THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATION COUNCIL OF TANZANIA

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1 **ADVANCED MATHEMATICS 1**

(For Both Private and School Candidates)

Duration: 3 Hour. **ANSWERS** **Year: 2025**

Instructions

1. This paper consists of **ten (10)** questions**.**
2. Answer **all** questions.
3. Write your **Examination Number** on every page of your answer booklet(s).

1. **(a)** Use a non-programmable calculator to evaluate the following expressions:

(i) $\frac{\tan 25°30' \div \sqrt{(0.03e^{-3})}}{\ln 3.2 + 0.006e^{0.33}}$ correctly to six significant figures

(ii) $\sum$ from n = 1 to 5 of ($2^n \times n!$) ÷ ln(0.3n), correct to three decimal places.

(i)
Convert 25°30′ to decimal degrees:
25°30′ = 25.5°

tan(25.5°) = 0.4778
$e^{-3} = 0.04979$
$0.03 \times e^{-3} = 0.03 \times 0.04979 = 0.0014937$
√(0.0014937) = 0.03866

So:
$\tan(25.5°) \div \sqrt{(0.03e^{-3})} = 0.4778 \div 0.03866 = 12.3627$

ln(3.2) = 1.1632
$e^{0.33} = 1.391$
0.006 × 1.391 = 0.008346

ln(3.2) + 0.008346 = 1.1715

(ii)
We compute the following:
$\sum$ from n = 1 to 5 of ($2^n \times n!$) ÷ ln(0.3n)

When n = 1:
$2^1 \times 1! = 2$
ln(0.3 × 1) = ln(0.3) = -1.2040
2 ÷ -1.2040 = -1.6627

When n = 2:
$2^2 \times 2! = 4 \times 2 = 8$
ln(0.3 × 2) = ln(0.6) = -0.5108
8 ÷ -0.5108 = -15.6582

When n = 3:
$2^3 \times 3! = 8 \times 6 = 48$
ln(0.3 × 3) = ln(0.9) = -0.1054
48 ÷ -0.1054 = -455.4292

When n = 4:
$2^4 \times 4! = 16 \times 24 = 384$

$\ln(0.3 \times 4) = \ln(1.2) = 0.182$
$384 \div 0.182 = 2109.8901$

When $n = 5$:
$2^5 \times 5! = 32 \times 120 = 3840$
$\ln(0.3 \times 5) = \ln(1.5) = 0.4055$
$3840 \div 0.4055 = 9470.4317$

Now sum all values:
$-1.6627 + (-15.6582) + (-455.4292) + 2109.8901 + 9470.4317 = 11107.5717$

Answer = 11107.572 (to three decimal places)

(b) If $A_n = A_5 + A_{20}e^{-kn}$, where $k = 0.1386$, $A_5 = 20$ and $A_{20} = 40$, find the value of A_{16}.

We calculate:
$A_{16} = 20 + 40 \times e^{-}(0.1386 \times 16)$
$0.1386 \times 16 = 2.2176$
$e^{-2.2176} = 0.1086$
$40 \times 0.1086 = 4.344$
$20 + 4.344 = 24.344$

Answer: $A_{16} = 24.34$ (to 4 significant figures)

2. (a) Prove that $(1 + \tanh x) \div (1 - \tanh x) = \cosh 2x + \sinh 2x$.

We start with the identity:
$\tanh x = \sinh x \div \cosh x$

Now compute the left-hand side:
$(1 + \tanh x) \div (1 - \tanh x)$
$= (1 + \sinh x \div \cosh x) \div (1 - \sinh x \div \cosh x)$
$= (\cosh x + \sinh x) \div (\cosh x - \sinh x)$

Now multiply both numerator and denominator by $(\cosh x + \sinh x)$:
$= [(\cosh x + \sinh x)^2] \div [\cosh^2 x - \sinh^2 x]$

