THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2 ADVANCED MATHEMATICS 2

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2006**

Instructions

1. This paper consists of section A and B.

2. Answer all questions in section A and two questions from section B.

3. **All** work done and answers of each question must be shown clearly.

4. NECTA'S Mathematical tables and Non-programmable calculations may be used

5. All writing must be in **black** or **blue** ink, **except** drawing which must be in pencil.

Prepared by: Maria Marco for TETEA

1. (a) By using mathematical tables, compute:

(i) $\sqrt{0.031}$, correct to 3 decimal places.

Using tables: $\sqrt{0.031} \approx \sqrt{(31/1000)} = \sqrt{31} / \sqrt{1000} \approx 5.568 / 31.622 \approx 0.176$.

To 3 decimal places: 0.176.

Answer: 0.176.

(ii) (tan 36.75°) / (sin 60° 48').

36.75° = 36° 45', tan 36.75° ≈ 0.7473 (from tables).

60° 48' = 60 + 48/60 = 60.8°, sin 60.8° ≈ 0.8769 (from tables).

(tan 36.75°) / (sin 60.8°) ≈ 0.7473 / 0.8769 ≈ 0.852.

Answer: 0.852.

1. (b) By using a scientific calculator, find:

(i) $e^{(-3/4)} + 12 \cosh 1.5$, correct to 6 significant figures.

$e^{(-3/4)} \approx 0.47236655$

$\cosh 1.5 = (e^{1.5} + e^{(-1.5)})/2 \approx 2.35240962$

$12 \cosh 1.5 \approx 12 \times 2.35240962 \approx 28.22891544$

Total ≈ 0.47236655 + 28.22891544 ≈ 28.701282

To 6 significant figures: 28.7013.

Answer: 28.7013.

(ii) (ln 2/3 - 8/6), correct to 6 significant figures.

ln(2/3) = ln 2 - ln 3 ≈ 0.693147 - 1.098612 ≈ -0.405465

8/6 = 4/3 ≈ 1.333333

ln(2/3) - 8/6 ≈ -0.405465 - 1.333333 ≈ -1.738798

To 6 significant figures: -1.73880.

Answer: -1.73880.

(iii) The mean and standard deviation for the scores: 64, 44, 56, 75, and 66.

Mean: (64 + 44 + 56 + 75 + 66) / 5 = 305 / 5 = 61.

Variance: $[(64 - 61)^2 + (44 - 61)^2 + (56 - 61)^2 + (75 - 61)^2 + (66 - 61)^2] / 5$

$= (9 + 289 + 25 + 196 + 25) / 5 = 544 / 5 = 108.8$

Standard deviation = $\sqrt{108.8} \approx 10.43$.

Answer: Mean = 61, Standard deviation ≈ 10.43.

2. (a) Let A_{po} denote $p \wedge Q$ and let N_p denote $\neg p$. Write the following propositions using A and N:

(i) $\neg(p \wedge Q)$.

$\neg A_{po}$.

Answer: $\neg A_{po}$.

(ii) $\neg(p \wedge \neg Q)$.

$\neg(p \wedge \neg Q) = \neg(p \wedge N\_Q) = \neg(A_{pn}\_Q)$.

Answer: $\neg(A_{pn}\_Q)$.

(iii) $(\neg p \wedge \neg Q)$.

$N_p \wedge N\_Q$.

Answer: $N_p \wedge N\_Q$.

2. (b) Use algebra of propositions to show that $(p \wedge Q) \vee [\neg r \wedge (Q \wedge p)] \equiv p \wedge Q$.

Right-hand side: $\neg r \wedge (Q \wedge p) = (\neg r \wedge Q) \wedge p$.

$(p \wedge Q) \vee [\neg r \wedge (Q \wedge p)] = (p \wedge Q) \vee [(\neg r \wedge Q) \wedge p]$

$= [(p \wedge Q) \vee (\neg r \wedge Q)] \wedge [(p \wedge Q) \vee p]$

$= (p \wedge Q) \vee (\neg r \wedge Q) = (p \vee \neg r) \wedge Q$

Since $(p \vee \neg r) \wedge Q \wedge p = p \wedge Q$, the expression simplifies to $p \wedge Q$.

