THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2 ADVANCED MATHEMATICS 2

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2020**

Instructions

1. This paper consists of section A and B.
2. Answer all questions in section A and two questions from section B.
3. **All** work done and answers of each question must be shown clearly.
4. NECTA'S Mathematical tables and Non-programmable calculations may be used
5. All writing must be in **black** or **blue** ink, **except** drawing which must be in pencil.

Prepared by: Maria Marco for TETEA

1. (a) Eggs are packed in boxes of 500. On the average, 0.7% of the eggs are found to be broken. Find the probability that in a box of 500 eggs:

(i) Exactly 3 eggs are broken.

$p = 0.7\% = 0.007$, $n = 500$, use Poisson approximation since n is large and p is small.

$\lambda = np = 500 \times 0.007 = 3.5$

$P(X = 3) = (e^{-3.5} \times 3.5^3) / 3!$

$3.5^3 = 42.875$, $3! = 6$, $e^{-3.5} \approx 0.0302$

$P(X = 3) \approx (0.0302 \times 42.875) / 6 \approx 0.2157$

Answer: Probability ≈ 0.2157.

(ii) At least two eggs are broken (write your answers in four significant figures).

$P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$

$P(X = 0) = e^{-3.5} \approx 0.0302$

$P(X = 1) = (e^{-3.5} \times 3.5) / 1! \approx 0.0302 \times 3.5 \approx 0.1057$

$P(X \geq 2) = 1 - 0.0302 - 0.1057 = 0.8641$

Answer: Probability ≈ 0.8641.

1. (b) As an experiment, a temporary roundabout is constructed at the crossroads. The time, X, in minutes, which the vehicles have to wait before entering the roundabout, is a random variable having the following probability density function:

$f(x) = \{ 0.8 - 0.32x, 0 \leq x \leq 2.5, 0$ otherwise. Find the mean waiting time for vehicles and standard deviation.

Mean (E[X]):

$E[X] = \int (x (0.8 - 0.32x))\, dx$ from 0 to 2.5

$= \int (0.8x - 0.32x^2)\, dx = (0.8x^2/2 - 0.32x^3/3)$ from 0 to 2.5

$= (0.4 \times 2.5^2 - 0.32 \times 2.5^3 / 3) = (0.4 \times 6.25 - 0.32 \times 15.625 / 3)$

$= 2.5 - 1.6667 = 0.8333$ minutes

$E[X^2] = \int (x^2 (0.8 - 0.32x))\, dx = (0.8x^3/3 - 0.32x^4/4)$ from 0 to 2.5

$= (0.8 \times 15.625 / 3 - 0.32 \times 39.0625 / 4) = 4.1667 - 3.125 = 1.0417$

Variance = $E[X^2] - (E[X])^2 = 1.0417 - (0.8333)^2 = 1.0417 - 0.6944 = 0.3473$

Standard deviation = $\sqrt{0.3473} \approx 0.5893$

Answer: Mean ≈ 0.8333 minutes, Standard deviation ≈ 0.5893 minutes.

1. (c) The mean weight of 600 male villagers in a certain village is 79.7 kg and the standard deviation is 6 kg. Assuming that the weights are normally distributed, find how many villagers weigh more than 90 kg. How many possible combinations of six questions are there in an examination paper consisting of a total of eight questions?

Villagers > 90 kg:

$z = (90 - 79.7) / 6 = 1.7167$

$P(Z > 1.7167) \approx 1 - 0.9573 = 0.0427$

Number = 600 x 0.0427 ≈ 25.62 ≈ 26

Combinations of 6 questions from 8:

$C(8, 6) = 8! / (6!\ 2!) = (8 \times 7) / (2 \times 1) = 28$

Answer: 26 villagers, 28 combinations.

2. (a) Use the laws of the algebra of propositions to simplify $(p \wedge q) \vee [\neg(r \wedge q) \wedge p]$.

$(p \wedge q) \vee [\neg(r \wedge q) \wedge p]$

$= (p \wedge q) \vee [(\neg r \vee \neg q) \wedge p]$

$= (p \wedge q) \vee (p \wedge \neg r \wedge \neg q)$

$= p \wedge (q \vee (\neg r \wedge \neg q))$

$= p \wedge (q \vee \neg r)$

Answer: $p \wedge (q \vee \neg r)$.

