Verify that $x^2 - 2x - 1 = 0$ has a root in the range $2 <= x <= 3$.

Test the endpoints:

At $x = 2$: $2^2 - 2 \times 2 - 1 = 4 - 4 - 1 = -1$ (negative).

At $x = 3$: $3^2 - 2 \times 3 - 1 = 9 - 6 - 1 = 2$ (positive).

Since the function changes sign from negative to positive between $x = 2$ and $x = 3$, by the Intermediate Value Theorem, there is a root in $(2, 3)$.

(b) By using the Secant method, perform four iterations to obtain an approximation for the root of the equation in 7(a) correct to three decimal places.

Secant method formula: $x_{n+1} = x_n - f(x_n) \times (x_n - x_{n-1}) / (f(x_n) - f(x_{n-1}))$.

Start with $x_0 = 2$, $x_1 = 3$.

$f(x) = x^2 - 2x - 1$.

$f(2) = 2^2 - 2 \times 2 - 1 = -1$.

$f(3) = 3^2 - 2 \times 3 - 1 = 2$.

Iteration 1:

$x_2 = 3 - 2 \times (3 - 2) / (2 - (-1)) = 3 - 2 \times 1 / 3 = 3 - 0.6667 = 2.3333$.

$f(2.3333) = (2.3333)^2 - 2 \times 2.3333 - 1 = 5.4444 - 4.6666 - 1 = -0.2222$.

Iteration 2:

$x_3 = 2.3333 - (-0.2222) \times (2.3333 - 3) / (-0.2222 - 2) = 2.3333 + 0.2222 \times (-0.6667) / (-2.2222) = 2.3333 + 0.2222 \times 0.3 = 2.3333 + 0.0667 = 2.4$.

$f(2.4) = (2.4)^2 - 2 \times 2.4 - 1 = 5.76 - 4.8 - 1 = 0.96$.

Iteration 3:

$x_4 = 2.4 - 0.96 \times (2.4 - 2.3333) / (0.96 - (-0.2222)) = 2.4 - 0.96 \times 0.0667 / 1.1822 = 2.4 - 0.0541 = 2.3459$.

$f(2.3459) = (2.3459)^2 - 2 \times 2.3459 - 1 = 5.5032 - 4.6918 - 1 = -0.1886$.

Iteration 4:

$x_5 = 2.3459 - (-0.1886) \times (2.3459 - 2.4) / (-0.1886 - 0.96) = 2.3459 + 0.1886 \times (-0.0541) / (-1.1486) = 2.3459 + 0.0089 = 2.3548$.

Approximation to three decimal places: 2.355.

(c)(i) Using the Trapezoidal rule with 5 strips, obtain an approximation to $\int_0^1 x^2 e^x \, dx$ correct to four significant figures.

Trapezoidal rule: Integral $\approx (h/2) \times (y_0 + 2(y_1 + y_2 + ... + y_{(n-1)}) + y_n)$, where $h = (b - a)/n$.

$a = 0$, $b = 1$, $n = 5$, $h = (1 - 0)/5 = 0.2$.

Points: $x = 0, 0.2, 0.4, 0.6, 0.8, 1$.

Compute $y = x^2 \times e^x$:

$x = 0$: $0^2 \times e^0 = 0$.

$x = 0.2$: $(0.2)^2 \times e^{(0.2)} = 0.04 \times 1.2214 = 0.0489$.

$x = 0.4$: $(0.4)^2 \times e^{(0.4)} = 0.16 \times 1.4918 = 0.2387$.

$x = 0.6$: $(0.6)^2 \times e^{(0.6)} = 0.36 \times 1.8221 = 0.6560$.

$x = 0.8$: $(0.8)^2 \times e^{(0.8)} = 0.64 \times 2.2255 = 1.4243$.

$x = 1$: $1^2 \times e^1 = 1 \times 2.7183 = 2.7183$.

Trapezoidal rule: $(0.2/2) \times (0 + 2(0.0489 + 0.2387 + 0.6560 + 1.4243) + 2.7183) = 0.1 \times (0 + 2(2.3679) + 2.7183) = 0.1 \times 7.4541 = 0.7454$.