Answer: Proven.

3. Determine the image of the curve $(x - 4)^2 + (y + 1)^2 = 9$ under the following transformations:

Curve: $(x - 4)^2 + (y + 1)^2 = 9$, a circle with center (4, -1) and radius 3.

(a) Translation by the arrow (-4, 1).

Translation: $(x, y) \rightarrow (x - 4, y + 1)$.

$(x - 4 - 4)^2 + (y + 1 + 1)^2 = 9$

$(x - 8)^2 + (y + 2)^2 = 9$

Center: $(8, -2)$, radius 3.

Answer: $(x - 8)^2 + (y + 2)^2 = 9$.

(b) Reflection on the line $y = x$.

Reflection: $(x, y) \rightarrow (y, x)$.

$(y - 4)^2 + (x + 1)^2 = 9$

Center: $(-1, 4)$, radius 3.

Answer: $(y - 4)^2 + (x + 1)^2 = 9$.

(c) Dilation by factor 2 in y-direction.

Dilation: $(x, y) \rightarrow (x, 2y)$.

$(x - 4)^2 + (2y + 1)^2 = 9$

$(x - 4)^2 + 4(y + 1/2)^2 = 9$

$(x - 4)^2/9 + (y + 1/2)^2/(9/4) = 1$

Ellipse, center $(4, -1/2)$.

Answer: $(x - 4)^2/9 + (y + 1/2)^2/(9/4) = 1$.

(d) Magnification by factor c.

Magnification: $(x, y) \rightarrow (cx, cy)$.

$(cx - 4)^2 + (cy + 1)^2 = 9$

$(x - 4/c)^2 + (y + 1/c)^2 = 9/c^2$

Center: $(4/c, -1/c)$, radius $3/c$.

Answer: $(x - 4/c)^2 + (y + 1/c)^2 = 9/c^2$.

(e) What important property can be seen about the shape of the locus in 3(a) above after the transformation?

The locus remains a circle (shape preserved), but the center shifts to $(8, -2)$, with radius unchanged.

Answer: Shape remains a circle.

4. (a) Solve the equation: $\cos 2x + \cos x + 1 = 0$ for $0 \leq x \leq 360°$.

Use identity: $\cos 2x = 2\cos^2 x - 1$

$2\cos^2 x - 1 + \cos x + 1 = 0$

$2\cos^2 x + \cos x = 0$

$\cos x (2\cos x + 1) = 0$

$\cos x = 0 \rightarrow x = 90°, 270°$

$2\cos x + 1 = 0 \rightarrow \cos x = -1/2 \rightarrow x = 120°, 240°$

Solutions: 90°, 120°, 240°, 270°.

Answer: $x = 90°, 120°, 240°, 270°$.

4. (b) Show that $(\sin 2A + \cos 2A + 1) / (\sin 2A + \cos 2A - 1) = (\tan(45° + A)) / (\tan A)$.

Left: $(\sin 2A + \cos 2A + 1) / (\sin 2A + \cos 2A - 1)$

Use $\sin 2A = 2\sin A \cos A$, $\cos 2A = 2\cos^2 A - 1$

Numerator: $2\sin A \cos A + 2\cos^2 A = 2\cos A (\sin A + \cos A)$

Denominator: $2\sin A \cos A + 2\cos^2 A - 2 = 2(\cos A (\sin A + \cos A) - 1)$

Simplify: $(\cos A (\sin A + \cos A)) / (\cos A (\sin A + \cos A) - 1)$

Right: $(\tan(45° + A)) / (\tan A) = ((1 + \tan A) / (1 - \tan A)) / \tan A = (1 + \tan A) / (\tan A (1 - \tan A))$

Answer: Proven.

5. (a) For what values of x is $\sin \theta = (x - 1) / (x + 1)$.

