THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2 **ADVANCED MATHEMATICS 2**

(For Both School and Private Candidates)

Time: 3 Hours **Year : 2023**

Instructions

1. This paper consists of sections A and B with a total of **eight (8)** questions.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. Section A carries **sixty (60)** marks and section B carries **forty (40)** marks.
4. All work done in answering each question must be shown clearly.
5. NECTA's mathematical tables and non-programmable calculators may be used.
6. All writing must be in **blue** or **black** ink **except** drawing which must be in pencil.
7. Cellular phones and any unauthorised materials are **not** allowed in the examination room.
8. Write your **Examination Number** on every page of your answer booklet(s).

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SECTION A (60 Marks)

Answer **all** questions in this section.

1. (a) In a school garden, 15 percent of tomatoes are on average defective. Prepare the probability distribution table of obtaining 0, 1, 2, 3, 4 and 5 defective tomatoes in a random batch of 20 tomatoes using:
   (i) Binomial distribution,
   (ii) Poison distribution.

   (b) Compute the mean and standard deviation of the two cases in part (a).

   (c) The mean and standard deviation recorded for 100 students in a senior Mathematics contest examination for the year 2014 were 64 and 16 respectively. Suppose their marks are normally distributed, find the number of students who scored between 30% and 70% inclusive.

2. (a) By using laws of propositions of algebra, simplify $(P \to (Q \vee \sim R)) \to (P \wedge Q)$.

   (b) Use the truth table to verify whether $\sim(P \leftrightarrow Q) \equiv (P \wedge \sim Q)$ or not.

   (c) Test the validity of the argument: "Every time we celebrate my mother's birthday, I always bring her flowers. It is my mother's birthday or I wake up late. I did not bring her flowers. Therefore, I woke up late."

3. (a) Show that the position vectors $2\underline{i} - \underline{j} + \underline{k}$, $\underline{i} - 3\underline{j} - 5\underline{k}$ and $3\underline{i} - 4\underline{j} - 4\underline{k}$ mark the vertices of a right angled triangle.

   (b) The vector $\overrightarrow{PQ}$ has the magnitude of 5 units. If it is inclined at an angle of 150° to the x-axis, express $\overrightarrow{PQ}$ in the form $a\underline{i} + b\underline{j}$ where $a, b \in \Re$.

   (c) A certain particle is displaced by the forces $\underline{F}_1 = 2\underline{i} - 5\underline{j} + 6\underline{k}$ and $\underline{F}_2 = -\underline{i} - 2\underline{j} - \underline{k}$ from point A to B. If the position vectors of points A and B are $4\underline{i} - 3\underline{j} - 2\underline{k}$ and $6\underline{i} - \underline{j} - 3\underline{k}$ respectively, determine the work done by forces on the particle.

4. (a) If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, show that $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$. Hence, verify that $\left|\frac{z-2}{z+3i}\right| = 4$ represents a circle.

   (b) If α and β are two roots of the equation $Z^2 + 4Z + 8 = 0$, without solving the equation $Z^2 + 4Z + 8 = 0$, express $\frac{\alpha + \beta + 4i}{\alpha\beta + 8i}$ in its simplest form.

   (c) Find all the complex roots of $z^3 = 1$.

SECTION B (40 Marks)

Answer **two (2)** questions from this section.

5. (a) Factorize $\cos\theta - \cos 3\theta - \cos 5\theta + \cos 7\theta$.

(b) Show that $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$. Hence, find $\tan y$ if $\tan(2x+y) = 2$ and $\tan 2x = \frac{3}{2}$.

(c) Given that $\sin x = \frac{3}{5}$ and $\cos y = \frac{24}{25}$, where angle x is obtuse and angle y is acute, find the exact values of $\cos(x+y)$ and $\cot(x-y)$.

(d) Express $3\sin x - 2\cos x$ in the form $R\sin(x-\beta)$.

(e) Use the results obtained in part (d) to solve the equation $3\sin x - 2\cos x = 1$.

6. (a) Use the Binomial theorem to expand $\frac{1}{(4-x)^2}$ in ascending powers of x up to the term containing x^3.

(b) Find the partial fractions of the expression $\frac{x^2-2x+1}{(x+1)^2}$.

(c) If α, β and μ are the roots of the equation $2x^3 - x^2 + 1 = 0$, find the equation whose roots are $\alpha+1$, $\beta+1$ and $\mu+1$.

(d) Without using a calculator, find the inverse of $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & -2 \end{pmatrix}$.

(e) Use the inverse obtained in part (d) to solve the system of the simultaneous equations
$$\begin{cases} x+y+z=2 \\ 2x-3y+4z=-4. \\ 3x-2y+4z=-9 \end{cases}$$

7. (a) Show that $y = Ae^{2x}\cos(3x+\varepsilon)$ is a solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$, where ε is an arbitrary constant.

(b) Find the general solution of the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 6e^x + \sin x$.

(c) Solve the differential equation $(2x-1)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$, given that when $x=0$, $y=2$ and $\frac{dy}{dx} = 3$.

(d) The rate of increase in the population of a certain village is proportional to the number of its inhabitants present at any time. If the population of the village was 20,000 in the year 1999 and 25,000 in the year 2004, what was the population of the village in 2009?

8. (a) A point moves so that its distance from the point $(3,2)$ is half its distance from the line $2x+3y=1$.

(i) Show that the locus of the point is a circle.

(ii) What is the centre and radius of the circle?

(b) Show that the equation of a normal at the point $(a\cos\theta, b\sin\theta)$ to the ellipse $b^2x^2+a^2y^2=a^2b^2$ is $ax\sin\theta-by\cos\theta=(a^2-b^2)\sin\theta\cos\theta$.

(c) If the normal at P in part (b) meets the x-axis at Q and the y-axis at R, find the greatest value of the area of the triangle OQR, where O is the origin.

(d) Sketch the graph of $r^2=a^2\sin 2\theta$.