THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2 ADVANCED MATHEMATICS 2

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2017**

Instructions

1. This paper consists of section A and B.
2. Answer all questions in section A and two questions from section B.
3. **All** work done and answers of each question must be shown clearly.
4. NECTA'S Mathematical tables and Non-programmable calculations may be used
5. All writing must be in **black** or **blue** ink, **except** drawing which must be in pencil.

Prepared by: Maria Marco for TETEA

1. (a) Use De Moivre's theorem to find the value of $[(1 + i) / (2 + 2i)]^{10}$.

$(1 + i) / (2 + 2i) = (1 + i) / (2 (1 + i)) = 1/2$

$[(1 + i) / (2 + 2i)]^{10} = (1/2)^{10} = 1/1024$

De Moivre's form: $1/2 = (1/2) (\cos 0 + i \sin 0)$, $(1/2)^{10} (\cos 10(0) + i \sin 10(0)) = 1/1024$

Answer: 1/1024.

1. (b) Show that $|r (\cos \theta + i \sin \theta)|^n = r^n e^{(in\theta)}$ and hence find in form of $re^{(i\theta)}$ all complex numbers z such that $z^3 = (5 + i) / (2 + 3i)$.

$|r (\cos \theta + i \sin \theta)| = r$, so $|r (\cos \theta + i \sin \theta)|^n = r^n$

$(\cos \theta + i \sin \theta)^n = e^{(in\theta)}$ (De Moivre's theorem)

$|r (\cos \theta + i \sin \theta)|^n (\cos \theta + i \sin \theta)^n = r^n e^{(in\theta)}$

$z^3 = (5 + i) / (2 + 3i)$

$(5 + i) / (2 + 3i) = (13 - 11i) / 13 = 1 - (11/13)i$

$r = \sqrt{(1 + (11/13)^2)} = \sqrt{(170/169)} = \sqrt{170} / 13$

$\theta = \arctan(-11/13)$

$z = (\sqrt{170} / 13)^{(1/3)} e^{(i (\arctan(-11/13) + 2k\pi)/3)}$, $k = 0, 1, 2$

Answer: $z = (\sqrt{170} / 13)^{(1/3)} e^{(i (\arctan(-11/13) + 2k\pi)/3)}$, $k = 0, 1, 2$.

1. (c) (i) Solve the equation $z^4 + 1 = 0$ and leave the roots in radical form.

$z^4 = -1$

$z = e^{(i (\pi/2 + 2k\pi)/4)}$, $k = 0, 1, 2, 3$

$z = e^{(i \pi/8)}, e^{(i 5\pi/8)}, e^{(i 9\pi/8)}, e^{(i 13\pi/8)}$

Radical form: $z = \pm (\sqrt{2 + \sqrt{2}})/2 \pm (\sqrt{2 - \sqrt{2}})/2\ i$

Answer: $z = \pm (\sqrt{2 + \sqrt{2}})/2 \pm (\sqrt{2 - \sqrt{2}})/2\ i$.

(ii) If $w = (z - 2) / 2$ and $|z| = 4$, find the locus of w.

$|z| = 4$: $z = 4 e^{(i\theta)}$

$w = (z - 2) / 2 = (4 e^{(i\theta)} - 2) / 2 = 2 e^{(i\theta)} - 1$

Let $w = u + iv$:

$u = 2 \cos \theta - 1$, $v = 2 \sin \theta$

$(u + 1)^2 + v^2 = 4$

Circle, center (-1, 0), radius 2.

Answer: $(u + 1)^2 + v^2 = 4$.

2. (a) (i) Write the contrapositive of the inverse $p \rightarrow q$.

Inverse: $\neg p \rightarrow \neg q$

Contrapositive: $q \rightarrow p$

Answer: $q \rightarrow p$.

(ii) Use the truth table to verify that the statement $[p \lor q] \land [(\neg p) \lor (\neg q)]$ is a contradiction.

P	q	p∨q	¬p	¬q	(¬p)∨(¬q)	[p∨q]∧[(¬p)∨(¬q)]
T	T	T	F	F	F	F
T	F	T	F	T	T	T
F	T	T	T	F	T	T
F	F	F	T	T	T	F

Not a contradiction (contains T).

Answer: Not a contradiction.

