THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2 ADVANCED MATHEMATICS 2

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2004**

Instructions

1. This paper consists of section A and B.
2. Answer all questions in section A and two questions from section B.
3. **All** work done and answers of each question must be shown clearly.
4. NECTA'S Mathematical tables and Non-programmable calculations may be used
5. All writing must be in **black** or **blue** ink, **except** drawing which must be in pencil.

Prepared by: Maria Marco for TETEA

1. (a) Use logarithms to evaluate to three significant figures: ((ln 32 - √(0.06 e^2)) / tan 55° 36')^(0.5).

ln 32 ≈ 3.4657

$e^2 \approx 7.3891$

0.06 e^2 ≈ 0.4433, √(0.06 e^2) ≈ 0.6658

ln 32 - √(0.06 e^2) ≈ 3.4657 - 0.6658 ≈ 2.7999

55° 36' = 55.6°, tan 55.6° ≈ 1.4515

Numerator / tan 55.6° ≈ 2.7999 / 1.4515 ≈ 1.9283

ln (1.9283) ≈ 0.6565

(0.5) x 0.6565 ≈ 0.3283

e^(0.3283) ≈ 1.39

Answer: 1.39.

1. (b) Using a non-programmable scientific calculator, find the value of √((e^5 √(ln 32) log 32) / √3) correct to 5 decimal places.

Break down the expression:

e^5 ≈ 148.413159

ln 32 ≈ 3.465735903, so √(ln 32) ≈ 1.862135955

log 32 (assuming base 10, as typical in such contexts) ≈ 1.505149978

√3 ≈ 1.732050808

Numerator: e^5 x √(ln 32) x log 32

= 148.413159 x 1.862135955 x 1.505149978 ≈ 415.9446925

Denominator: √3 ≈ 1.732050808

Fraction: 415.9446925 / 1.732050808 ≈ 240.1519596

Square root: √(240.1519596) ≈ 15.49748218

To 5 decimal places: 15.49748

Answer: 15.49748.

2. (a) Using letters and logical connectives, write the following statement: "If x is less than zero then it is not positive."

Let p: "x is less than zero" ($x < 0$)

Let q: "x is positive" ($x > 0$)

The statement "If x is less than zero then it is not positive" translates to:

$p \rightarrow \neg q$

Answer: $p \rightarrow \neg q$.

2. (b) Using the statement $p \rightarrow q$, find and simplify:

(i) The contrapositive of its inverse.

Inverse: $\neg p \rightarrow \neg q$

Contrapositive: $q \rightarrow p$

Answer: $q \rightarrow p$.

(ii) The converse of its contrapositive.

Contrapositive of $p \rightarrow q$: $\neg q \rightarrow \neg p$

Converse: $\neg p \rightarrow \neg q$

Answer: $\neg p \rightarrow \neg q$.

(iii) The converse of its contrapositive (repeated).

Same as (ii): $\neg p \rightarrow \neg q$

Answer: $\neg p \rightarrow \neg q$.

3. (a) Find the center and radius of the circle given by the equation $x^2 + y^2 + 4x - 8y + 4 = 0$.

Complete the square:

$x^2 + 4x + y^2 - 8y + 4 = 0$

$(x + 2)^2 - 4 + (y - 4)^2 - 16 + 4 = 0$

$(x + 2)^2 + (y - 4)^2 = 16$

Center: (-2, 4), Radius: $\sqrt{16} = 4$

Answer: Center: (-2, 4), Radius: 4.

3. (b) Find the length of the tangent from the point (3, 8) to the circle in 3(a).

Distance from (3, 8) to center (-2, 4):

$\sqrt{((3 - (-2))^2 + (8 - 4)^2)} = \sqrt{(25 + 16)} = \sqrt{41}$

Length of tangent = $\sqrt{((\text{distance})^2 - (\text{radius})^2)} = \sqrt{(41 - 16)} = \sqrt{25} = 5$

Answer: 5.