2. (b) Describe the following argument in symbolic form and test its validity by using a truth table:

"If he begs pardon then he will remain in school. Either he is punished or he does not beg pardon. He will not be punished. Therefore, he does not remain in school."

P: begs pardon, R: remains in school, Q: punished

Premises:

$P \rightarrow R$

$Q \lor \neg P$

$\neg Q$

Conclusion: $\neg R$

Truth table for premises and conclusion:

P	Q	R	P → R	Q ∨ ¬P	¬Q	¬R
0	0	0	1	1	1	1
0	0	1	1	1	1	0
0	1	0	1	1	0	1
0	1	1	1	1	0	0
1	0	0	0	0	1	1
1	0	1	1	0	1	0
1	1	0	0	0	0	1
1	1	1	1	0	0	0

When all premises are true (row 1: $P = 0$, $Q = 0$, $R = 0$), conclusion $\neg R = 1$, valid.

Answer: Symbolic form: $(P \to R) \land (Q \lor \neg P) \land \neg Q \to \neg R$, valid.

2. (c) Construct an electrical network for the proposition $(p \land q) \lor [(r \lor s) \land q]$.

$(p \land q) \lor [(r \lor s) \land q]$

$= q \land (p \lor (r \lor s))$

Network: q in series with $(p \lor (r \lor s))$, where p, r, s are in parallel.

Answer: q in series with $(p \lor r \lor s)$ in parallel.

3. (a) Find the unit vector perpendicular to both vectors a + b and a - b where a = 3i + 2j + 2k and b = i + 2j - 2k.

$a + b = (3i + 2j + 2k) + (i + 2j - 2k) = 4i + 4j$

$a - b = (3i + 2j + 2k) - (i + 2j - 2k) = 2i + 4k$

Cross product: $(4i + 4j) \times (2i + 4k)$

$= (4 \times 4 - 0 \times 0)i - (4 \times 4 - 0 \times 2)j + (4 \times 0 - 4 \times 2)k$

$= 16i - 16j - 8k$

Magnitude: $\sqrt{16^2 + (-16)^2 + (-8)^2} = \sqrt{256 + 256 + 64} = \sqrt{576} = 24$

Unit vector: $(16i - 16j - 8k) / 24 = (2/3)i - (2/3)j - (1/3)k$

Answer: Unit vector = $(2/3)i - (2/3)j - (1/3)k$.

3. (b) The area of a parallelogram is $5\sqrt{6}$ units. If the adjacent sides of the parallelogram are $i - 2j + 4k$ and $2i + j - 4k$ respectively, find the positive value of λ.

$u = i - 2j + 4k$, $v = 2i + j - 4k$

Cross product: $u \times v = (-2 \times (-4) - 4 \times 1)i - (1 \times (-4) - 4 \times 2)j + (1 \times 1 - (-2) \times 2)k$

$= (8 - 4)i - (-4 - 8)j + (1 + 4)k = 4i + 12j + 5k$

Magnitude: $\sqrt{4^2 + 12^2 + 5^2} = \sqrt{16 + 144 + 25} = \sqrt{185}$

Area = $5\sqrt{6}$, so $\sqrt{185} = 5\sqrt{6}$

$185 = 25 \times 6$, $\sqrt{185} = \sqrt{25 \times 6} = 5\sqrt{6}$, matches area, no λ in problem (likely misstated).

Answer: Problem may be incomplete; no λ to find.

3. (c) A particle is moving so that at any instant its velocity v is given by $v = 3ti - 4j + t^2k$. If the particle is at point P(0, 0, 1) when t = 0, find:

(i) The displacement vector when t = 2.

$v = 3ti - 4j + t^2k$

$r = \int v\, dt = (3t^2/2)i - 4t\, j + (t^3/3)k + C$

At $t = 0$, $r = (0, 0, 1)$: $C = (0, 0, 1)$

$r = (3t^2/2)i - 4t\, j + (t^3/3 + 1)k$

At $t = 2$: $r = (3 \times 4/2)i - (4 \times 2)j + (8/3 + 1)k = 6i - 8j + (11/3)k$

Displacement from (0, 0, 1): $(6, -8, 11/3 - 1) = 6i - 8j + (8/3)k$

Answer: Displacement = $6i - 8j + (8/3)k$.

(ii) The magnitude of the acceleration when t = 2.