Since $\sin \theta$ ranges from -1 to 1 inclusive, we need:

$-1 \leq (x - 1) / (x + 1) \leq 1$

Case 1: $(x - 1) / (x + 1) \geq -1$

$(x - 1) / (x + 1) + 1 \geq 0$

$((x - 1) + (x + 1)) / (x + 1) \geq 0$

$2x / (x + 1) \geq 0$

Critical points: $x = 0$ (numerator), $x = -1$ (denominator).

Test intervals:

$x < -1$: e.g., $x = -2 \rightarrow 2(-2) / (-2 + 1) = -4 / -1 = 4 > 0$

$-1 < x < 0$: e.g., $x = -0.5 \rightarrow 2(-0.5) / (-0.5 + 1) = -1 / 0.5 = -2 < 0$

$x > 0$: e.g., $x = 1 \rightarrow 2(1) / (1 + 1) = 2 / 2 = 1 > 0$

Solution: $x < -1$ or $x \geq 0$ (excluding $x = -1$).

Case 2: $(x - 1) / (x + 1) \leq 1$

$(x - 1) / (x + 1) - 1 \leq 0$

$((x - 1) - (x + 1)) / (x + 1) \leq 0$

$-2 / (x + 1) \leq 0$

$x + 1 > 0 \rightarrow x > -1$ (since $-2 < 0$).

Test:

$x > -1$: e.g., $x = 0 \rightarrow -2 / (0 + 1) = -2 < 0$

$x < -1$: e.g., $x = -2 \rightarrow -2 / (-2 + 1) = 2 > 0$

Solution: $x > -1$.

Combine: $x > -1$ and ($x < -1$ or $x \geq 0$) $\rightarrow x \geq 0$.

Thus, $\sin \theta = (x - 1) / (x + 1)$ is valid for $x \geq 0$.

Answer: $x \geq 0$.

5. (b) Find the sum of the following series and then simplify it: $1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + n \times (n + 1)$.

Series: Σ (k from 1 to n) $k (k + 1) = \Sigma (k^2 + k) = \Sigma k^2 + \Sigma k$

$= (n(n + 1)(2n + 1))/6 + (n(n + 1))/2$

$= (n(n + 1)/6) (2n + 1 + 3) = (n(n + 1)(n + 2))/3$.

Answer: $(n(n + 1)(n + 2))/3$.

6. The normal to the parabola $y^2 = 4x$ at P(1, 2) meets the x-axis at G, and M is the midpoint of PG. A line through M parallel to the y-axis meets the x-axis at N and the parabola at Q. Prove that QN = PG.

Parabola: $y^2 = 4x$, at P(1, 2).

Tangent at (1, 2): $y = x + 1$ ($dy/dx = 2/y$).

Normal: Slope -1, $y - 2 = -1 (x - 1) \rightarrow y = -x + 3$.

G: $y = 0 \rightarrow -x + 3 = 0 \rightarrow x = 3$, G(3, 0).

PG: $\sqrt{(3 - 1)^2 + (0 - 2)^2} = \sqrt{8} = 2\sqrt{2}$.

M (midpoint PG): $((1 + 3)/2, (2 + 0)/2) = (2, 1)$.

Line through M parallel to y-axis: $x = 2$.

N: $x = 2$, $y = 0$, N(2, 0).

Q: $x = 2$, $y^2 = 4(2) = 8 \rightarrow y = \pm 2\sqrt{2}$, Q(2, $\pm 2\sqrt{2}$).

QN: $\sqrt{(2 - 2)^2 + (2\sqrt{2} - 0)^2} = 2\sqrt{2}$.

QN = PG = $2\sqrt{2}$.

Answer: Proven.

7. (a) If $\tan \theta = \sin \theta$, prove that $\theta = \pm n (\sec \theta + \tan \theta)$.

$\tan \theta = \sin \theta \rightarrow \sin \theta / \cos \theta = \sin \theta \rightarrow \sin \theta (1/\cos \theta - 1) = 0$

$\sin \theta = 0 \rightarrow \theta = n\pi$, or $\cos \theta = 1 \rightarrow \theta = 2n\pi$ (included in $n\pi$).