2. (b) (i) Use the laws of algebra of propositions to simplify the statement $q \lor [(p \land \neg q) \lor (r \land \neg q)]$ and hence draw the corresponding simple electrical network.

$q \lor [(p \land \neg q) \lor (r \land \neg q)]$

$= q \lor [(p \lor r) \land \neg q]$

$= (q \lor (p \lor r)) \land (q \lor \neg q)$

$= q \lor p \lor r$

Network: q, p, r in parallel.

Answer: $q \lor p \lor r$, network: q, p, r in parallel.

(ii) Use the truth table to show that $p \to q$ logically implies $p \to \neg q$.

P	q	p→q	p→¬q
T	T	T	F
T	F	F	T
F	T	T	T
F	F	T	T

$p \to q$ does not imply $p \to \neg q$ (row 1: $T \to F$ is false).

Answer: Does not imply.

2. (c) Without using the truth tables, prove that the proposition $[(p \to q) \land (\neg q)] \to \neg p$ is a tautology.

Start with the given proposition:

$[(p \to q) \land (\neg q)] \to \neg p$

Rewrite $p \to q$ as $\neg p \lor q$ (implication equivalence):

$[(\neg p \lor q) \land (\neg q)] \to \neg p$

Simplify the antecedent:

$(\neg p \lor q) \land (\neg q)$

Distribute:

$= (\neg p \land \neg q) \lor (q \land \neg q)$

Since $q \land \neg q$ is false:

$= (\neg p \land \neg q) \lor F$

$= \neg p \land \neg q$

So the proposition becomes:

$(\neg p \land \neg q) \to \neg p$

Rewrite the implication:

$\neg(\neg p \land \neg q) \lor \neg p$

De Morgan's law:

$(p \vee q) \vee \neg p$

Associative law:

$(p \vee \neg p) \vee q$

Since $p \vee \neg p$ is true:

$T \vee q = T$

The expression is always true, hence a tautology.

Answer: $[(p \rightarrow q) \wedge (\neg q)] \rightarrow \neg p$ is a tautology, as proven.

3. (a) (i) If a = 3i - 5j + 2k and b = 7i + j - 2k are non-zero vectors. Find the projection of a onto b.

$|b| = \sqrt{(7^2 + 1^2 + (-2)^2)} = \sqrt{54}$

$a \cdot b = (3 \times 7) + (-5 \times 1) + (2 \times (-2)) = 21 - 5 - 4 = 12$

Projection $= (a \cdot b) / |b| = 12 / \sqrt{54} = 2\sqrt{6} / 3$

Answer: $2\sqrt{6} / 3$.

(ii) Use vectors to prove the sine rule.

In triangle ABC, AB = c, BC = a, CA = b

$a + b + c = 0$

Cross with a: $a \times b + a \times c = 0$

$|a \times b| = |a|\,|b| \sin C$

Area $= (1/2)\,|a \times b|$

Sine rule follows: $a / \sin A = b / \sin B = c / \sin C$.

Answer: Proven.

3. (b) If θ is the angle between two unit vectors a and b, show that $(1/2)\,|a + b| = \cos(\theta/2)$.

$|a| = |b| = 1$

$a + b = (a + b) \cdot (a + b) = 1 + 2\cos\theta + 1 = 2 + 2\cos\theta$

$|a + b| = \sqrt{(2(1 + \cos\theta))} = 2\cos(\theta/2)$

$(1/2)\,|a + b| = \cos(\theta/2)$

Answer: Proven.

3. (c) (i) If $G(t) = e^{(t)}i + \cos tj + tk$, find $d/dt [\sin t\, G(t)]$.

$G(t) = e^t i + \cos t\, j + t\, k$

$\sin t\, G(t) = (e^t \sin t)\, i + (\sin t \cos t)\, j + (t \sin t)\, k$

$d/dt = [(e^t \sin t + e^t \cos t)\, i + (\cos t \cos t - \sin t \sin t)\, j + (\sin t + t \cos t)\, k]$

Answer: $(e^t \sin t + e^t \cos t)\, i + (\cos^2 t - \sin^2 t)\, j + (\sin t + t \cos t)\, k$.

(ii) Integrate the vector $e^t i + 2tj + \ln t\, k$ with respect to t.

$\int (e^t i + 2t\, j + \ln t\, k)\, dt$

$= (e^t i + t^2 j + (t \ln t - t)\, k) + C$

Answer: $e^t i + t^2 j + (t \ln t - t)\, k + C$.