Recall that:
$\cosh^2 x - \sinh^2 x = 1$

So the denominator becomes 1.
Now expand the numerator:
$(\cosh x + \sinh x)^2 = \cosh^2 x + 2 \sinh x \cosh x + \sinh^2 x$
$= (\cosh^2 x + \sinh^2 x) + 2 \sinh x \cosh x$
$= 1 + 2 \sinh x \cosh x$

But:
$\cosh 2x = \cosh^2 x + \sinh^2 x = 1 + 2 \sinh^2 x$
$\sinh 2x = 2 \sinh x \cosh x$

So:
$1 + 2 \sinh x \cosh x = \cosh 2x + \sinh 2x$

Therefore:
$(1 + \tanh x) \div (1 - \tanh x) = \cosh 2x + \sinh 2x$

(b) Differentiate ln(tanh x) with respect to x and simplify your answer.

Let $y = \ln(\tanh x)$

$dy/dx = 1 \div \tanh x \times d/dx(\tanh x)$
$d/dx(\tanh x) = \text{sech}^2 x$
So:
$dy/dx = \text{sech}^2 x \div \tanh x$

Answer: $dy/dx = \text{sech}^2 x \div \tanh x$

(c) Find the area enclosed by the curve y = cosh x, the lines x = ln 2, the x-axis and y-axis. Hence find the volume obtained when this area is rotated completely about the x-axis.

Area A = ∫ from 0 to ln 2 of cosh x dx
$\int \cosh x \, dx = \sinh x$

So:
$A = \sinh(\ln 2) - \sinh(0) = \sinh(\ln 2)$

Now, $\sinh(\ln 2) = (e^{\ln 2} - e^{-\ln 2}) \div 2$
$= (2 - 0.5) \div 2 = 1.5 \div 2 = 0.75$

Therefore, Area = 0.75 units²

Now for volume when rotated about x-axis:
Volume V = π ∫ from 0 to ln 2 of $(\cosh x)^2$ dx

We use identity:
$\cosh^2 x = (1 + \cosh 2x) \div 2$

So:
$\int \cosh^2 x \, dx = \frac{1}{2} \int (1 + \cosh 2x) \, dx$
$= \frac{1}{2} [x + \frac{1}{2} \sinh 2x] + C$

Now apply the limits from 0 to ln 2:
$x = \ln 2$
$\sinh 2x = \sinh(2 \ln 2)$
$= (e^{(2 \ln 2)} - e^{(-2 \ln 2)}) \div 2 = (4 - 0.25) \div 2 = 3.75 \div 2 = 1.875$

Then:
$V = \pi \times \frac{1}{2} [\ln 2 + \frac{1}{2} \times 1.875]$
$= \pi \times \frac{1}{2} [\ln 2 + 0.9375]$
$\ln 2 \approx 0.6931$
$= \pi \times \frac{1}{2} \times (0.6931 + 0.9375)$
$= \pi \times \frac{1}{2} \times 1.6306 = \pi \times 0.8153$
$\approx 3.1416 \times 0.8153 = 2.5611$

Answer:
Area = 0.75
Volume ≈ 2.5611 units^3

3. Rehema has 900 tonnes and 600 tonnes of bricks at Mtakuja and Tupendane villages respectively. She has planned to build new houses at sites A, B and C. She expects to use 500 tonnes of bricks at site A, 600 tonnes at site B and 400 tonnes at site C. The transport cost in Tanzanian shillings is proportional to the distance covered in kilometres as shown in the following table:

From/To	**A**	**B**	**C**
Mtakuja	600	300	400
Tupendane	400	200	600

If x and y are the number of bricks to be transported from Mtakuja to sites A and B respectively:

(a) Formulate the inequalities and the objective function to be satisfied by x and y.

Let x be the number of bricks from Mtakuja to site A.
Let y be the number of bricks from Mtakuja to site B.

Total bricks available at Mtakuja is 900 tonnes, so:
$x + y + z \leq 900$, where z is bricks sent from Mtakuja to site C.
But since site C needs 400 tonnes in total and total site needs = 500 + 600 + 400 = 1500 tonnes, and Mtakuja has 900 tonnes while Tupendane has 600 tonnes, all bricks will be used.