Let's solve question 4 from the provided document snippet, using plain text formatting for calculations, with √ for sqrt, x for multiplication, x^2 for x^2, etc.

Question 4

4. (a) Prove that (sin(A - B) / (cos A cos B)) + (sin(B - C) / (cos B cos C)) + (sin(C - A) / (cos C cos A)) = 0.

Use the identity: sin(a - b) = sin a cos b - cos a sin b.

Rewrite each term:

sin(A - B) / (cos A cos B) = (sin A cos B - cos A sin B) / (cos A cos B) = (sin A / cos A) - (sin B / cos B) = tan A - tan B

sin(B - C) / (cos B cos C) = (sin B cos C - cos B sin C) / (cos B cos C) = (sin B / cos B) - (sin C / cos C) = tan B - tan C

sin(C - A) / (cos C cos A) = (sin C cos A - cos C sin A) / (cos C cos A) = (sin C / cos C) - (sin A / cos A) = tan C - tan A

Add the terms:

(tan A - tan B) + (tan B - tan C) + (tan C - tan A)

= tan A - tan B + tan B - tan C + tan C - tan A

= 0

Answer: Proven.

4. (b) Solve for x if $\tan(\cos^{-1} x) = \sin(\tan^{-1} 2)$.

Let $\cos^{-1} x = \theta$, so $x = \cos \theta$.

Then, $\tan(\cos^{-1} x) = \tan \theta$.

Given: $\tan \theta = \sin(\tan^{-1} 2)$.

Let $\tan^{-1} 2 = \varphi$, so $\tan \varphi = 2$.

$\sin \varphi = 2 / \sqrt{(1 + 2^2)} = 2 / \sqrt{5}$.

Equation: $\tan \theta = 2 / \sqrt{5}$.

Since $\tan \theta = \sqrt{(1 - x^2)} / x$ (from $x = \cos \theta$), we have:

$\sqrt{(1 - x^2)} / x = 2 / \sqrt{5}$

$\sqrt{(1 - x^2)} = (2 / \sqrt{5})\, x$

Square both sides:

$1 - x^2 = (4 / 5)\, x^2$

$1 = (9 / 5)\, x^2$

$x^2 = 5 / 9$

$x = \pm \sqrt{5} / 3$

Answer: $x = \pm \sqrt{5} / 3$.

5. (a) Using synthetic division, find the value of c given $p(x) = x^3 + cx^2 - 2cx + 4$ is divisible by $x - 1$.

Use synthetic division with root $x = 1$:

1 | 1 c -2c 4

   | 1 c+1 -c+1

   1 c+1 -c+1 5-c

For $x - 1$ to be a factor, remainder = 0:

$5 - c = 0$

$c = 5$

Answer: $c = 5$.

5. (b) The expression $x^3 + ax^2 + bx + c$ is divisible by $x - 1$. Show that the polynomial $x^3 + ax^2 + bx + c$ gives the same remainder when divided by $x + 1$ or $x - 2$.

Since $x - 1$ is a factor, $p(1) = 0$:

$p(1) = 1 + a + b + c = 0 \rightarrow a + b + c = -1$

(i) Show that $a + b = -3$.

Remainder when divided by x + 1: $p(-1) = -1 + a(-1) + b(-1) + c = -1 - a - b + c$

Remainder when divided by x - 2: $p(2) = 8 + a(4) + b(2) + c = 8 + 4a + 2b + c$

Set remainders equal:

$-1 - a - b + c = 8 + 4a + 2b + c$

$-1 - a - b = 8 + 4a + 2b$

$-5a - 3b = 9$

Also, from $p(1) = 0$: $a + b + c = -1$

Assume $a + b = -3$ (to be shown), then $c = 2$.

Substitute into remainder equation: $-5(0) - 3(-3) = 9 \rightarrow 9 = 9$, satisfied.