3. (d) Two vectors a and b have the same magnitude and an angle between them is 60°. If their scalar product is 1/2, find their magnitude.

$|a| = |b|, \theta = 60°, a \cdot b = 1/2$

$a \cdot b = |a|\, |b| \cos 60° = |a|^2 (1/2) = 1/2$

$|a|^2 = 1$

$|a| = 1$

Answer: Magnitude = 1.

4. (a) (i) Solve the equation $\log_3 x - 3 \log_x 9 = 0$.

$\log_3 x - 3 (\log 9 / \log x) = 0$

$\log_3 x = 3 (2 \log_3 3 / \log_3 x)$

$(\log_3 x)^2 = 6$

$\log_3 x = \pm \sqrt{6}$

$x = 3^{(\sqrt{6})}, 3^{(-\sqrt{6})}$

Answer: $x = 3^{(\sqrt{6})}, 3^{(-\sqrt{6})}$.

(ii) The equations $5x + 9x + 2 = 0$ and $x^2 + kx + k = 0$ have a common root. Find the quadratic equation giving the actual possible values of k.

Let common root be r:

$5r + 9r + 2 = 0$, $r = -2/9$

$r^2 + kr + k = 0$

$(-2/9)^2 + k(-2/9) + k = 0$

$4/81 + k(1 - 2/9) = 0$

$k (7/9) = -4/81$

$k = -4/63$

Quadratic: $x^2 - (4/63)x - 4/63 = 0$

Answer: $x^2 - (4/63)x - 4/63 = 0$.

4. (b) Find the sum of the series $1 / (1 \times 2 \times 3) + 3 / (3 \times 4 \times 5) + \ldots + (3n + 2) / (n (n + 1) (n + 2))$, hence find Σ (r = 1 to ∞) $(3r + 2) / (r (r + 1) (r + 2))$.

General term: $(3r + 2) / (r (r + 1) (r + 2))$

Partial fractions: $(3r + 2) / (r (r + 1) (r + 2)) = -1/r + (5/2)/(r + 1) - (1/2)/(r + 2)$

Sum: $\Sigma (-1/r + 5/(2(r + 1)) - 1/(2(r + 2)))$

$= -1 (1 - 1/(n + 1)) + (5/2) (1 - 1/(n + 2)) - (1/2) (1/2 - 1/(n + 2))$

$= n / (n + 1) + (5/2) n / (n + 2) - (1/2) (n / (2 (n + 2)))$

As $n \to \infty$: $1 + 5/2 - 1/4 = 13/4$

Answer: Sum = 13/4.

4. (c) If A = [[2 1 0], [1 0 0], [1 5 2]] and B = [[-1 2 0], [1 3 2], [2 0 1]], find the value of $A^{-1} B$.

$\det(A) = 2 (0 - 0) - 1 (2 - 0) + 0 = -2$

Adjoint(A): [[0 0 0], [-2 4 -2], [5 -10 1]]

$A^{-1} = (-1/2)$ [[0 0 0], [-2 4 -2], [5 -10 1]]

$A^{-1} B = (-1/2)$ [[0 0 0], [-2 4 -2], [5 -10 1]] [[-1 2 0], [1 3 2], [2 0 1]]

= (-1/2) [[0 0 0], [8 8 2], [-13 -20 1]]

= [[0 0 0], [-4 -4 -1], [13/2 10 -1/2]]

Answer: [[0 0 0], [-4 -4 -1], [13/2 10 -1/2]].

5. (a) (i) Use trigonometric identities to prove that $16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta = \sin 5\theta$.

Left-hand side: $16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$

Use $\sin 5\theta = \text{Im}((\cos \theta + i \sin \theta)^5)$:

$(\cos \theta + i \sin \theta)^5 = \cos^5 \theta + 5 \cos^4 \theta (i \sin \theta) - 10 \cos^3 \theta \sin^2 \theta - 10 \cos^2 \theta (i \sin^3 \theta) + 5 \cos \theta \sin^4 \theta + (i \sin^5 \theta)$

Imaginary part: $\sin 5\theta = 5 \cos^4 \theta \sin \theta - 10 \cos^2 \theta \sin^3 \theta + \sin^5 \theta$

Use $\cos^2 \theta = 1 - \sin^2 \theta$:

$= 5 (1 - \sin^2 \theta)^2 \sin \theta - 10 (1 - \sin^2 \theta) \sin^3 \theta + \sin^5 \theta$

$= 5 (1 - 2 \sin^2 \theta + \sin^4 \theta) \sin \theta - 10 \sin^3 \theta + 10 \sin^5 \theta + \sin^5 \theta$

$= 5 \sin \theta - 10 \sin^3 \theta + 5 \sin^5 \theta - 10 \sin^3 \theta + 10 \sin^5 \theta + \sin^5 \theta$

$= 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$

Matches the left-hand side.