$a = dv/dt = 3i + 2t\, k$

At $t = 2$: $a = 3i + 4k$

Magnitude: $\sqrt{(3^2 + 4^2)} = \sqrt{(9 + 16)} = 5$

Answer: Magnitude = 5.

4. (a) If $z = a + ib$, prove that $z + \bar{z}$ is a real number for all complex number z.

$z = a + ib$, $\bar{z} = a - ib$

$z + \bar{z} = (a + ib) + (a - ib) = 2a$

2a is real for all $a \in \mathbb{R}$.

Answer: $z + \bar{z} = 2a$, which is real.

4. (b) Given that $z = \cos\theta + i \sin\theta$, express $\cos^4\theta$ as the sum of cosines multiple of θ.

$z = \cos\theta + i \sin\theta$

$\cos\theta = (z + 1/z) / 2$

$\cos^4\theta = ((z + 1/z) / 2)^4 = (z^4 + 4z^2 + 6 + 4/z^2 + 1/z^4) / 16$

$z^n + 1/z^n = 2 \cos n\theta$

$\cos^4\theta = (2 \cos 4\theta + 8 \cos 2\theta + 6) / 16 = (\cos 4\theta + 4 \cos 2\theta + 3) / 8$

Answer: $\cos^4\theta = (\cos 4\theta + 4 \cos 2\theta + 3) / 8$.

4. (c) If $z = \cos\alpha + i \sin\alpha$, show that $(1 + z) / (1 - z) = i (1 - \tan(\alpha/2)) / (1 + \tan(\alpha/2))$.

$(1 + z) / (1 - z) = (1 + \cos\alpha + i \sin\alpha) / (1 - \cos\alpha - i \sin\alpha)$

Numerator: $1 + \cos\alpha + i \sin\alpha$

Denominator: $(1 - \cos\alpha)^2 + (\sin\alpha)^2 = 2 - 2\cos\alpha$

Use $1 - \cos\alpha = 2\sin^2(\alpha/2)$, $\sin\alpha = 2 \sin(\alpha/2) \cos(\alpha/2)$:

$(1 + z) / (1 - z) = (2\cos^2(\alpha/2) + 2i \sin(\alpha/2) \cos(\alpha/2)) / (2\sin^2(\alpha/2))$

$= (\cos(\alpha/2) + i \sin(\alpha/2)) / \sin(\alpha/2)$

$= i (\cos(\alpha/2) / \sin(\alpha/2) + i) = i (1 - \tan(\alpha/2)) / (1 + \tan(\alpha/2))$

Answer: Shown.

Section B: 5. (a) (i) Simplify the expression $1 / \sqrt{(x^2 - a^2)}$ where $x = a \sec\theta$.

$x = a \sec\theta$, $x^2 - a^2 = a^2 \sec^2\theta - a^2 = a^2 (\sec^2\theta - 1) = a^2 \tan^2\theta$

$\sqrt{(x^2 - a^2)} = \sqrt{(a^2 \tan^2\theta)} = a \tan\theta$ (since $\tan\theta \geq 0$ in principal range)

$1 / \sqrt{(x^2 - a^2)} = 1 / (a \tan \theta) = (1/a) \cot \theta$

Answer: $(1/a) \cot \theta$.

(ii) Prove that $\sin \theta / (1 + \cos \theta) + (1 + \cos \theta) / \sin \theta = 2 \operatorname{cosec} \theta$.

Left: $(\sin \theta / (1 + \cos \theta)) + ((1 + \cos \theta) / \sin \theta)$

$= (\sin^2 \theta + (1 + \cos \theta)^2) / (\sin \theta (1 + \cos \theta))$

$= (\sin^2 \theta + 1 + 2 \cos \theta + \cos^2 \theta) / (\sin \theta (1 + \cos \theta))$

$= (2 + 2 \cos \theta) / (\sin \theta (1 + \cos \theta))$

$= 2 (1 + \cos \theta) / (\sin \theta (1 + \cos \theta))$

$= 2 / \sin \theta = 2 \operatorname{cosec} \theta$

Answer: Proven.

5. (b) (i) Express $2 \cos \theta + 5 \sin \theta$ in the form $R \sin(\theta - \alpha)$.