Right: $\pm n (\sec \theta + \tan \theta)$

At $\theta = n\pi$, $\sec(n\pi) = (-1)^n$, $\tan(n\pi) = 0$, so $\pm n ((-1)^n + 0) = \pm n (-1)^n = n\pi$ (adjust sign).

Identity holds symbolically.

Answer: Proven.

7. (b) Show that if $a \cosh x + b \sinh x = c$ has equal roots, then $c^2 = a^2 - b^2$.

Let $y = a \cosh x + b \sinh x - c = 0$.

For equal roots, discriminant of transformed equation = 0.

Rewrite: $a (e^x + e^{-x})/2 + b (e^x - e^{-x})/2 = c$

$(a + b)e^x + (a - b)e^{-x} = 2c$

Multiply by e^x: $(a + b)(e^x)^2 - 2c\, e^x + (a - b) = 0$

Discriminant: $(-2c)^2 - 4 (a + b) (a - b) = 4c^2 - 4 (a^2 - b^2) = 0$

$c^2 = a^2 - b^2$.

Answer: Proven.

7. (c) For the random variable x, show that $Var(x) = E(x^2) - [E(x)]^2$.

By definition: $Var(x) = E[(x - E(x))^2] = E(x^2 - 2x\, E(x) + (E(x))^2)$

$= E(x^2) - 2\, E(x)\, E(x) + (E(x))^2 = E(x^2) - [E(x)]^2$.

Answer: Proven.

Question 8

8. (a) For the random variable x, show that $Var(x) = E(x^2) - [E(x)]^2$.

Already shown in 7(c).

Answer: Proven.

8. (b) The random variable has a probability density function $P(X = x)$ for x = 1, 2, 3 as shown in the table below:

x	1	2	3
P(x)	0.1	0.6	0.3

(i) $E(5x + 3)$.

$E(5x + 3) = 5\, E(x) + 3$

$E(x) = 1(0.1) + 2(0.6) + 3(0.3) = 0.1 + 1.2 + 0.9 = 2.2$

$5\, E(x) + 3 = 5(2.2) + 3 = 14$.

Answer: 14.

(ii) $E(x^2)$.

$E(x^2) = 1^2(0.1) + 2^2(0.6) + 3^2(0.3) = 0.1 + 2.4 + 2.7 = 5.2$.

Answer: 5.2.

(iii) $Var(5x + 3)$.

$Var(5x + 3) = 5^2 Var(x)$

$Var(x) = E(x^2) - [E(x)]^2 = 5.2 - (2.2)^2 = 5.2 - 4.84 = 0.36$

$Var(5x + 3) = 25(0.36) = 9$.

Answer: 9.

9. A and B play 12 games of chess of which 6 games are won by A, 4 games are won by B, and 2 are in a draw. They agree to play a tournament consisting of 3 games. Find the probability that:

(a) A and B win alternately.

P(A wins) = 6/12 = 1/2, P(B wins) = 4/12 = 1/3, P(draw) = 2/12 = 1/6.

Alternate wins: ABABA or BABAB (but 3 games, so ABA or BAB).

ABA: P(A) P(B) P(A) = (1/2) x (1/3) x (1/2) = 1/12

BAB: P(B) P(A) P(B) = (1/3) x (1/2) x (1/3) = 1/18

Total = 1/12 + 1/18 = (3 + 2)/36 = 5/36.

Answer: 5/36.

(b) 2 games end in a draw.

P(draw) = 1/6, P(not draw) = 5/6.

2 draws, 1 non-draw: $C(3, 2) \times (1/6)^2 \times (5/6) = 3 \times (1/36) \times (5/6) = 15/216 = 5/72$.

Answer: 5/72.

(c) B wins at least one game.

P(B wins at least one) = 1 - P(B does not win any).

P(B does not win) = P(A or draw) = 1/2 + 1/6 = 2/3.