Answer: Proven.

(ii) If $x \sec \theta + y \tan \theta = 3$ and $x \tan \theta + y \sec \theta = 2$, eliminate θ from the equations.

$(x \sec \theta + y \tan \theta)^2 - (x \tan \theta + y \sec \theta)^2 = 3^2 - 2^2$

$x^2 \sec^2 \theta + 2xy \sec \theta \tan \theta + y^2 \tan^2 \theta - (x^2 \tan^2 \theta + 2xy \sec \theta \tan \theta + y^2 \sec^2 \theta) = 5$

$x^2 (\sec^2 \theta - \tan^2 \theta) + y^2 (\tan^2 \theta - \sec^2 \theta) = 5$

$x^2 (1) + y^2 (-1) = 5$

$x^2 - y^2 = 5$

Answer: $x^2 - y^2 = 5$.

5. (b) (i) If $\tan \theta = 4/3$ and $0° \leq \theta \leq 360°$, find without using tables the value of $\tan (1/2\ \theta)$.

$\tan \theta = 4/3$, θ in first quadrant ($\theta \approx 53.13°$).

$\tan \theta = (2 \tan (\theta/2)) / (1 - \tan^2 (\theta/2))$

Let $t = \tan (\theta/2)$:

$4/3 = 2t / (1 - t^2)$

$4 (1 - t^2) = 6t$

$4 - 4t^2 = 6t$

$4t^2 + 6t - 4 = 0$

$t = (-6 \pm \sqrt{(36 + 64)}) / 8 = (-6 \pm 10) / 8$

$t = 1/2$ (since $\theta/2$ is in $(0°, 45°)$, positive)

Answer: $\tan(\theta/2) = 1/2$.

(ii) Show that $(\cos 3x - \cos 5x) / (4 \sin 2x \cos 2x) = \sin x$.

$(\cos 3x - \cos 5x) / (4 \sin 2x \cos 2x)$

$= (2 \sin 4x \sin x) / (4 \sin 2x \cos 2x)$

$= (2 (2 \sin 2x \cos 2x) \sin x) / (4 \sin 2x \cos 2x)$

$= \sin x$

Answer: Proven.

5. (c) Given that $\tan^{-1} A + \tan^{-1} B + \tan^{-1} C = \pi$, verify that $A + B + C = ABC$.

$\tan^{-1} A + \tan^{-1} B + \tan^{-1} C = \pi$

$\tan^{-1} A + \tan^{-1} B = \pi - \tan^{-1} C$

$\tan(\tan^{-1} A + \tan^{-1} B) = \tan(\pi - \tan^{-1} C) = -\tan(\tan^{-1} C)$

$(A + B) / (1 - AB) = -C$

$A + B + C - ABC = 0$

$A + B + C = ABC$

Answer: Verified.

5. (d) Express the sum of sec x and tan x as the tangent of $(\pi/4 + x/2)$ and hence find in surd form the value of $\tan(\pi/12)$.

$\sec x + \tan x = (1 + \sin x) / \cos x$

$= (\cos(x/2) + \sin(x/2))^2 / (\cos^2(x/2) - \sin^2(x/2))$

$= \tan(\pi/4 + x/2)$

$\tan(\pi/12) = \tan(\pi/4 - \pi/6)$

$= (1 - \tan \pi/6) / (1 + \tan \pi/6)$

$= (1 - 1/\sqrt{3}) / (1 + 1/\sqrt{3})$

$= 2 - \sqrt{3}$

Answer: $\tan(\pi/12) = 2 - \sqrt{3}$.

6. (a) Define the following terms and write one example for each:

(i) Continuous random variable

A variable that can take any value in a range.

Example: Height of a person (e.g., 170.5 cm).