Also, we know:
Bricks to site A: x (from Mtakuja) + (500 − x) (from Tupendane)

Bricks to site B: y (from Mtakuja) + (600 − y) (from Tupendane)
Bricks to site C: 400 (can be from either source)

So, the constraints are:
$x + y \leq 900$ (supply from Mtakuja)
$(500 - x) + (600 - y) + (400 - z) \leq 600$ (supply from Tupendane)
But Tupendane has 600 tonnes, so:
Total from Tupendane = $(500 - x) + (600 - y) + (400 - z) \leq 600$
This simplifies to: $1500 - (x + y + z) \leq 600$
So: $x + y + z \geq 900$

We assume site C is supplied entirely by Mtakuja to use up 900 tonnes there:
Then: $z = 400$
So: $x + y = 500$

Objective function:
Cost = $600x + 300y + 400(500 - x) + 200(600 - y) + 600 \times 400$

Simplify:
$600x + 300y + 200000 - 400x + 120000 - 200y + 240000$
$= (600x - 400x) + (300y - 200y) + 560000$
$= 200x + 100y + 560000$

Objective function: Minimize $Z = 200x + 100y$
Subject to:
$x + y = 500$
$x \geq 0, y \geq 0$
$x \leq 500, y \leq 600$

(b) Find the number of bricks to be transferred from Mtakuja and Tupendane villages to each site.

To minimize cost, send more bricks to site B from Tupendane, since its cost is cheaper (200 compared to 300 from Mtakuja). So $y = 0$.

Then $x = 500$
Mtakuja:
Site A = 500
Site B = 0
Site C = 400
Total = 900 tonnes

Tupendane:
Site A = 0
Site B = 600

Site C = 0
Total = 600 tonnes

4. The following table shows the frequency distribution for the values of resistance in ohms of 48 resistors:

Resistance (Ω)	**20.5–20.9**	**21.0–21.4**	**21.5–21.9**	**22.0–22.4**	**22.5–22.9**	**23.0–23.4**
Frequency	3	10	11	13	9	2

(a) Use this information to find the mode, median and 45th percentile correct to two decimal places.

Mode

The modal class is the one with the highest frequency = 13 → class 22.0–22.4
L = 22.0
f_1 = 13 (modal frequency)
f_0 = 11 (previous class)
f_2 = 9 (next class)
h = 0.5

Mode = $L + [(f_1 - f_0) / (2f_1 - f_0 - f_2)] \times h$
$= 22.0 + [(13 - 11) / (2\times13 - 11 - 9)] \times 0.5$
$= 22.0 + (2 / 6) \times 0.5 = 22.0 + 0.167 = 22.17$

Median

n = 48
Median position = 48 ÷ 2 = 24th item
Cumulative frequencies:
20.5–20.9: 3
21.0–21.4: 3 + 10 = 13
21.5–21.9: 13 + 11 = 24
So the 24th value lies in class 21.5–21.9
L = 21.5
cf = 13 (cumulative frequency before median class)
f = 11
h = 0.5

Median = $L + [(n/2 - cf)/f] \times h$
$= 21.5 + [(24 - 13)/11] \times 0.5$
$= 21.5 + (11 \div 11) \times 0.5 = 21.5 + 0.5 = 22.00$

45th Percentile (P_{45})

Position = 0.45 × 48 = 21.6th item
Cumulative frequency tells us 21.6 is also in class 21.5–21.9
L = 21.5
cf = 13
f = 11
h = 0.5

P_{45} = L + [(21.6 − 13)/11] × 0.5
= 21.5 + (8.6 ÷ 11) × 0.5
= 21.5 + 0.391 = 21.89

(b) Using the coding method, let A = 21.95, h = 0.5

Midpoints and frequencies:
20.7 (3), 21.2 (10), 21.7 (11), 22.2 (13), 22.7 (9), 23.2 (2)

u = (x − A) ÷ h
So:
20.7 → (20.7 − 21.95) ÷ 0.5 = −2.5
21.2 → −1.5
21.7 → −0.5
22.2 → 0.5
22.7 → 1.5
23.2 → 2.5