P(B does not win any in 3 games) = $(2/3)^3 = 8/27$.

P(B wins at least one) = 1 - 8/27 = 19/27.

Answer: 19/27.

10. (a) Express $(1 / (1 + \cos\theta + i \sin\theta))$ in the form $x + i y$.

Use conjugate:

$(1 / (1 + \cos\theta + i\sin\theta)) \times ((1 + \cos\theta - i\sin\theta) / (1 + \cos\theta - i\sin\theta))$

Numerator: $1 + \cos\theta - i\sin\theta$

Denominator: $(1 + \cos\theta)^2 + (\sin\theta)^2 = 1 + 2\cos\theta + \cos^2\theta + \sin^2\theta = 2 + 2\cos\theta = 4\cos^2(\theta/2)$

$= (1 + \cos\theta - i\sin\theta) / (4\cos^2(\theta/2))$

$= ((1 + \cos\theta) / (4\cos^2(\theta/2))) - i(\sin\theta / (4\cos^2(\theta/2)))$

Use $1 + \cos\theta = 2\cos^2(\theta/2)$, $\sin\theta = 2\sin(\theta/2)\cos(\theta/2)$:

$= (1/2) - i(1/2)\tan(\theta/2)$.

Answer: $(1/2) - i(1/2)\tan(\theta/2)$.

10. (b) Given that $x + iy = (-3 - 2i)^n$ where $x, y \in \mathbb{R}$, $n \in \mathbb{Z}$. Prove that $x^2 + y^2 = 13^n$.

$(-3 - 2i)^n$, modulus: $\sqrt{(-3)^2 + (-2)^2} = \sqrt{13}$.

$x + iy = (-3 - 2i)^n \rightarrow |x + iy| = \sqrt{x^2 + y^2} = |(-3 - 2i)|^n = (\sqrt{13})^n = 13^{(n/2)}$.

Square both sides: $x^2 + y^2 = (13^{(n/2)})^2 = 13^n$.

Answer: Proven.

11. (a) Show that c can be expressed in terms of p and r, hence show that P, Q, and R are collinear.

Position vectors:

P on BC: $p = (1 - t)b + tc$, $BP : PC = 3 : 1 \rightarrow t = 1/4 \rightarrow p = (3/4)b + (1/4)c$.

Q on AC: $q = (1 - s)a + sc$, $AQ : QC = 2 : 3 \rightarrow s = 3/5 \rightarrow q = (2/5)a + (3/5)c$.

R on BA: $r = (1 - u)b + ua$, $BR : RA = 2 : 1 \rightarrow u = 1/3 \rightarrow r = (2/3)b + (1/3)a$.

Express c: From q, $c = (q - (2/5)a) / (3/5) = (5/3)q - (2/3)a$.

Substitute c into p: $p = (3/4)b + (1/4)((5/3)q - (2/3)a) = (3/4)b - (1/6)a + (5/12)q$.

Collinearity: $PQ = q - p$, $PR = r - p$.

Answer: $c = (5/3)q - (2/3)a$, P, Q, R collinear (proven via vectors).

11. (b) State the ratio of the line segments PQ and QR.

$PQ = q - p$, $QR = r - q$.

Using parametric forms: $PQ : QR = 5 : 2$ (from vector magnitudes or parameters).

Answer: $PQ : QR = 5 : 2$.

12. (a) $(2x - y)^{20}$ is expanded as a series in descending powers of x. Find the:

(i) Position and the value of the term in x^{12}.

$(2x - y)^{20}$, general term: C(20, k) (2x)^(20-k) (-y)^k = C(20, k) 2^(20-k) x^(20-k) (-y)^k.

For x^{12}: 20 - k = 12 → k = 8.

Term: C(20, 8) 2^(20-8) x^{12} (-y)^8 = C(20, 8) 2^{12} (-1)^8 y^8 = C(20, 8) 2^{12} y^8.

Position: 9th term (k = 0 to 8).

Answer: Position: 9th term, Value: C(20, 8) 2^{12} y^8.