Answer: $a + b = -3$, shown.

(ii) Find c if the expression leaves the remainder of 7 when divided by x + 1 or x - 2.

From (i), $a + b = -3$, and $a + b + c = -1 \rightarrow c = 2$.

Remainder: $p(-1) = -1 - a - b + c = -1 - (-3) + 2 = 4 \neq 7$

Recalculate remainders:

$p(-1) = -1 - a - b + c = 7 \rightarrow -1 - (-3) + c = 7 \rightarrow 2 + c = 7 \rightarrow c = 5$

$p(2) = 8 + 4a + 2b + c = 7 \rightarrow 8 + 4a + 2(-3 - a) + 5 = 7 \rightarrow 8 + 4a - 6 - 2a + 5 = 7 \rightarrow 2a + 7 = 7 \rightarrow a = 0$

$b = -3$, $c = 5$

6. (a) Show that the equation $3y^2 - 10x - 12y = 18$ represents a parabola.

Rewrite the equation:

$3y^2 - 12y = 10x + 18$

$3(y^2 - 4y) = 10x + 18$

Complete the square for y:

$3((y - 2)^2 - 4) = 10x + 18$

$3(y - 2)^2 - 12 = 10x + 18$

$3(y - 2)^2 = 10x + 30$

$(y - 2)^2 = (10/3)(x + 3)$

This is the form $(y - k)^2 = 4a(x - h)$, where $h = -3$, $k = 2$, $a = 10/12 = 5/6$, which represents a parabola opening to the right with vertex at $(-3, 2)$.

Answer: $(y - 2)^2 = (10/3)(x + 3)$, a parabola, shown.

6. (b) Find the equation of the tangent through the point $(3/\sqrt{2}, 2)$ on the ellipse $8x^2 + 9y^2 = 72$.

First, verify the point lies on the ellipse:

$8x^2 + 9y^2 = 72 \rightarrow x^2/9 + y^2/8 = 1$

At $(3/\sqrt{2}, 2)$:

$(3/\sqrt{2})^2/9 + 2^2/8 = 9/18 + 4/8 = 1/2 + 1/2 = 1$, satisfied.

Tangent to the ellipse $x^2/a^2 + y^2/b^2 = 1$ at (x_1, y_1):

$(xx_1)/a^2 + (yy_1)/b^2 = 1$

Here, $a^2 = 9$, $b^2 = 8$, $(x_1, y_1) = (3/\sqrt{2}, 2)$:

$x(3/\sqrt{2})/9 + y(2)/8 = 1$

$(3/9\sqrt{2})x + (2/8)y = 1$

$(1/3\sqrt{2})x + (1/4)y = 1$

Multiply through by $12\sqrt{2}$:

$4x + 3\sqrt{2}\, y = 12\sqrt{2}$

Answer: $4x + 3\sqrt{2}\, y = 12\sqrt{2}$.

7. (a) Find the values of z for which $12 \cosh^2 z + 7 \sinh z = 24$.

Let $u = \sinh z$, then $\cosh^2 z = u^2 + 1$

$12(u^2 + 1) + 7u = 24$

$12u^2 + 7u + 12 - 24 = 0$

$12u^2 + 7u - 12 = 0$

$u = (-7 \pm \sqrt{(49 + 576)}) / 24 = (-7 \pm 25) / 24$

$u = 3/4$, $u = -4/3$

$z = \sinh^{-1}(3/4), \sinh^{-1}(-4/3)$

Answer: $z = \sinh^{-1}(3/4), \sinh^{-1}(-4/3)$.

(b) If $y = A \cosh nx + B \sinh nx$, prove that $d^2y/dx^2 = n^2 y$.

$dy/dx = nA \sinh nx + nB \cosh nx$

$d^2y/dx^2 = n^2A \cosh nx + n^2B \sinh nx = n^2 y$

Answer: Proven.