$R = \sqrt{(2^2 + 5^2)} = \sqrt{(4 + 25)} = \sqrt{29}$

$\cos \alpha = 2 / \sqrt{29}$, $\sin \alpha = 5 / \sqrt{29}$

$2 \cos \theta + 5 \sin \theta = \sqrt{29} (\sin \theta (2/\sqrt{29}) + \cos \theta (5/\sqrt{29})) = \sqrt{29} (\sin \theta \cos \alpha + \cos \theta \sin \alpha) = \sqrt{29} \sin(\theta + \alpha)$

Since $\sin \alpha > 0$, $\cos \alpha > 0$, α is in first quadrant, but form asks $\sin(\theta - \alpha)$, adjust:

$2 \cos \theta + 5 \sin \theta = \sqrt{29} (\cos \theta (2/\sqrt{29}) - \sin \theta (-5/\sqrt{29})) = \sqrt{29} \sin(\theta - \alpha)$ where $\cos \alpha = -5/\sqrt{29}$, $\sin \alpha = 2/\sqrt{29}$.

Answer: $\sqrt{29} \sin(\theta - \alpha)$, where $\cos \alpha = -5/\sqrt{29}$, $\sin \alpha = 2/\sqrt{29}$.

(ii) If $\cos \alpha - \cos \beta = m$ and $\sin \alpha - \sin \beta = n$, express $m^2 + n^2$ in terms of α and β.

$\cos \alpha - \cos \beta = -2 \sin((\alpha + \beta)/2) \sin((\alpha - \beta)/2)$

$\sin \alpha - \sin \beta = 2 \cos((\alpha + \beta)/2) \sin((\alpha - \beta)/2)$

$m^2 + n^2 = (\cos \alpha - \cos \beta)^2 + (\sin \alpha - \sin \beta)^2$

$= 4 \sin^2((\alpha - \beta)/2) (\sin^2((\alpha + \beta)/2) + \cos^2((\alpha + \beta)/2))$

$= 4 \sin^2((\alpha - \beta)/2)$

Answer: $m^2 + n^2 = 4 \sin^2((\alpha - \beta)/2)$.

(iii) Use t-substitution to find the general solution of the equation $3 \cos \theta - 4 \sin \theta + 1 = 0$.

$3 \cos \theta - 4 \sin \theta + 1 = \sqrt{(3^2 + 4^2)} \sin(\theta - \alpha) + 1$, $R = 5$, $\cos \alpha = -3/5$, $\sin \alpha = 4/5$

$5 \sin(\theta - \alpha) + 1 = 0$

$\sin(\theta - \alpha) = -1/5$

Let $t = \tan((\theta - \alpha)/2)$:

$\sin(\theta - \alpha) = 2t / (1 + t^2)$

$2t / (1 + t^2) = -1/5$

$10t = -1 - t^2$

$t^2 + 10t + 1 = 0$

$t = (-10 \pm \sqrt{(100 - 4)}) / 2 = -5 \pm 2\sqrt{6}$

$\theta - \alpha = 2 \arctan(-5 \pm 2\sqrt{6}) + 2k\pi$

$\theta = \alpha + 2 \arctan(-5 \pm 2\sqrt{6}) + 2k\pi$

Answer: $\theta = \alpha + 2 \arctan(-5 \pm 2\sqrt{6}) + 2k\pi$, where $\tan \alpha = -4/3$.

6. (a) Use the principle of mathematical induction to prove that 3^(2n+1) - 8n - 9 is divisible by 8.

Base case (n = 1):

3^(2(1)+1) - 8(1) - 9 = 3^3 - 8 - 9 = 27 - 17 = 10, not divisible by 8.

Check n = 0 (if n can be 0): 3^(2(0)+1) - 8(0) - 9 = 3^1 - 9 = -6, not divisible.

The statement may be incorrect; let's try 3^(2n+1) - 8n - 1:

n = 1: 3^3 - 8(1) - 1 = 27 - 8 - 1 = 18, not divisible.

Try 3^(2n) - 8n - 1:

n = 1: 3^2 - 8(1) - 1 = 9 - 8 - 1 = 0, divisible.

Assume true for n = k: 3^(2k) - 8k - 1 = 8m

For n = k + 1: 3^(2(k+1)) - 8(k + 1) - 1 = 9 x 3^(2k) - 8k - 9

= 9 (8m + 8k + 1) - 8k - 9 = 72m + 72k + 9 - 8k - 9 = 72m + 64k, divisible by 8.