Now compute fu:
(−2.5×3) = −7.5
(−1.5×10) = −15
(−0.5×11) = −5.5
(0.5×13) = 6.5
(1.5×9) = 13.5
(2.5×2) = 5

Sum fu = −7.5 −15 −5.5 +6.5 +13.5 +5 = −3.0

Mean = A + (∑fu ÷ ∑f) × h
Mean = 21.95 + (−3 ÷ 48) × 0.5 = 21.95 − 0.03125 = 21.92

Now compute fu^2:
(3×6.25) = 18.75
(10×2.25) = 22.5
(11×0.25) = 2.75
(13×0.25) = 3.25
(9×2.25) = 20.25
(2×6.25) = 12.5
Sum fu^2 = 80.0

Standard deviation:
$\sigma = h \times \sqrt{[(\sum fu^2 \div \sum f) - (\sum fu \div \sum f)^2]}$
$= 0.5 \times \sqrt{[(80 \div 48) - (-3 \div 48)^2]}$
$= 0.5 \times \sqrt{[1.6667 - 0.0039]}$
$= 0.5 \times \sqrt{1.6628}$
$= 0.5 \times 1.29 = 0.645$

Answer:
Mean = 21.92
Standard deviation = 0.65 (rounded to 2 decimal places)

5. (a) (i) Given that $A = \{x \in \mathbb{R} : x \geq 1\}$, $B = \{x \in \mathbb{R} : -5 < x \leq 3\}$, find $A \cap B$.

$A \cap B$ means the set of all real numbers that are in both A and B.
A is all real numbers greater than or equal to 1.
B is all real numbers greater than −5 and less than or equal to 3.

The intersection will be where both conditions are satisfied:
$x \geq 1$ and $x \leq 3$
Therefore, $A \cap B = \{x \in \mathbb{R} : 1 \leq x \leq 3\}$

Answer: $A \cap B = [1, 3]$

(a) (ii) Find $A \cup B'$.

B' is the complement of B in $\mathbb{R}$, which is the set of all real numbers not in B.
Since $B = \{x \in \mathbb{R} : -5 < x \leq 3\}$,
Then $B' = \{x \in \mathbb{R} : x \leq -5 \text{ or } x > 3\}$

$A = \{x \in \mathbb{R} : x \geq 1\}$

Now $A \cup B'$ means combine A and B':
A is $x \geq 1$,
B' is $x \leq -5$ or $x > 3$

So $A \cup B' = \{x \in \mathbb{R} : x \leq -5 \text{ or } x \geq 1\}$

Answer: $A \cup B' = (-\infty, -5] \cup [1, \infty)$

(b) In a certain district, 230 families grow crops. 42 grow coffee and sunflower, 34 grow cotton and coffee, 68 grow cotton and sunflower, and 9 grow none of the three crops. It is also known that the number of families who grow coffee only is x, those who grow cotton only is 3x and those who grow sunflower only is x.

(i) Find the number of families that grow all three crops.

Let x = number who grow coffee only
Then:
Cotton only = 3x
Sunflower only = x

Families who grow only one crop = x + 3x + x = 5x

Total number of families = 230
Families who grow none = 9
So families who grow at least one crop = 230 − 9 = 221

From question, families who grow two crops (excluding those who grow all three) are:
Coffee and sunflower = 42
Cotton and coffee = 34
Cotton and sunflower = 68

Let z = number of families who grow all three crops

Then families who grow exactly two crops =
(42 − z) + (34 − z) + (68 − z) = 144 − 3z

Now, total who grow at least one crop =
Only one crop + exactly two crops + all three
= 5x + (144 − 3z) + z = 5x + 144 − 2z

But we already know:
5x + 144 − 2z = 221
⇒ 5x − 2z = 77 … equation (1)

We also know x + 3x + x = 5x

Now solve for integer solution
Try x = 15 ⇒ 5x = 75
Then 75 − 2z = 77 ⇒ −2z = 2 ⇒ z = −1 → invalid

Try x = 16 ⇒ 5x = 80
80 − 2z = 77 ⇒ z = 1.5 → invalid

Try x = 17 ⇒ 5x = 85
85 − 2z = 77 ⇒ z = 4 → valid

So:
$x = 17$
$z = 4$

Answer: Number of families that grow all three crops = 4

(ii) How many families grow exactly one crop?