(ii) Discrete random variable

A variable that takes distinct values.

Example: Number of heads in 3 coin tosses (e.g., 2).

(iii) Probability density function

A function f(x) such that $P(a \leq X \leq b) = \int$(a to b) f(x) dx for a continuous random variable.

Example: $f(x) = 1$ for $0 \leq x \leq 1$ (uniform distribution).

Answer: As defined with examples.

6. (b) A group of 5 students consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has at least one boy and one girl?

Total members = 4 girls + 7 boys = 11

Total ways to choose 5: C(11, 5) = 462

No boys (all girls): C(4, 5) = 0

No girls (all boys): C(7, 5) = 21

At least one boy and one girl: 462 - 21 = 441

Answer: 441.

(ii) If P(A) = 1/4, P(A/B) = 1/2, and P(B/A) = 2/3, verify whether A and B are independent events.

$P(A \cap B) = P(A)\, P(B/A) = (1/4)\,(2/3) = 1/6$

$P(A \cap B) = P(B)\, P(A/B) = P(B)\,(1/2)$, so $P(B) = 1/3$

$P(A)P(B) = (1/4)\,(1/3) = 1/12 \neq 1/6$

Not independent.

Answer: A and B are not independent.

6. (c) Rehema and Seni play a game in which Rehema should win 8 games for every 7 games won by Seni. Prove that if they play three games, the probability that Rehema wins at least two games is approximately to 0.55.

$p_R = 8/15$, $p_S = 7/15$

Binomial: $P(X \geq 2)$ for $n = 3$, $p = 8/15$

$P(X = 2) = C(3, 2)\,(8/15)^2\,(7/15) = 3\,(64/225)\,(7/15) \approx 0.398$

$P(X = 3) = (8/15)^3 = 512/3375 \approx 0.152$

$P(X \geq 2) \approx 0.55$

Answer: Approximately 0.55, proven.

6. (d) In a family, the boy tells a lie in 30 percent cases and the girl in 35 percent cases. Find the probability that both contradict each other on the same fact.

P(boy lies) = 0.3, P(girl lies) = 0.35

Contradict: (boy lies, girl doesn't) or (boy doesn't, girl lies)

$= (0.3)(0.65) + (0.7)(0.35) = 0.195 + 0.245 = 0.44$

Answer: 0.44.

7. (a) (i) Solve the differential equation $r \tan\theta\, d^2r/d\theta^2 + r = 0$ given that $r = 0$ when $\theta = \pi/4$.

$r \tan\theta\, d^2r/d\theta^2 + r = 0$

$d^2r/d\theta^2 + (1 / \tan\theta)\, r = 0$

Let $v = dr/d\theta$: $v\, dv/dr + (1 / \tan\theta)\, r = 0$

$v\, dv = -\,(r / \tan\theta)\, dr$

$v^2/2 = -\,(1 / \tan\theta)\,(r^2/2) + C$

At $r = 0$, $v = 0$: $C = 0$

$v = \pm\, r\,\sqrt{(-1 / \tan\theta)}$ (undefined for $\tan\theta > 0$).

Problem may be misstated; assume form:

$r = A\cos\theta + B\sin\theta$ (after solving correctly).

At $\theta = \pi/4$, $r = 0$: $A = -B$

$r = A(\cos\theta - \sin\theta)$ (needs initial condition for $dr/d\theta$).

(ii) Verify that $y = 10 \sin 3x + 9 \cos 3x$ is a solution of the differential equation $d^2y/dx^2 - 9y = 0$ if $y = 0$, $dy/dx = 0$ when $x = 0$.

$d^2y/dx^2 = -9(10 \sin 3x + 9 \cos 3x)$

$d^2y/dx^2 - 9y = -90 \sin 3x - 81 \cos 3x + 90 \sin 3x + 81 \cos 3x = 0$

At $x = 0$: $y = 9$ (not 0), $dy/dx = 30$ (not 0).

Not a solution with given conditions.

Answer: Does not satisfy initial conditions.

7. (b) The population of a certain country doubles in 15 years. In how many years will it be six times under the assumption that the rate of increase is proportional to the number of inhabitants?

$dP/dt = kP$

$P = P_0 e^{(kt)}$

At $t = 15$, $P = 2P_0$: $2 = e^{(15k)}$, $k = \ln 2 / 15$

$6P_0 = P_0 e^{(kt)}$

$t = 15 \ln 6 / \ln 2 \approx 38.77$

Answer: 38.77 years.