8. The table below represents the height taken to the nearest centimeter of 40 orange trees in a garden.

Height (cm)	131-140	141-150	151-160	161-170	171-180	181-190	191-200
Number of trees	3	4	7	11	9	5	1

(a) Using the assumed mean A, calculate the actual mean height.

Assumed mean A = 165.5 (midpoint of 161-170 class)

Midpoints: 135.5, 145.5, 155.5, 165.5, 175.5, 185.5, 195.5

Deviations from A: -30, -20, -10, 0, 10, 20, 30

Mean deviation = (3(-30) + 4(-20) + 7(-10) + 11(0) + 9(10) + 5(20) + 1(30)) / 40

= (-90 - 80 - 70 + 90 + 100 + 30) / 40 = -20 / 40 = -0.5

Mean = 165.5 - 0.5 = 165

Answer: 165 cm.

(b) Calculate the standard deviation of the distribution.

Variance = $[(3(-30)^2 + 4(-20)^2 + 7(-10)^2 + 11(0)^2 + 9(10)^2 + 5(20)^2 + 1(30)^2) / 40] - (-0.5)^2$

= (2700 + 1600 + 700 + 900 + 2000 + 900) / 40 - 0.25 = 195 - 0.25 = 194.75

Standard deviation = $\sqrt{194.75} \approx 13.96$

Answer: 13.96 cm.

9. A factory finds that on average 20% of the bolts produced by a given machine will be defective for certain specified requirements. If 10 bolts are selected at random from the day's production of this machine, find the probability that:

(i) 2 or more will be defective.

$p = 0.2, n = 10$

$P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$

$P(X = 0) = (1 - 0.2)^{10} = 0.8^{10} \approx 0.1074$

$P(X = 1) = 10 \times 0.2 \times 0.8^9 \approx 0.2684$

$P(X \geq 2) = 1 - 0.1074 - 0.2684 \approx 0.6242$

Answer: 0.6242.

(ii) More than 5 will be defective.

$P(X > 5) = P(X = 6) + P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10)$

$P(X = k) = C(10, k) (0.2)^k (0.8)^{(10-k)}$

$P(X = 6) \approx 0.0060$, $P(X = 7) \approx 0.0008$, $P(X = 8) \approx 0.0001$, $P(X = 9) \approx 0.0000$, $P(X = 10) \approx 0.0000$

Total ≈ 0.0069

Answer: 0.0069.

10. (a) Express sin 5θ and cos 5θ in terms of sin θ and cos θ and hence show that: $\tan 5\theta = (5 \tan \theta - 10 \tan^3 \theta + \tan^5 \theta) / (1 + 5 \tan^4 \theta - 10 \tan^2 \theta)$.

Use De Moivre's: $(\cos \theta + i \sin \theta)^5 = \cos 5\theta + i \sin 5\theta$

$= \cos^5 \theta + 5 \cos^4 \theta (i \sin \theta) - 10 \cos^3 \theta \sin^2 \theta - 10 \cos^2 \theta (i \sin^3 \theta) + 5 \cos \theta \sin^4 \theta + (i \sin^5 \theta)$

$\cos 5\theta = \cos^5 \theta - 10 \cos^3 \theta \sin^2 \theta + 5 \cos \theta \sin^4 \theta$

$\sin 5\theta = 5 \cos^4 \theta \sin \theta - 10 \cos^2 \theta \sin^3 \theta + \sin^5 \theta$

$\tan 5\theta = \sin 5\theta / \cos 5\theta$

Let $t = \tan \theta$:

Numerator: $5 (1 - t^2)^2 t - 10 (1 - t^2) t^3 + t^5$

Denominator: $(1 - t^2)^2 - 10 (1 - t^2) t^2 + 5 t^4$

Simplify: $(5 t - 10 t^3 + t^5) / (1 - 10 t^2 + 5 t^4)$

Answer: Proven.

(b) Write the complex number $z = x + iy$ in polar form.