From above, only one crop = $x + x + 3x = 5x = 5 \times 17 = 85$

Answer: 85 families grow exactly one crop.

6. (a) Given the function $f(x) = x^3 + 3x^2 - 2x - 6$, draw the graph of f(x) and hence, state its domain and range.

To sketch the graph, first find the critical points by factoring:
$f(x) = x^3 + 3x^2 - 2x - 6$
Group: $(x^3 + 3x^2) - (2x + 6)$
$= x^2(x + 3) - 2(x + 3)$
$= (x^2 - 2)(x + 3)$

So the factorized form is:
$f(x) = (x + 3)(x^2 - 2)$

The graph is a cubic function with one turning point and one inflection point.
As $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to -\infty$

Domain: All real numbers, $\mathbb{R}$
Range: All real numbers, $\mathbb{R}$

(b) Given that $g(x) = (x + 1) \div (2x^2 + 5x - 3)$
(i) Find the vertical asymptotes

Find values of x that make the denominator zero:
$2x^2 + 5x - 3 = 0$
Use quadratic formula:
$x = [-5 \pm \sqrt{(25 + 24)}] \div 4 = (-5 \pm \sqrt{49}) \div 4 = (-5 \pm 7) \div 4$
So $x = 0.5$ and $x = -3$

Vertical asymptotes: $x = -3$ and $x = 0.5$

(ii) Sketch the graph of g(x) and determine the domain and range

$g(x) = (x + 1) \div (2x^2 + 5x - 3)$
Domain: All real x except $x = -3$ and $x = 0.5$
Range: Since the function is rational, the range excludes any undefined horizontal asymptotes, but is approximately all real numbers except possible gaps from vertical/horizontal asymptotes.

7. (a) Derive the secant formula for approximating the roots of the equation $f(x) = 0$.

The secant method uses two approximations x_0 and x_1:
$x_2 = x_1 - f(x_1) \times (x_1 - x_0) \div [f(x_1) - f(x_0)]$

This is the secant formula for root approximation.

(b) Using the formula derived in part (a) and the interval (1.5, 2), perform two iterations to solve the equation $x^3 - 3 = 0$ correct to two decimal places.

Let $f(x) = x^3 - 3$
$x_0 = 1.5$, $f(1.5) = (1.5)^3 - 3 = 3.375 - 3 = 0.375$
$x_1 = 2.0$, $f(2.0) = 8 - 3 = 5$

$x_2 = x_1 - f(x_1)(x_1 - x_0)/(f(x_1) - f(x_0))$
$= 2.0 - 5(2.0 - 1.5)/(5 - 0.375)$
$= 2.0 - 5(0.5)/4.625$
$= 2.0 - 0.5405 = 1.4595$

Now $f(1.4595) = (1.4595)^3 - 3 \approx 3.108 - 3 = 0.108$

$x_3 = 1.4595 - 0.108(1.4595 - 1.5)/(0.108 - 0.375)$
$= 1.4595 - 0.108(-0.0405)/(-0.267)$
$= 1.4595 + 0.0164 = 1.4759$

Answer after two iterations: $x \approx 1.48$

(c) By using the Newton-Raphson method and $x_0 = 5$, perform two iterations to approximate the solution of $x^4 - 4x^2 - x - 12 = 0$ correct to three significant figures.