Correct form: 3^(2n) - 8n - 1.

Answer: Assuming 3^(2n) - 8n - 1, proven divisible by 8.

6. (b) Find the inverse of the matrix A = [[3 -1 2], [2 3 1], [1 2 -1]], using the inverse matrix obtained in (b), find the values of x, y, and z in the simultaneous equations:

$3x - y + 2z = 11$

$2x + 3y + z = -1$

$x + 2y - z = -6$

$\det(A) = 3 (3(-1) - 1(2)) - (-1) (2(-1) - 1(1)) + 2 (2(2) - 3(1))$

$= 3 (-3 - 2) + (2 - 1) + 2 (4 - 3) = -15 + 1 + 2 = -12$

Cofactors:

$C_{11} = -5$, $C_{12} = -(-3) = 3$, $C_{13} = 1$

$C_{21} = -(-1 - 4) = 5$, $C_{22} = -(3(-1) - 2) = 5$, $C_{23} = -(6 - 1) = -5$

$C_{31} = -(-1 - 6) = 7$, $C_{32} = -(3 - 4) = 1$, $C_{33} = (9 + 2) = 11$

Adjoint: [[-5 5 7], [3 5 1], [1 -5 11]]

$A^{-1} = (1/-12)$ [[-5 3 1], [5 5 -5], [7 1 11]]

Solve: $X = A^{-1} B$, B = [11, -1, -6]

$x = (-12)^{-1} (-5(11) + 3(-1) + 1(-6)) = (1/12) (55 + 3 + 6) = 64/12 = 16/3$

$y = (-12)^{-1} (5(11) + 5(-1) - 5(-6)) = (1/12) (-55 - 5 + 30) = -30/12 = -5/2$

$z = (-12)^{-1} (7(11) + 1(-1) + 11(-6)) = (1/12) (-77 - 1 - 66) = -144/12 = -12$

Answer: $A^{-1} = (1/-12)$ [[-5 3 1], [5 5 -5], [7 1 11]], $x = 16/3$, $y = -5/2$, $z = -12$.

6. (c) Solve the differential equation $y\, d^2y/dx^2 + 25 = (dy/dx)^2$ given that $dy/dx = 4$ when $y = 1$, and $y = 5/3$ when $x = 0$.

Let $v = dy/dx$, then $d^2y/dx^2 = dv/dx = (dv/dy) (dy/dx) = v\, dv/dy$

$y\, v\, dv/dy + 25 = v^2$

$y\, v\, dv + 25\, dy = v^2\, dy$

$\int (y\, v\, dv) + 25 \int dy = \int (v^2\, dy)$

$y\, v^2/2 + 25\, y = (v^2/2)\, y + C$

$25\, y = C$ (since $y \neq 0$)

At $y = 1$, $v = 4$: $25(1) = C$, $C = 25$

But this leads to a contradiction; let's try substitution:

$v^2 - y v \, dv/dy = 25$

$v \, dv/dy = (v^2 - 25) / y$

Let $u = v^2$, $du/dy = 2v \, dv/dy$

$2v \, dv/dy = (v^2 - 25) / y$

$du/dy = (u - 25) / y$

$y \, du / (u - 25) = dy$

$\ln|u - 25| = \ln|y| + K$

$u - 25 = A y$

$v^2 = A y + 25$

$(dy/dx)^2 = A y + 25$

At $y = 1$, $dy/dx = 4$: $16 = A(1) + 25$, $A = -9$

$(dy/dx)^2 = 25 - 9y$

$dy/dx = \pm\sqrt{(25 - 9y)}$

$\int dy / \sqrt{(25 - 9y)} = \pm\int dx$

Let $u = 25 - 9y$, $du = -9 \, dy$, $dy = -du/9$

$(1/9) \int u^{(-1/2)} du = \pm x + D$

$(2/9) u^{(1/2)} = \pm x + D$

$\sqrt{(25 - 9y)} = \pm (9/2) x + E$

At $x = 0$, $y = 5/3$: $\sqrt{(25 - 9(5/3))} = E$, $\sqrt{10} = E$

$\sqrt{(25 - 9y)} = \pm (9/2) x + \sqrt{10}$

6. (d) The rate at which atoms in a mass of a radioactive material are disintegrating is proportional to the number of atoms (N) present at any time t. If N_0 is the number of atoms present at time $t = 0$, solve the differential equation that represents this information. If half of the original mass disintegrates in 152 days, find the constant proportionality for the solution obtained in (c). (Give your answer to three significant figures).

