THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2

ADVANCED MATHEMATICS 2
(For Both School and Private Candidates)

Time: 3 Hours *Wednesday, March 16, 2005 p.m.*

Instructions

1. This paper consists of sections A and B.
2. Answer ***all*** questions in section A and ***four (4)*** questions from section B.
3. All necessary steps in answering each question must be shown clearly.
4. Mathematical tables, mathematical formulae, slide rules and non-programmable pocket calculators may be used.
5. Cellular phones are ***not*** allowed in the examination room.
6. Write your ***Examination Number*** on every page of your answer booklet(s).

PPU This paper consists of 4 printed pages.

SECTION A (60 marks)

Answer **all** questions in this section showing **all** necessary steps and answers.

1. (a) Use logarithms to evaluate:

$$\sqrt[3]{e^2\sqrt{\ell n3-\cos 300^0}}$$ (3 marks)

(b) Using a non-programmable scientific calculator, evaluate:

$$\sqrt{\frac{e^{\log 3}+\sqrt{\log\sqrt{5}}}{\ell n23}}$$

Correct to 6 decimal points. (3 marks)

2. (a) Let p ≡ He goes abroad.

q ≡ He has a passport.

Write down verbal sentences representing the statements:

(i) $p \leftrightarrow \sim q$

(ii) $\sim p \leftrightarrow q$ (2 marks)

(b) Construct a truth table for the proposition:

$$[(\sim p \wedge q) \wedge \sim q] \leftrightarrow [q \rightarrow \sim(\sim p \wedge q)].$$ (4 marks)

3. (a) Find an equation of a circle through the points A (3, −5) and B (2, 6) and the line segment $\overline{AB}$ as its diameter. (2 marks)

(b) Compute the distance of the tangent line from the point P (9, 8) to the circle which passes through the points A (5, 7), B (− 2, 6) and C (6, 0). (4 marks)

4. (a) Show, by synthetic division that $x = -2$ is a root of a polynomial $P(x) = 3x^4 - 8x^3 + 31x^2 + 72x - 92$. (2 marks)

(b) If m and n are the roots of the equation $3x^2 - 7x + 8 = 0$, form quadratic equations whose roots are:

(i) m^2n and n^2m

(ii) m^2 and n^2. (4 marks)

5. (a) For small angles θ, the approximate values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ are θ, $1-\frac{1}{2}\theta^2$ and θ respectively. Obtain an expression involving θ that $\frac{7\tan\theta - 20\cos\theta + 21}{1+\sin 2\theta}$ approximates to, for small values of θ. (3 marks)

(b) Calculate the size of angle A in a triangle for which: a = 10, b = 12 and c = 9. (3 marks)

6. (a) Show that the shortest distance of the tangent line from the point (10, 10) to the curve $4x^2+9y^2=25$ at (2, 1) is $\sqrt{145}$. **(3 marks)**

(b) Show that the equation:
$16x^2+25y^2-64x+150y-111=0$ is an equation of an ellipse. **(3 marks)**

7. (a) By using logarithmic form of hyperbolic functions, evaluate $\text{Sinh}^{-1}(3)$. **(2 marks)**

(b) Show that a condition for the equation:
$a\ \text{Cosh}\,x+\text{Sinh}x+b=0$, where 'a' and 'b' are real constants, to have real roots is $a^2-b^2\leq 1$.
(4 marks)

8. (a) The probability that a person supports a certain political party is 0.6. Find the probability that in a randomly selected sample of 8 voters there are exactly 3 persons who support a certain political party. **(2½ marks)**

(b) If the probability that it is a fine day is 0.4, find the expected number of fine days in a week, and the standard deviation. **(3½ marks)**

9. (a) Events A and B are such that $P(A)=\frac{1}{3}$ and $P(A\cap B)=\frac{1}{12}$. If A and B are independent events, find (i) P(B)
(ii) $P(A\cup B)$ **(3½ marks)**

(b) When a die is thrown, an odd number occurs. What is the probability that the number is prime? **(2½ marks)**

10. (a) Solve for z and w in the following system of simultaneous equations.
$$\begin{cases} iz-w=2i \\ iz+iw=1 \end{cases}$$
(3 marks)

(b) Describe the locus of a complex variable z such that $|z-2+3i|\geq 4$ **(3 marks)**

SECTION B (40 marks)

Answer **four (4)** questions from this section.

11. (a) Find the shortest distance between the two lines defined by the following equations:
$\underline{r}=(1-t)\underline{i}+(t-2)\underline{j}+(3-2t)\underline{k}$ and
$\underline{r}=(s+1)\underline{i}+(2s-1)\underline{j}-(2s+1)\underline{k}$ **(5 marks)**

(b) Find the cartesian equation of a plane through the points A (−1, 1, 1) B (2, 1, 0) and C (−2, 0, 3). **(5 marks)**

12. Find A^{-1} if $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{pmatrix}$, hence use the result to solve the following system of equations:

$$\begin{cases} x+2y-3z=-4 \\ 2x+3y+2z=2 \\ 3x-3y-4z=11 \end{cases}$$

(10 marks)

13. (a) (i) Change the equation $(x^2+y^2)^2 = x^2-y^2$ in polar form.

(ii) Express $r = 4\sin\theta\cos\theta$ in cartesian form. **(5 marks)**

(b) Show that $y = mx + c$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ when $c^2 = a^2m^2 - b^2$ **(5 marks)**

14. (a) Evaluate $\int_0^2 \frac{dx}{\sqrt{4x^2+8x+13}}$ **(5 marks)**

(b) Show that $\int_1^8 \frac{dx}{\sqrt{x^2-2x+2}} = \int_1^2 \frac{3dx}{\sqrt{x^2-2x+2}}$ **(5 marks)**

15. (a) Below is the truth table containing truth values of two propositions labelled (m) and (n).

P	q	(m)	(n)
T	T	F	F
T	F	T	F
F	T	T	F
F	F	F	T

(i) Using basic conjunctions, find the propositions (m) and (n), expressing them in terms of connectives $\sim$ and $\rightarrow$ only.

(ii) Hence using a truth table show that (m) is a tautology and (n) is not.

(5½ marks)

(b) Check the validity of the argument: "If I like logic, I will study arguments. I will study arguments if and only if I have logical mind. I don't like logic. Therefore I will not study arguments."

(4½ marks)

16. (a) Evaluate the following integral:

$$\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx$$

(2½ marks)

(b) Use Taylor's theorem to obtain a series expansion for $\cos\left(x+\frac{\pi}{3}\right)$ stating terms up to and including that in x^3.

(3½ marks)

(c) Find the minimum value of $3\cosh x + 2\sinh x$.

(4 marks)