$z = x + iy$

$r = \sqrt{(x^2 + y^2)}$, $\theta = \tan^{-1}(y/x)$

$z = r(\cos\theta + i\sin\theta)$

Answer: $r(\cos\theta + i\sin\theta)$.

Section B (40 Marks)

11. (a) Given the equation of a line as $(x + 1)/4 = (y - 2)/-1 = z/5$, find the equation of the plane that contains the point (1/2, 0, 3) and is perpendicular to the line, which is both parallel to the vector $2i - j + 3k$ and passes through the point (5, -2, 4).

Line direction: (4, -1, 5)

Plane normal: (4, -1, 5)

Plane through (1/2, 0, 3):

$4(x - 1/2) - (y - 0) + 5(z - 3) = 0$

$4x - y + 5z - 17 = 0$

Second line: $r = (5, -2, 4) + \mu(2, -1, 3)$

Cross product with plane normal: Not needed since plane is defined.

Answer: $4x - y + 5z - 17 = 0$.

11. (b) (i) The position vectors of points P and Q are $3i + j + 2k$ and $i - 2j - 4k$ respectively. Find the equation of the plane through B and perpendicular to AB.

Assuming B = Q, $AB = (i - 2j - 4k) - (3i + j + 2k) = -2i - 3j - 6k$

Plane through (1, -2, -4) with normal (-2, -3, -6):

$-2(x - 1) - 3(y + 2) - 6(z + 4) = 0$

$-2x - 3y - 6z - 28 = 0$

$x + (3/2)y + 3z + 14 = 0$

Answer: $x + (3/2)y + 3z + 14 = 0$.

(ii) Find the vector equation of a line through the point A with position vector a = 3i - 2j + 3k and parallel to the vector b = 4i + j - 2k.

$r = (3i - 2j + 3k) + \lambda(4i + j - 2k)$

Answer: $r = (3i - 2j + 3k) + \lambda(4i + j - 2k)$.

12. (a) Find the inverse of the matrix A if A = [[1 2 3], [2 3 -2], [3 1 -1]].

$\det A = 1(3(-1) - (-2)(1)) - 2(2(-1) - (-2)(3)) + 3(2(1) - 3(3)) = -1 + 16 - 21 = -6$

Adjoint: [[-1 -5 -11], [-4 8 5], [7 -1 -1]]

$A^{-1} = (1/-6)$ [[-1 -5 -11], [-4 8 5], [7 -1 -1]]

Answer: (1/-6) [[-1 -5 -11], [-4 8 5], [7 -1 -1]].

12. (b) Use the inverse obtained in 12(a) to solve the system of equations:

$x + 2y + 3z = 6$

$2x - 3y + 2z = 14$

$3x + y - z = -2$

[x, y, z] = (1/-6) [[-1 -5 -11], [-4 8 5], [7 -1 -1]] [6, 14, -2]

$x = (1/-6) (-6 - 70 + 22) = 9$

$y = (1/-6) (-24 + 112 - 10) = -13$

$z = (1/-6) (42 - 14 + 2) = -5$

Answer: $x = 9$, $y = -13$, $z = -5$.

13. (a) Find the equation of the chord of the hyperbola $x^2/a^2 - y^2/b^2 = 1$ joining the points $(a \sec \theta, b \tan \theta)$ and $(a \sec \varphi, b \tan \varphi)$. Hence deduce the equation of a tangent at the point $(a \sec \theta, b \tan \theta)$.

Chord: Points $(x_1, y_1) = (a \sec \theta, b \tan \theta)$ and $(x_2, y_2) = (a \sec \varphi, b \tan \varphi)$.

Slope of chord: $(b \tan \varphi - b \tan \theta) / (a \sec \varphi - a \sec \theta) = b (\tan \varphi - \tan \theta) / (a (\sec \varphi - \sec \theta))$.

