THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION
EXAMINATION

142/2

ADVANCED MATHEMATICS 2
(For Both School and Private Candidates)

Time: 3 Hours   Year : 2021

Instructions

1. This paper consists sections A and B with a total of **eight (8)** questions.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. Section A carries **sixty (60)** marks and section B carries **forty (40)** marks.
4. All work done in answering each question must be shown clearly.
5. NECTA'S mathematical tables and non-programmable calculators may be used.
6. Cellular phones and any unauthorised materials are **not** allowed in the examination room.
7. Write your **Examination Number** on every page of your answer booklet(s).

SECTION A (60 Marks)

Answer **all** questions in this section.

1. (a) If A and B are such that $P(A)=\frac{1}{3}$, $P(B)=\frac{1}{4}$ and $P(A\cup B)=\frac{1}{2}$, calculate;

   (i) $P(A\cap B')$.

   (ii) $P(A/B')$.

   (b) Two dices are thrown simultaneously.

   (i) List the sample space for this event.

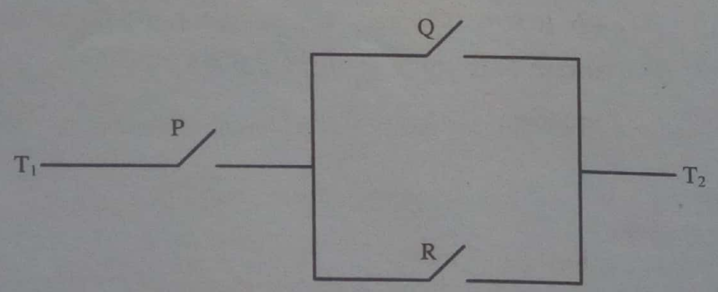
   (ii) Find the probability that the sum of the numbers obtained on the dice is neither a multiple of 2 nor a multiple of 3.

   (c) If X is binomially distributed, the probability that the event will happen exactly x times in n trials is given by the function $P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$. Establish the validity of the Poisson approximation to the binomial distribution.

2. (a) The contrapositive of the statement Y is given by $\sim(Q\wedge P)\rightarrow\sim P$. By using the laws of algebra of propositions, show that its inverse is a tautology.

   (b) Test the validity of the argument whose conclusion is $\sim Q$ and premises are $P\rightarrow(\sim P\rightarrow Q)$, $Q\rightarrow\sim P$ and P.

   (c) (i) Construct a truth table for the compound statement that corresponds to the following circuit:



   (ii) Draw a simple network diagram for the statement $(P\rightarrow Q)\wedge(P\vee Q)$.

3. (a) If $\underline{a}=\underline{i}-\underline{j}+2\underline{k}$ and $\underline{b}=\underline{i}+\underline{j}$, find the unit vector orthogonal to both $\underline{a}$ and $\underline{b}$.

   (b) The position vectors of the points A and B are $2\underline{i}+3\underline{j}-\underline{k}$ and $-\underline{i}+5\underline{j}+7\underline{k}$ respectively. If C divides $\overline{AB}$ internally in the ratio $2:1$, find the position vector of point C.

   (c) Using the cosine rule, show that in the triangle ABC, $\underline{c}=\underline{b}\cos A+\underline{a}\cos B$.

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4. (a) If $2x+10yi-4y=-12+5i$, find the values of x and y.

(b) Express $(\cos\theta+i\sin\theta)^{-n}$ in the form $a+ib$.

(c) Given that $z=x+iy$, express the complex number $\frac{z+i}{iz+2}$ in polynomial form and hence find the resulting complex number when $z=1+2i$.

SECTION B (40 Marks)

Answer **two (2)** questions from this section.

5. (a) If A, B and C are angles of a right angled triangle such that $\cos A=\frac{3}{5}$ and $\cos B=\frac{5}{13}$, find the value of $\tan 2A$, $\cos(A+B)$ and $\operatorname{cosec}(A-B)$ in the form $\frac{x}{y}$.

(b) (i) Show that $\cot\left(x+\frac{\pi}{2}\right)-\tan\left(x-\frac{\pi}{2}\right)=\frac{2\cos 2x}{\sin 2x}$.

(ii) Solve the equation $4\cos 2\theta-2\cos\theta+3=0$, for $0^\circ\leq\theta\leq 360^\circ$.

(c) Express $\cos^4\theta$ in terms of cosines multiples of θ.

6. (a) By using the first five terms in the expansion of $(1+x)^n$, find the value of $(1.98)^{10}$ correct to three decimal places.

(b) The polynomial x^5+4x^2+ax+b leaves the remainder of $2x+3$ when it is divided by x^2-1. Use the remainder theorem to find the values of a and b.

(c) The roots of the quadratic equation $x^2+2mx+n=0$ differ by 2. Show that $m^2=1+n$.

(d) If $A=\begin{pmatrix}4 & -1 & 1\\ 0 & 0 & 2\\ m & -1 & 1\end{pmatrix}$ is singular, find the value of m.

(e) Use Cramer's rule to solve the following system of equations: $\begin{cases}5x+6y+4z=5\\ 7x-4y-3z=8\\ 2x+3y+2z=2\end{cases}$

7. (a) Form a differential equation whose solution is $x=\tan(Ay)$.

(b) Solve the differential equation $\frac{d^2\theta}{dt^2}-4\frac{d\theta}{dt}+4\theta=\frac{3}{7}$.

(c) A biologist is researching the population of a specie. She tries a number of different models for the rate of growth of the population and solves them to compare with observed data. Her first model is $\frac{dn}{dt} = kn\left(1-\frac{n}{a}\right)$ where n is the population at time t years, k is a constant and a is the maximum population sustainable by the environment. Given that $k = 0.2$, $a = 100000$ and the initial population is 30000;

(i) find the general solution of the differential equation.

(ii) estimate the population after 5 years to 2 significant figurers.

8. (a) Express $x^2 + y^2 = 2x + 2y$ in polar form.

(b) Find the equation of the chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ joining the points whose eccentric angles are θ and ϕ.

(c) Show that $P(a\sec\theta, b\tan\theta)$ lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, hence find the equation of the tangent line at point P on the given hyperbola.

(d) Show whether the equation of a normal to the parabola $y^2 = 4ax$ at point (x_1, y_1) is $(x - x_1)y_1 + 2a(y - y_1) = 0$.

(e) (i) Change the polar equation $r^2(b^2\cos^2\theta + a^2\sin^2\theta) = a^2b^2$ into the Cartesian equation.

(ii) Draw the graph of $r = 2(1 + \cos\theta)$.