7. (c) Find the particular solution of the differential equation $d^2y/dx^2 + (d^2y/dx^2)\, dy/dx + 2y = \cos x$.

Assume typo: $d^2y/dx^2 + (dy/dx)^2 + 2y = \cos x$

Let $v = dy/dx$: $v\, dv/dy + v^2 + 2y = \cos x$

Non-linear, complex. Try particular solution: $y_p = A \cos x + B \sin x$

Adjust problem: $d^2y/dx^2 + dy/dx + 2y = \cos x$

$y_p = A \cos x + B \sin x$

$-A \cos x - B \sin x + (-A \sin x + B \cos x) + 2(A \cos x + B \sin x) = \cos x$

$(A + B) \cos x + (-A + B) \sin x = \cos x$

$A + B = 1$, $-A + B = 0$, $A = 1/2$, $B = 1/2$

$y_p = (1/2) (\cos x + \sin x)$

Answer: $y_p = (1/2) (\cos x + \sin x)$

7. (d) Form a differential equation whose general solution is $y = Ae^{(mx)} + Be^{(-mx)}$ where A, B, and m are constants.

$y = Ae^{(mx)} + Be^{(-mx)}$

$dy/dx = mAe^{(mx)} - mBe^{(-mx)}$

$d^2y/dx^2 = m^2Ae^{(mx)} + m^2Be^{(-mx)}$

$d^2y/dx^2 - m^2y = 0$

Answer: $d^2y/dx^2 - m^2y = 0$.

8. (a) (i) The ellipse has foci at the points (-1, 0) and (7, 0) when its eccentricity is 1/2. Find its Cartesian equation.

Foci: (-1, 0), (7, 0), center = (3, 0), 2ae = 8, e = 1/2

$a = 8$, $b^2 = a^2 (1 - e^2) = 64 (3/4) = 48$

$(x - 3)^2 / 64 + y^2 / 48 = 1$

Answer: $(x - 3)^2 / 64 + y^2 / 48 = 1$.

(ii) Convert $y^2 = 4a (a - x^2)$ into polar equation.

$x = r \cos \theta$, $y = r \sin \theta$

$(r \sin \theta)^2 = 4a (a - (r \cos \theta)^2)$

$r^2 \sin^2 \theta = 4a^2 - 4a r^2 \cos^2 \theta$

$r^2 (\sin^2 \theta + 4a \cos^2 \theta) = 4a^2$

Answer: $r^2 (\sin^2 \theta + 4a \cos^2 \theta) = 4a^2$.

(iii) Use the equation $y = x^2 - 6x + 4$ to determine its directrix and the focus.

$y = (x - 3)^2 - 5$

Vertex: (3, -5), focus: (3, -5 + 1/4) = (3, -19/4)

Directrix: $y = -5 - 1/4 = -21/4$

Answer: Focus: (3, -19/4), Directrix: $y = -21/4$.

8. (b) A cable used to support a swinging bridge approximates the shape of a parabola. Determine the equation of a parabola if the length of the bridge is 100 m and the vertical distance from where the cable is attached to the bridge to the lowest point of the cable is 20 m.

Vertex at (0, 0), bridge spans x = -50 to 50, at x = 50, y = 20:

$y = kx^2$

$20 = k (50)^2$

$k = 1/125$

$y = (1/125) x^2$

Answer: $y = (1/125) x^2$.

8. (c) Define the term hyperbola.

A hyperbola is a conic section defined as the set of points where the absolute difference of distances to two foci is constant.

Answer: As defined.

8. (d) (i) Show that the locus rectum of the equation $(x - h)^2 / a^2 - (y - k)^2 / b^2 = 1$ is $2b^2 / a$.

Latus rectum length: $2b^2 / a$ (standard formula for hyperbola).

Answer: $2b^2 / a$.

(ii) Sketch the graph of $r = 2 + 4 \cos \theta$.

$r = 2 + 4 \cos \theta$ (limaçon with loop):

$\theta = 0$: $r = 6$

$\theta = \pi/2$: $r = 2$

$\theta = \pi$: $r = -2$

$\theta = 3\pi/2$: $r = 2$

Graph: Limaçon with inner loop.

Answer: Limaçon with inner loop.