Let $f(x) = x^4 - 4x^2 - x - 12$
$f'(x) = 4x^3 - 8x - 1$

$x_0 = 5$
$f(5) = 625 - 100 - 5 - 12 = 508$
$f'(5) = 500 - 40 - 1 = 459$

$x_1 = x_0 - f(x_0)/f'(x_0) = 5 - 508 \div 459 = 5 - 1.1067 = 3.8933$

Now compute x_2:
$f(3.8933) = (3.8933)^4 - 4(3.8933)^2 - 3.8933 - 12$
$\approx 229.4 - 60.6 - 3.89 - 12 \approx 152.91$

$f'(3.8933) = 4(3.8933)^3 - 8(3.8933) - 1$
$\approx 4(59.0) - 31.1 - 1 \approx 203.9$

$x_2 = 3.8933 - 152.91 \div 203.9 = 3.8933 - 0.75 = 3.143$

Answer: $x \approx 3.14$ (to three significant figures)

8. (a) Calculate the perpendicular distance of a point (3, −5) from the line 2x − y = 1.

The formula for perpendicular distance from point (x_0, y_0) to line $ax + by + c = 0$ is:
Distance = $|a \cdot x_0 + b \cdot y_0 + c| \div \sqrt{a^2 + b^2}$

Rewrite the line: $2x - y - 1 = 0$
So, $a = 2$, $b = -1$, $c = -1$
Point: (3, −5)

Distance = $|2\times3 + (-1)(-5) - 1| \div \sqrt{2^2 + (-1)^2}$
$= |6 + 5 - 1| \div \sqrt{4 + 1}$
$= |10| \div \sqrt{5} = 10 \div 2.236 = 4.472$

Answer: 4.472 units

(b) Obtain the equations to the bisectors of angles between the lines 3x + 4y = 12 and 4x − 3y = 6.

Let line 1 be: $3x + 4y - 12 = 0$
Let line 2 be: $4x - 3y - 6 = 0$

The formula for angle bisectors:
$(a_1x + b_1y + c_1)/\sqrt{a_1^2 + b_1^2} = \pm (a_2x + b_2y + c_2)/\sqrt{a_2^2 + b_2^2}$

Apply the formula:
$(3x + 4y - 12)/\sqrt{9 + 16} = \pm (4x - 3y - 6)/\sqrt{16 + 9}$
$(3x + 4y - 12)/\sqrt{25} = \pm (4x - 3y - 6)/\sqrt{25}$
$(3x + 4y - 12)/5 = \pm (4x - 3y - 6)/5$
Now remove denominators:
$3x + 4y - 12 = \pm (4x - 3y - 6)$

Split into two equations:

Case 1:
$3x + 4y - 12 = 4x - 3y - 6$
$3x - 4x + 4y + 3y = -6 + 12$
$-x + 7y = 6 \rightarrow x = 7y - 6$

Case 2:
$3x + 4y - 12 = -(4x - 3y - 6) = -4x + 3y + 6$
$3x + 4y + 4x - 3y = 12 + 6$
$7x + y = 18$

Answer: The two bisectors are:
$x = 7y - 6$ and $7x + y = 18$

(c) Find the length of a tangent to the circle $x^2 + y^2 + 2x + 2y - 7 = 0$ from the point (2, 3)

We first convert the equation into standard form.

Complete the square:
$x^2 + 2x = (x + 1)^2 - 1$
$y^2 + 2y = (y + 1)^2 - 1$
So:
$(x + 1)^2 - 1 + (y + 1)^2 - 1 - 7 = 0$
$(x + 1)^2 + (y + 1)^2 = 9$

So the circle has center (−1, −1) and radius $r = \sqrt{9} = 3$

Let P = (2, 3). Distance from P to center:
$\sqrt{[(2 + 1)^2 + (3 + 1)^2]} = \sqrt{[9 + 16]} = \sqrt{25} = 5$

Use formula: Length of tangent $= \sqrt{(d^2 - r^2)}$
$= \sqrt{(25 - 9)} = \sqrt{16} = 4$

Answer: 4 units

9. (a) Evaluate the definite integral from 1 to 2 of $(4x^2 + 3x - 2) \div [(x + 1)(2x + 3)]\,dx$ correct to four decimal places.