To four significant figures: 0.7454.

(ii) By using the integration by parts technique, evaluate the exact value of the definite integral $\int_0^1 x^2 e^x\, dx$ correct to four significant figures.

Integration by parts: $\int u\, dv = uv - \int v\, du$.

Let $u = x^2$, $dv = e^x dx$, so $du = 2x\, dx$, $v = e^x$.

$\int x^2 e^x dx = x^2 e^x - \int e^x \times 2x\, dx$.

Apply integration by parts again on $\int 2x e^x dx$:

Let $u = 2x$, $dv = e^x dx$, so $du = 2\, dx$, $v = e^x$.

$\int 2x e^x dx = 2x e^x - \int 2 e^x dx = 2x e^x - 2e^x$.

So, $\int x^2 e^x dx = x^2 e^x - (2x e^x - 2e^x) = x^2 e^x - 2x e^x + 2e^x$.

Evaluate from 0 to 1:

At $x = 1$: $1^2 e^1 - 2 \times 1 \times e^1 + 2e^1 = e - 2e + 2e = e$.

At $x = 0$: $0^2 e^0 - 2 \times 0 \times e^0 + 2e^0 = 0 - 0 + 2 = 2$.

Integral $= e - 2 = 2.7183 - 2 = 0.7183$.

To four significant figures: 0.7183.

(c)(iii) Use the results obtained in (c)(i) and (c)(ii) to find an absolute error in the value of $\int_0^1 x^2 e^x\, dx$.

Approximate value (Trapezoidal): 0.7454.

Exact value (Integration by parts): 0.7183.

Absolute error $= |0.7454 - 0.7183| = 0.0271$.

8. (a) If the perpendicular distance of the point $(1, k)$ to the line $6x + 8y = 5$ is 5.5, find the possible value of k.

Line: $6x + 8y - 5 = 0$. Point: $(1, k)$.

Distance formula: $|ax_0 + by_0 + c| / \sqrt{a^2 + b^2} = |6 \times 1 + 8 \times k - 5| / \sqrt{6^2 + 8^2} = |1 + 8k| / 10$.

Given distance = 5.5: $|1 + 8k| / 10 = 5.5$.

$|1 + 8k| = 55$.

$1 + 8k = 55$ or $1 + 8k = -55$.

$1 + 8k = 55$: $8k = 54$, $k = 54/8 = 6.75$.

$1 + 8k = -55$: $8k = -56$, $k = -56/8 = -7$.

Possible values of k: 6.75 or -7.

(b) The curves $y = 2x^2 - 3$ and $y = x^2 - 5x + 3$ intersect at points U and W, of which W is in the fourth quadrant. Find the tangent of the acute angle between these curves at W.

Find intersection: $2x^2 - 3 = x^2 - 5x + 3$.

$x^2 + 5x - 6 = 0$.

$(x + 6)(x - 1) = 0$, $x = -6$ or $x = 1$.

At $x = 1$: $y = 2 \times 1^2 - 3 = -1$. Point: $(1, -1)$ (fourth quadrant, so $W = (1, -1)$).

At $x = -6$: $y = 2 \times (-6)^2 - 3 = 69$. Point U: $(-6, 69)$.

Slopes:

$y = 2x^2 - 3$: $dy/dx = 4x$.

$y = x^2 - 5x + 3$: $dy/dx = 2x - 5$.

At $x = 1$ (W):

$m_1 = 4 \times 1 = 4$.

$m_2 = 2 \times 1 - 5 = -3$.

Tangent of angle: $\tan \theta = |(m_1 - m_2) / (1 + m_1 \times m_2)| = |(4 - (-3)) / (1 + 4 \times (-3))| = |7 / (1 - 12)| = |7 / -11| = 7/11$.

Tangent of acute angle at W: 7/11.

8. (c) Find the equation of a circle which passes through points A(0, 1), B(4, 3) and C(1, -1). Write your answer in the form $x^2 + y^2 + 2gx + 2fy + c = 0$.

General form: $x^2 + y^2 + 2gx + 2fy + c = 0$.