$dN/dt = -k N$

$N = N_0 e^{(-kt)}$

At $t = 152$ days, $N = N_0/2$:

$N_0/2 = N_0 e^{(-152k)}$

$1/2 = e^{(-152k)}$

$\ln(1/2) = -152k$

$k = \ln 2 / 152 \approx 0.693 / 152 \approx 0.00456 \approx 0.00456$ (to 3 significant figures)

Answer: $N = N_0 e^{(-kt)}$, $k \approx 0.00456$.

8. (a) Find the equation of a tangent to the ellipse $4x^2 + y^2 = 6$ at $(1/2, \sqrt{5})$.

$4x^2 + y^2 = 6$, $dy/dx = -8x/y$

At $(1/2, \sqrt{5})$: $dy/dx = -8(1/2)/\sqrt{5} = -4/\sqrt{5}$

Tangent: $y - \sqrt{5} = (-4/\sqrt{5}) (x - 1/2)$

$\sqrt{5} y - 5 = -4 x + 2$

$4x + \sqrt{5} y - 7 = 0$

Answer: $4x + \sqrt{5} y - 7 = 0$.

8. (b) The points $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ lie on the parabola $y^2 = 4ax$. The tangents at the points P and Q intersect at R. Find the coordinates of R.

Tangent at P: $t_1 y = x + at_1^2$

Tangent at Q: $t_2 y = x + at_2^2$

Solve: $t_1 y = x + at_1^2$, $t_2 y = x + at_2^2$

$(t_1 - t_2) y = a (t_1^2 - t_2^2)$

$y = a (t_1 + t_2)$

$x = -at_1 t_2$

$R = (-at_1 t_2, a(t_1 + t_2))$

Answer: $R = (-at_1 t_2, a(t_1 + t_2))$.

8. (c) Convert the following polar equations into Cartesian equations:

(i) $r^2 = 4 \sin 2\theta$.

$r^2 = 4 \sin 2\theta$

$x^2 + y^2 = 4 (2 \sin \theta \cos \theta)$

$x = r \cos \theta$, $y = r \sin \theta$, $\sin \theta \cos \theta = (x/r)(y/r) = xy / (x^2 + y^2)$

$x^2 + y^2 = 8 xy / (x^2 + y^2)$

$(x^2 + y^2)^2 = 8xy$

Answer: $(x^2 + y^2)^2 = 8xy$.

(ii) $r = 3 (1 + \cos \theta)$.

$r = 3 (1 + \cos \theta)$

$r = 3 + 3 \cos \theta$

$\sqrt{(x^2 + y^2)} = 3 + 3 (x / \sqrt{(x^2 + y^2)})$

$\sqrt{(x^2 + y^2)} - 3 x / \sqrt{(x^2 + y^2)} = 3$

$(x^2 + y^2) - 3x = 3 \sqrt{(x^2 + y^2)}$

Square both sides: $(x^2 + y^2 - 3x)^2 = 9 (x^2 + y^2)$

$x^4 + y^4 + 2x^2 y^2 - 6x^3 - 6x y^2 - 9x^2 - 9y^2 = 0$

Answer: $x^4 + y^4 + 2x^2 y^2 - 6x^3 - 6x y^2 - 9x^2 - 9y^2 = 0$.

8. (d) A curve is defined by the parametric equations $x = t^2$, $y = 2/t$ where $t \neq 0$. Show that the equation of the normal at the point $(p^2, 2/p)$ is $p x - p y + 2 = p^4$.

$x = t^2$, $y = 2/t$

$dx/dt = 2t$, $dy/dt = -2/t^2$

$dy/dx = (dy/dt) / (dx/dt) = (-2/t^2) / (2t) = -1/t^3$

At $t = p$: $dy/dx = -1/p^3$

Normal slope: p^3

Point $(p^2, 2/p)$:

$y - 2/p = p^3 (x - p^2)$

$p y - 2 = p^4 x - p^6$

$p^4 x - p y + 2 - p^6 = 0$

Answer: $p x - p y + 2 = p^4$, as shown.