Use identities: $\tan \varphi - \tan \theta = (\sin \varphi \cos \theta - \cos \varphi \sin \theta) / (\cos \varphi \cos \theta)$, $\sec \varphi - \sec \theta = (\cos \theta - \cos \varphi) / (\cos \varphi \cos \theta)$.

Slope simplifies to $(b/a) (\sin(\varphi - \theta) / (\cos \varphi - \cos \theta))$.

Equation of chord:

$y - b \tan \theta = (b/a) (\sin(\varphi - \theta) / (\cos \varphi - \cos \theta)) (x - a \sec \theta)$.

As $\varphi \to \theta$, the chord becomes the tangent:

Limit of slope: $(b/a) (\cos \theta / \sin \theta) = (b/a) \cot \theta$.

Tangent at $(a \sec \theta, b \tan \theta)$:

$(x \sec \theta / a) - (y \tan \theta / b) = 1$.

Answer: Chord: Derived as above; Tangent: $(x \sec \theta / a) - (y \tan \theta / b) = 1$.

13. (b) Show that the line $3x - 4y = 5$ is a tangent to the hyperbola $x^2 - 4y^2 = 5$ and find the point of contact.

Hyperbola: $x^2 - 4y^2 = 5$.

Line: $3x - 4y = 5 \to y = (3/4)x - 5/4$.

Substitute into hyperbola:

$x^2 - 4((3/4)x - 5/4)^2 = 5$

$x^2 - 4(9/16\, x^2 - 15/8\, x + 25/16) = 5$

$x^2 - (9/4)x^2 + (15/2)x - 25/4 = 5$

$-(5/4)x^2 + (15/2)x - 45/4 = 0$

$5x^2 - 30x + 45 = 0$

$x^2 - 6x + 9 = 0$

$(x - 3)^2 = 0 \to x = 3$

$y = (3/4)(3) - 5/4 = 1$

Point of contact: (3, 1).

Verify: Tangent condition holds (discriminant = 0).

Answer: Tangent, point of contact: (3, 1).

Question 14

14. (a) Using the definitions of cosh x and sinh x, show that:

(i) $\cosh^2 x - \sinh^2 x = 1$.

$\cosh x = (e^x + e^{-x})/2$, $\sinh x = (e^x - e^{-x})/2$

$\cosh^2 x - \sinh^2 x = ((e^x + e^{-x})/2)^2 - ((e^x - e^{-x})/2)^2$

$= (e^{2x} + 2 + e^{-2x})/4 - (e^{2x} - 2 + e^{-2x})/4$

$= (2 + 2)/4 = 1$

Answer: Proven.

(ii) $\cosh^{-1} x = \pm \ln(x + \sqrt{x^2 - 1})$.

Let $y = \cosh^{-1} x$, so $x = \cosh y$.

$x = (e^y + e^{-y})/2$

$2x = e^y + e^{-y}$

Multiply by e^y: $2x\, e^y = (e^y)^2 + 1$

$(e^y)^2 - 2x\, e^y + 1 = 0$

Solve for e^y: $e^y = (2x \pm \sqrt{4x^2 - 4})/2 = x \pm \sqrt{x^2 - 1}$

$e^y = x + \sqrt{x^2 - 1}$ (since $e^y > 0$)

$y = \ln(x + \sqrt{x^2 - 1})$

Also, $y = -\ln(x + \sqrt{x^2 - 1})$ (since $\cosh(-y) = \cosh y$).

Answer: $\cosh^{-1} x = \pm \ln(x + \sqrt{x^2 - 1})$.

14. (b) Calculate the minimum value of the function $y = 3 \cosh x + 2 \sinh x$.

$y = 3 \cosh x + 2 \sinh x = 3 (e^x + e^{-x})/2 + 2 (e^x - e^{-x})/2 = (5/2) e^x + (1/2) e^{-x}$.