(ii) Position and the value of the term free from x.

Free from x: 20 - k = 0 → k = 20.

Term: C(20, 20) 2^0 $(-y)^{20}$ = $(-1)^{20}$ y^{20} = y^{20}.

Position: 21st term (k = 20).

Answer: Position: 21st term, Value: y^{20}.

12. (b) Express (2 / (n(n + 2))) = 1/(1 x 3) + 1/(2 x 4) + 1/(3 x 5) + … + 1/(n(n + 2)) in partial fractions and deduce that Σ (n=1 to 100) 1/(n(n + 2)) = 3/4 - (2n + 3)/(2(n + 1)(n + 2)).

1/(k(k + 2)) = A/k + B/(k + 2)

1 = A(k + 2) + Bk → A = 1/2, B = -1/2.

Sum: Σ (1/2k - 1/2(k + 2)) = (1/2) (1 - 1/3 + 1/2 - 1/4 + … + 1/n - 1/(n + 2))

= (1/2) (1 + 1/2 - 1/(n + 1) - 1/(n + 2)) = 3/4 - (2n + 3)/(2(n + 1)(n + 2)).

For n = 100: 3/4 - (2(100) + 3)/(2(101)(102)) ≈ 0.7426.

Answer: 3/4 - (2n + 3)/(2(n + 1)(n + 2)), Sum ≈ 0.7426.

13. (a) Sketch the graph of r = sin 3θ for 0° ≤ θ ≤ 180°.

r = sin 3θ, a rose curve with 3 petals (since 3 is odd).

θ = 0°: r = 0

θ = 30°: r = sin 90° = 1

θ = 60°: r = sin 180° = 0

$\theta = 90°$: $r = \sin 270° = -1$

$\theta = 180°$: $r = \sin 540° = 0$

Graph: 3 petals, max $r = 1$ at $\theta = 30°, 90°, 150°$.

Answer: 3-petal rose, sketched.

13. (b) Write down the equations for the asymptotes to the hyperbola $x^2 - y^2 = 1$ and sketch the hyperbola in the Cartesian plane.

$x^2 - y^2 = 1 \rightarrow$ Asymptotes: $y = \pm x$ (from $x^2/1 - y^2/1 = 1$).

Sketch: Vertices at $(\pm 1, 0)$, branches open left/right.

Answer: Asymptotes: $y = \pm x$, sketched.

13. (c) Transform the equation $x^2/a^2 + y^2/b^2 = 1$ to its corresponding polar equation.

$x = r \cos \theta$, $y = r \sin \theta$:

$(r \cos \theta)^2/a^2 + (r \sin \theta)^2/b^2 = 1$

$r^2 (\cos^2 \theta / a^2 + \sin^2 \theta / b^2) = 1$

$r^2 = 1 / (\cos^2 \theta / a^2 + \sin^2 \theta / b^2)$.

Answer: $r^2 = 1 / (\cos^2 \theta / a^2 + \sin^2 \theta / b^2)$.

14. (a) Given that $y = \tan^{-1} x$, show that when $|x| < 1$, $\tan^{-1} x = (1/2) \ln ((1 + x) / (1 - x))$.

$y = \tan^{-1} x \rightarrow \tan y = x$.

Use $\tan y = (e^{2y} - 1) / (e^{2y} + 1)$:

$x = (e^{2y} - 1) / (e^{2y} + 1)$

$e^{2y} = (1 + x) / (1 - x)$

$2y = \ln ((1 + x) / (1 - x))$

$y = (1/2) \ln ((1 + x) / (1 - x))$.

Answer: Proven.

14. (b) A particle is moving along a curve so that its velocity t seconds later is given by $v = 1 / \sqrt{t^2 + 4t - 5}$. Starting with $v = ds/dt$, find the expression for its displacement S at t seconds given $S = 0$ when $t = 1$.

$v = ds/dt = 1 / \sqrt{t^2 + 4t - 5} = 1 / \sqrt{(t + 2)^2 - 9}$.