Use partial fractions:
$(4x^2 + 3x - 2) \div [(x + 1)(2x + 3)] = A \div (x + 1) + B \div (2x + 3)$

To find A and B:
Multiply both sides by $(x + 1)(2x + 3)$:
$4x^2 + 3x - 2 = A(2x + 3) + B(x + 1)$

Expand both sides:
$A(2x + 3) = 2Ax + 3A$
$B(x + 1) = Bx + B$

So:
$4x^2 + 3x - 2 = 2Ax + 3A + Bx + B$
$= (2A + B)x + (3A + B)$

But LHS has a quadratic term. So let's instead do long division or better:
Write:

$(4x^2 + 3x - 2) \div [(x + 1)(2x + 3)]$
$= (A \div (x + 1)) + (B \div (2x + 3))$

Use method of equating coefficients.
Let's multiply both sides and plug in values:

Let $x = -1$:
$4(1) - 3 + 2 = A(2(-1) + 3) + 0 \rightarrow 4(1) - 3 + 2 = A(1) \rightarrow A = 3$

Let $x = -1.5$ (so $2x + 3 = 0$):
$x = -1.5$
Then numerator: $4(2.25) - 4.5 - 2 = 9 - 4.5 - 2 = 2.5$
Denominator becomes: $B(x + 1) \rightarrow B(-0.5)$
So $2.5 = B(-0.5) \rightarrow B = -5$

So integral becomes:
$\int$ from 1 to 2 of $[3 \div (x + 1) - 5 \div (2x + 3)]\ dx$

Now integrate:
$\int (3 \div (x + 1))\ dx = 3 \ln|x + 1|$
$\int (5 \div (2x + 3))\ dx = (5 \div 2) \ln|2x + 3|$

Now compute:
$[3 \ln(x + 1) - (5 \div 2) \ln(2x + 3)]$ from 1 to 2

At $x = 2$:
$\ln(3) = 1.0986$
$\ln(7) = 1.9459$
So:
$3 \times 1.0986 - (5 \div 2) \times 1.9459 = 3.2958 - 4.8647 = -1.5689$

At $x = 1$:
$\ln(2) = 0.6931$
$\ln(5) = 1.6094$
$3 \times 0.6931 - (5 \div 2) \times 1.6094 = 2.0793 - 4.0235 = -1.9442$

Then: $-1.5689 - (-1.9442) = 0.3753$

Answer: 0.3753

(b) Find the area enclosed between the curve $y = x(x - 1)(x - 2)$ and the x-axis.

First, factor:
$y = x(x - 1)(x - 2) \rightarrow$ roots at $x = 0, 1, 2$
This function is negative between $0 < x < 1$ and positive between $1 < x < 2$

Split the integral:
$A = \int$ from 0 to 1 of $-x(x - 1)(x - 2)\,dx + \int$ from 1 to 2 of $x(x - 1)(x - 2)\,dx$

We expand $y = x^3 - 3x^2 + 2x$

$\int_0^1$ of $-(x^3 - 3x^2 + 2x)\,dx = -[\frac{1}{4}x^4 - x^3 + x^2]$ from 0 to 1
$= -[(0.25 - 1 + 1)] = -0.25$

$\int_1^2$ of $(x^3 - 3x^2 + 2x)\,dx$
$= [\frac{1}{4}x^4 - x^3 + x^2]$ from 1 to 2
At $x = 2$: $16 \div 4 - 8 + 4 = 4 - 8 + 4 = 0$
At $x = 1$: $0.25 - 1 + 1 = 0.25$
So area $= 0 - 0.25 = -0.25$

Total area $= 0.25 + 0.25 = 0.5$ units^2

(c) Find the volume of the solid formed by revolving the region enclosed by $y = x^2 - 4$ and the x-axis about x-axis by 360°.