Point $A(0, 1)$: $0 + 1 + 2g \times 0 + 2f \times 1 + c = 0 \rightarrow 1 + 2f + c = 0 \rightarrow 2f + c = -1$.

Point $B(4, 3)$: $16 + 9 + 2g \times 4 + 2f \times 3 + c = 0 \rightarrow 25 + 8g + 6f + c = 0$.

Point $C(1, -1)$: $1 + 1 + 2g \times 1 + 2f \times (-1) + c = 0 \rightarrow 2 + 2g - 2f + c = 0 \rightarrow 2g - 2f + c = -2$.

Solve:

Eq 1: $2f + c = -1$.

Eq 2: $8g + 6f + c = -25$.

Eq 3: $2g - 2f + c = -2$.

Subtract Eq 1 from Eq 2: $(8g + 6f + c) - (2f + c) = -25 - (-1) \rightarrow 8g + 4f = -24 \rightarrow 2g + f = -6$.

Subtract Eq 1 from Eq 3: $(2g - 2f + c) - (2f + c) = -2 - (-1) \rightarrow 2g - 4f = -1$.

Now: $2g + f = -6$ (Eq 4), $2g - 4f = -1$ (Eq 5).

Subtract Eq 4 from Eq 5: $(2g - 4f) - (2g + f) = -1 - (-6) \rightarrow -5f = 5 \rightarrow f = -1$.

Substitute $f = -1$ into Eq 4: $2g - 1 = -6 \rightarrow 2g = -5 \rightarrow g = -5/2$.

Substitute $f = -1$ into Eq 1: $2 \times (-1) + c = -1 \rightarrow -2 + c = -1 \rightarrow c = 1$.

Equation: $x^2 + y^2 + 2 \times (-5/2) x + 2 \times (-1) y + 1 = 0 \rightarrow x^2 + y^2 - 5x - 2y + 1 = 0$.

9. (a) If I_m = ∫_(-m)^m sin x dx, show that I_m = (1/m) x (-cos x sin^(m-1) x + (m-1) I_(m-2)), hence find ∫_(-1)^1 sin x dx.

Compute I_m directly: ∫_(-m)^m sin x dx = [-cos x]_(-m)^m = -cos m - (-cos(-m)) = -cos m + cos m = 0.

Derive the formula using integration by parts:

Let u = sin^(m-1) x, dv = sin x dx, so du = (m-1) sin^(m-2) x cos x dx, v = -cos x.

I_m = ∫_(-m)^m sin x x sin^(m-1) x dx = [-cos x sin^(m-1) x](-m)^m + ∫(-m)^m cos x x (m-1) sin^(m-2) x cos x dx.

First term: [-cos m sin^(m-1) m + cos(-m) sin^(m-1)(-m)] = [-cos m sin^(m-1) m + cos m x (-sin^(m-1) m)] = -2 cos m sin^(m-1) m.

Second term: (m-1) ∫_(-m)^m cos² x sin^(m-2) x dx = (m-1) ∫_(-m)^m (1 - sin² x) sin^(m-2) x dx = (m-1) [I_(m-2) - I_m].

So, I_m = -2 cos m sin^(m-1) m + (m-1) I_(m-2) - (m-1) I_m.

Rearrange: $m I_m = -2 \cos m \sin^{(m-1)} m + (m-1) I_{(m-2)}$.

Thus, $I_m = (1/m) \times (-2 \cos m \sin^{(m-1)} m + (m-1) I_{(m-2)})$, which matches the given form.

For $m = 1$: $\int_{(-1)}^{1} \sin x \, dx = [-\cos x]_{(-1)}^{1} = -\cos 1 + \cos 1 = 0$.

9. (b) Evaluate $\int_0^1 (1 + x)^{(-2)} (2x + 2)^{(-2)} dx$ correct to three decimal places.

Simplify: $(1 + x)^{(-2)} (2x + 2)^{(-2)} = (1 + x)^{(-2)} \times (2(x + 1))^{(-2)} = (1 + x)^{(-2)} \times 2^{(-2)} (x + 1)^{(-2)} = (1/4) (x + 1)^{(-4)}$.

$\int_0^1 (1/4) (x + 1)^{(-4)} dx = (1/4) \int_0^1 (x + 1)^{(-4)} dx$.