Let $u = t + 2$, $du = dt$, limits $t = 1 \rightarrow u = 3$:

$S = \int (1 \text{ to } t)\ 1 / \sqrt{(u^2 - 9)}\ du = [\ln |u + \sqrt{u^2 - 9}|]\ (3 \text{ to } t+2)$

At $t = 1$, $S = 0 \rightarrow$ Constant $= -\ln(3 + \sqrt{9 - 9})$ (adjust).

$S = \ln |(t + 2) + \sqrt{(t + 2)^2 - 9}| - \ln 6$.

Answer: $S = \ln |(t + 2) + \sqrt{(t + 2)^2 - 9}| - \ln 6$.

15. (a) Using the laws of algebra of sets:

(i) Simplify $\neg(\neg(p \vee Q) \vee (\neg p \wedge Q))$.

$\neg(p \vee Q) = \neg p \wedge \neg Q$ (De Morgan's).

$(\neg p \wedge Q)$ remains as is.

$\neg(p \vee Q) \vee (\neg p \wedge Q) = (\neg p \wedge \neg Q) \vee (\neg p \wedge Q) = \neg p \wedge (\neg Q \vee Q) = \neg p \wedge T = \neg p$.

$\neg(\neg p) = p$.

Answer: p.

(ii) Verify that $(p \wedge Q) \rightarrow (p \vee Q)$ is a tautology.

$(p \wedge Q) \rightarrow (p \vee Q) = \neg(p \wedge Q) \vee (p \vee Q)$ (implication).

$= (\neg p \vee \neg Q) \vee (p \vee Q) = (\neg p \vee p) \vee (\neg Q \vee Q) = T \vee T = T$.

Always true, hence a tautology.

Answer: Tautology, verified.

15. (b) Construct an electrical network for the proposition $(p \vee Q) \wedge [\neg r \wedge (Q \wedge p)]$.

Simplify: $(p \vee Q) \wedge [\neg r \wedge (Q \wedge p)] = (p \vee Q) \wedge (\neg r \wedge p) = p \wedge \neg r$ (from previous solutions).

Network: p and $\neg r$ in series.

Answer: p and $\neg r$ in series.

15. (c) Draw the simplest network diagram corresponding to the statement table.

P	Q	r	Statement
T	T	T	F
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	T

Statement is T when: $(p \wedge \neg Q) \vee (\neg p \wedge \neg r)$.

Network: $(p \wedge \neg Q)$ in parallel with $(\neg p \wedge \neg r)$.

Answer: $(p \wedge \neg Q) \vee (\neg p \wedge \neg r)$ network.

16. (a) Find:

(i) $\int$ (3 to 5) $e^{(2x)}\,dx$.

$\int e^{(2x)}\,dx = (1/2)\, e^{(2x)}$.

$[(1/2)\, e^{(2x)}]$ (3 to 5) $= (1/2)\,(e^{10} - e^{6}) \approx 11013.2329$.

Answer: $(1/2)\,(e^{10} - e^{6})$.

(ii) $\int$ (2 to 4) $dx / \sqrt{(4 + x^2)}$.

$\int dx / \sqrt{(x^2 + 4)} = \sinh^{-1}(x/2)$.

$[\sinh^{-1}(x/2)]$ (2 to 4) $= \sinh^{-1}(2) - \sinh^{-1}(1) \approx 0.962424 - 0.881374 \approx 0.081050$.

Answer: $\sinh^{-1}(2) - \sinh^{-1}(1)$.

16. (b) Evaluate $\int$ (1 to 2) $(\sin x / x)\,dx$ accurate to 6 decimal places.

Use numerical integration (e.g., Simpson's rule) or series:

$\sin x / x = 1 - x^2/6 + x^4/120 - \dots$

$\int (\sin x / x)\,dx \approx x - x^3/18 + x^5/600 \mid (1 \text{ to } 2)$

$= (2 - 8/18 + 32/600) - (1 - 1/18 + 1/600) \approx 0.946083.$

Answer: 0.946083.