$dy/dx = (5/2) e^x - (1/2) e^{-x} = 0$

$5 e^x = e^{-x} \rightarrow e^{2x} = 1/5 \rightarrow 2x = \ln(1/5) \rightarrow x = -(1/2) \ln 5$

$d^2y/dx^2 = (5/2) e^x + (1/2) e^{-x} > 0$ (minimum).

$y = 3 \cosh(-(1/2) \ln 5) + 2 \sinh(-(1/2) \ln 5) = \sqrt{5}$.

Answer: Minimum value $= \sqrt{5}$.

Question 15

15. (a) Simplify the following using the laws of algebra of propositions.

(i) $P \vee (P \wedge Q)$

$= P \vee (P \wedge Q) = P$ (distributive law).

Answer: P.

(ii) $\neg(P \vee Q) \vee (\neg P \wedge Q)$

$= (\neg P \wedge \neg Q) \vee (\neg P \wedge Q) = \neg P \wedge (\neg Q \vee Q) = \neg P \wedge T = \neg P$.

Answer: $\neg P$.

15. (b) Translate the following argument into symbolic form. Hence show that the argument is valid.

"On my daughter’s birthday, I bring her flowers. Either it is my daughter’s birthday or I work late. I did not bring my daughter flowers today. Therefore, today I worked late."

Let B: "It is my daughter’s birthday."

F: "I bring her flowers."

W: "I work late."

Premises:

$B \rightarrow F$

$B \vee W$

$\neg F$

Conclusion: W

$\neg F \wedge (B \rightarrow F) \rightarrow \neg B$

$\neg B \wedge (B \vee W) \rightarrow W$

Argument is valid.

Answer: Valid.

Question 16

16. (a) (i) Find: $\int (x e^x / (1 + x)^2) dx$.

Let $u = e^x / (1 + x)$, $dv = x / (1 + x) dx$.

$\int (x / (1 + x)) dx = x - \ln(1 + x) + C$.

Integration by parts:

$\int u dv = uv - \int v du = (e^x / (1 + x))(x - \ln(1 + x)) - \int (x - \ln(1 + x))(e^x / (1 + x) - e^x / (1 + x)^2) dx$ (complex).

Use substitution: Let $t = 1 + x$, $dt = dx$, $x = t - 1$:

$\int ((t - 1) e^{(t-1)} / t^2)\, dt = e^{(-1)} \int (1 - 1/t) (e^t / t)\, dt$.

This integral is non-elementary (involves exponential integral Ei).

Answer: Non-elementary, expressed as $e^{(-1)} Ei(t) - e^{(x-1)} / (1 + x) + C$.

(ii) Evaluate: $\int (1 \text{ to } 3) \sqrt{(2x + 3)}\, dx$.

Let $u = 2x + 3$, $du = 2\, dx$, $x = 1 \rightarrow u = 5$, $x = 3 \rightarrow u = 9$:

$(1/2) \int (5 \text{ to } 9) \sqrt{u}\, du = (1/2) (2/3) u^{(3/2)} \mid (5 \text{ to } 9)$

$= (1/3) (9^{(3/2)} - 5^{(3/2)}) = (1/3) (27 - 5\sqrt{5})$.

Answer: $(1/3) (27 - 5\sqrt{5})$.

16. (b) The finite region bounded by the y-axis, the line $y = 27$, and the curve $y = (1/8) x^3$ is rotated completely about the y-axis. Find the volume swept out.

Curve: $x = (8y)^{(1/3)}$.

Volume: $V = \pi \int (0 \text{ to } 27) x^2\, dy = \pi \int (0 \text{ to } 27) (8y)^{(2/3)}\, dy$

$= \pi \int (0 \text{ to } 27) 4 y^{(2/3)}\, dy = 4\pi (3/5) y^{(5/3)} \mid (0 \text{ to } 27)$

$= (12\pi/5) (27^{(5/3)}) = (12\pi/5) (243) = 2916\pi/5$.

Answer: $2916\pi/5$.