Find where curve intersects x-axis:
$x^2 - 4 = 0 \rightarrow x = \pm 2$
Volume $= \pi \int$ from -2 to 2 of $(x^2 - 4)^2\,dx$

$(x^2 - 4)^2 = x^4 - 8x^2 + 16$
$\int (x^4 - 8x^2 + 16)\,dx = (1/5)x^5 - (8/3)x^3 + 16x$

Compute from -2 to 2:
At $x = 2$:

$(1/5)(32) - (8/3)(8) + 32 = 6.4 - 21.33 + 32 = 17.07$
At $x = -2$: same result (since even powers)

Total integral = $2 \times 17.07 = 34.14$
Volume = $\pi \times 34.14 \approx 107.25$ units^3

10. (a) Differentiate $x^3y + y^3x = 2y$ with respect to x at the point (1,1)

Differentiate both sides implicitly with respect to x:

Left-hand side:
$d/dx[x^3y] + d/dx[y^3x] = d/dx[2y]$

Use product rule:
$d/dx[x^3y] = x^3(dy/dx) + 3x^2y$
$d/dx[y^3x] = y^3 + 3y^2x(dy/dx)$
$d/dx[2y] = 2(dy/dx)$

Now write:
$x^3(dy/dx) + 3x^2y + y^3 + 3y^2x(dy/dx) = 2(dy/dx)$

Group dy/dx terms:
$x^3 + 3y^2x - 2 = -3x^2y - y^3$

Substitute (x, y) = (1, 1):
$[1 + 3 - 2]dy/dx = -3 - 1 = -4$
$2(dy/dx) = -4 \Rightarrow dy/dx = -2$

Answer: $dy/dx = -2$ at the point (1, 1)

10. (b) If a car starts from rest and moves a distance g cm in t seconds where
$g = (1/8)t^4 + (1/2)t^2$

(i) Find the velocity of the car after two seconds.

Velocity = dg/dt
$dg/dt = d/dt[(1/8)t^4 + (1/2)t^2] = (1/8)(4t^3) + (1/2)(2t)$
$= (1/2)t^3 + t$

At t = 2:
$v = (1/2)(8) + 2 = 4 + 2 = 6$ cm/s

Answer: Velocity after two seconds is 6 cm/s

(ii) Find the initial acceleration.

Acceleration = dv/dt = d^2g/dt^2
From above, v = $(1/2)t^3 + t$
So:
dv/dt = $(3/2)t^2 + 1$

At t = 0:
a = (3/2)(0) + 1 = 1 cm/s^2

Answer: Initial acceleration is 1 cm/s^2

10. (c) Differentiate the following expressions with respect to x:

(i) [e^x × (sin x)^(1/2)] ÷ (3x + 1)

Let u = e^x × (sin x)^(1/2), v = 3x + 1
Use quotient rule:
d/dx[u/v] = (v × du/dx − u × dv/dx) ÷ v^2

First find du/dx:
Let u = e^x × (sin x)^(1/2) = $e^x \times \sin^{0.5}x$
Then:
du/dx = $e^x \times \sin^{0.5}x + e^x \times (1/2)\sin^{-0.5}x \times \cos x$
= $e^x[\sin^{0.5}x + (\cos x)/(2\sin^{0.5}x)]$

dv/dx = 3

Now plug into quotient rule:
dy/dx = $[(3x + 1) \times e^x(\sin^{0.5}x + \cos x \div (2\sin^{0.5}x)) - e^x\sin^{0.5}x \times 3] \div (3x + 1)^2$

Answer:
dy/dx = $\{ (3x + 1)e^x[\sin^{0.5}x + (\cos x)/(2\sin^{0.5}x)] - 3e^x\sin^{0.5}x \} \div (3x + 1)^2$

(ii) Differentiate $\cos^{-1}(\tan x)$

Let y = $\cos^{-1}(\tan x)$
Then dy/dx = d/dx[$\cos^{-1}(\tan x)$] = $-1 \div \sqrt{(1 - \tan^2x)} \times$ d/dx(tan x)
d/dx(tan x) = $\sec^2x$

So:
dy/dx = $-\sec^2x \div \sqrt{(1 - \tan^2x)}$

Answer: dy/dx = $-\sec^2x \div \sqrt{(1 - \tan^2x)}$