Integrate: $\int (x + 1)^{(-4)} dx = (x + 1)^{(-3)} / (-3) = -(1/3) (x + 1)^{(-3)}$.

Evaluate: $(1/4) [-(1/3) (x + 1)^{(-3)}]_0^1 = (1/4) [-(1/3)(2)^{(-3)} + (1/3)(1)^{(-3)}] = (1/4) [(1/3) - (1/24)] = (1/4) \times (7/24) = 7/96 = 0.072916...$

To three decimal places: 0.073.

10. (a) Differentiate $y = \cos^{-1} [(1 - x^2) / (1 + x^2)]$ with respect to x.

Let $u = (1 - x^2) / (1 + x^2)$, so $y = \cos^{-1} u$.

$dy/dx = (dy/du) \times (du/dx)$.

$dy/du = -1 / \text{sqrt}(1 - u^2)$.

du/dx: Quotient rule on u:

Numerator derivative: $-2x$.

Denominator derivative: $2x$.

$du/dx = [(-2x)(1 + x^2) - (1 - x^2)(2x)] / (1 + x^2)^2 = [-2x - 2x^3 - 2x + 2x^3] / (1 + x^2)^2 = -4x / (1 + x^2)^2$.

$1 - u^2 = [(1 + x^2)^2 - (1 - x^2)^2] / (1 + x^2)^2 = 4x^2 / (1 + x^2)^2$.

$\text{sqrt}(1 - u^2) = 2|x| / (1 + x^2)$.

$dy/dx = [-1 / (2|x| / (1 + x^2))] \times [-4x / (1 + x^2)^2] = (4x / 2|x|) \times (1 / (1 + x^2))$.

$dy/dx = 2 / (1 + x^2)$ if $x > 0$, or $-2 / (1 + x^2)$ if $x < 0$.

(b)(i) Use Maclaurin's theorem to find the series expansion of $\ln(1 + x)$ up to the term containing x^4.

Maclaurin series: $f(x) = f(0) + f'(0)x + (f''(0)/2!)x^2 + (f'''(0)/3!)x^3 + (f^{(4)}(0)/4!)x^4 + \ldots$

$f(x) = \ln(1 + x)$.

$f(0) = \ln(1) = 0$.

$f'(x) = 1 / (1 + x)$, $f'(0) = 1$.

$f''(x) = -1 / (1 + x)^2$, $f''(0) = -1$.

$f'''(x) = 2 / (1 + x)^3$, $f'''(0) = 2$.

$f^{(4)}(x) = -6 / (1 + x)^4$, $f^{(4)}(0) = -6$.

Series: $0 + 1x + (-1/2)x^2 + (2/6)x^3 + (-6/24)x^4 = x - (1/2)x^2 + (1/3)x^3 - (1/4)x^4$.

(ii) By using the results obtained in (b)(i), compute $\ln(1.1)$ to four decimal places.

From (b)(i): $\ln(1 + x) \approx x - (1/2)x^2 + (1/3)x^3 - (1/4)x^4$ for $x = 0.1$.

$\ln(1.1) = 0.1 - (1/2) \times (0.1)^2 + (1/3) \times (0.1)^3 - (1/4) \times (0.1)^4$.

$0.1 = 0.1$.

$(1/2) \times 0.01 = 0.005$.

$(1/3) \times 0.001 = 0.0003333$.

$(1/4) \times 0.0001 = 0.000025$.

$\ln(1.1) \approx 0.1 - 0.005 + 0.0003333 - 0.000025 = 0.0953083$.

To four decimal places: 0.0953.

10(c)

If $f(x, y) = \sin xy$, find $\partial^2 f / (\partial x\, \partial y)$.

$f(x, y) = \sin(xy)$.

$\partial f/\partial y = \cos(xy) \times x$.

$\partial/\partial x\, (\partial f/\partial y) = \partial/\partial x\, [x \cos(xy)] = \cos(xy) + x \times [-\sin(xy) \times y] = \cos(xy) - xy \sin(xy)$.

So, $\partial^2 f / (\partial x\, \partial y) = \cos(xy) - xy \sin(xy)$.