THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2 **ADVANCED MATHEMATICS 2**

(For Both Private and School Candidates)

Time: 3 Hours **Year : 2024**

Instructions

1. This paper consists of sections A and B with a total of **eight (8)** questions.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. All work done in answering each question must be shown clearly.
4. NECTA's mathematical tables and non-programmable calculators may be used.
5. All writing must be in **black** or **blue** ink, **except** drawings which must be in pencil.
6. Communication devices and any unauthorised materials are **not** allowed in the examination room.
7. Write your **Examination Number** on every page of your answer booklet(s).

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SECTION A (60 Marks)

Answer **all** questions in this section.

1. (a) Find the probability of getting between 2 and 5 heads inclusive in 9 tosses of a fair coin by using:

   (i) the binomial distribution,

   (ii) the normal approximation to the binomial distribution.

   (b) An envelope contains 48 office pins and 60 optical pins with one third of the office pins rusted and one quarter of the optical pins rusted. If one item is chosen at random from the envelope, find the probability that the item selected is:

   (i) a rusted office pin.

   (ii) a rusted optical pin.

   (iii) a rusted pin or an office pin.

   (c) In how many ways can an escort of six policemen be chosen from ten policemen and in how many of the escorts will a particular policeman be included?

2. (a) Use a truth table to show that $(p \vee q) \rightarrow p$ is logically equivalent to $(\sim p \wedge \sim q) \vee p$.

   (b) Draw the simplest electric network for the proposition $q \vee (p \wedge q') \vee (r \wedge p')$.

   (c) Simplify the statement $\sim (p \vee q) \vee (\sim p \wedge q)$ using laws of propositions.

   (d) Test the validity of the argument: "If there are remedial classes, standard IV pupils will understand lessons well. If standard IV pupils understand lessons well, there will be no failure in assessments, but there is failure in assessments. Therefore, there are no remedial classes."

3. (a) (i) Find the projection of vector $\underline{a} = \underline{i} + 3\underline{j} - 5\underline{k}$ onto $\underline{b} = 4\underline{i} + 4\underline{j} - 7\underline{k}$.

   (ii) If the position vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are $\underline{i} + 2\underline{j} + 4\underline{k}$ and $3\underline{i} + \underline{j} - 7\underline{k}$ respectively, find the position vector $\overrightarrow{OR}$ which divides $\overrightarrow{PQ}$ in the ratio 2:3 internally.

   (b) The position vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are $5\underline{i} - 6\underline{j} + \underline{k}$, $\underline{i} - 3\underline{k}$ and $-\underline{i} + \underline{j} + 2\underline{k}$ respectively. Find the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$ correct to two significant figures.

(c) Determine the area of a parallelogram formed by vectors $\underset{\sim}{a} = 4\underset{\sim}{i} + 9\underset{\sim}{j} - 6\underset{\sim}{k}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + 5\underset{\sim}{j} - 2\underset{\sim}{k}$ correct to two decimal places.

4. (a) If $3m + 10ni + 5n = 13 + 20i$ where $m, n \in \mathbb{R}$ and $i^2 = -1$, calculate the values of m and n.

(b) Use de Moivre's theorem to prove that $\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$.

(c) (i) Given that $(x+iy)^3 = u + iv$, show that $\dfrac{u}{x} + \dfrac{v}{y} = 4(x^2 - y^2)$.

(ii) If z is a complex number, show that $\left|\dfrac{z-2}{z+3i}\right| = 4$ is an equation of a circle.

(d) If $x + iy = (-3-2i)^n$, where $x, y \in \mathbb{R}$, $n \in \mathbb{Z}$, prove that $x^2 + y^2 = 13^n$.

SECTION B (40 Marks)

Answer **two (2)** questions from this section.

5. (a) (i) Express $\sqrt{\dfrac{1-\sin 2\beta}{1+\sin 2\beta}}$ in terms of $\tan\beta$.

(ii) Use compound angle formulae to prove that $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$.

(b) Find the general solution of $\sin\theta + \sqrt{3}\cos\theta = 1$.

(c) (i) If $\tan x = \operatorname{cosec} x - \sin x$, prove that $\tan^2\left(\dfrac{1}{2}x\right) = -2 \pm \sqrt{5}$.

(ii) Find a positive value of angle α which satisfies the equation $\tan^{-1} 3\alpha + \tan^{-1}\alpha = \dfrac{\pi}{4}$. Give the answer correct to three decimal places.

(d) Express $3\cos\theta - 4\sin\theta$ in the form $R\cos(\theta - \alpha)$ and hence solve the equation $3\cos\theta - 4\sin\theta$ in the interval $0 \le \theta \le 2\pi$.

6. (a) Find the values of x for which $\dfrac{x^2 + x - 2}{x^2 - 2x - 3} < 0$.

(b) Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$.

(c) Find the inverse of matrix $A=\begin{pmatrix} 1 & 2 & 1 \\ -1 & 3 & 2 \\ 2 & 1 & -3 \end{pmatrix}$.

(d) Use the inverse matrix obtained in (c) to solve the following simultaneous equations:

$$\begin{cases} x+2y+z=1 \\ -x+3y+2z=-3 \\ 2x+y-3z=0 \end{cases}$$

7. (a) (i) Solve the differential equation $(1+x^2)\frac{dy}{dx}+1+y^2=0$ given that $y(0)=1$.

(ii) Find the general solution of the differential equation $(x-y)dy-(x+y)dx=0$.

(b) The slope of a curve defined by $M=h(x)$ at any point is proportional to the expression x^2+1. If the curve passes through points $(3,0)$ and $(0,36)$, find the equation of the curve.

(c) The temperature y of a body at time t satisfies the differential equation $6\frac{d^2y}{dt^2}+\frac{dy}{dt}=0$.

(i) Find y in terms of t given that $y=63°C$ when $t=0$ and $y=33°C$ when $t=6\ln 6$ minutes.

(ii) How cool does the body get after 30 minutes? Give the answer correct to two decimal places.

(d) Verify whether the following equations belong to a family of the exact differential equations:

(i) $y^2dx+(4xy+1)dy=0$.

(ii) $(2x\cos y+3x^2y)dx+(x^3 - x^2\sin y - y)dy=0$.

8. (a) (i) Find the equation of an ellipse whose center is at the origin of the xy-plane, major axis is on the y-axis and passes through the points $(3,2)$ and $(1,6)$.

(ii) Change $r=\frac{16}{5-3\cos\theta}$ into Cartesian equation.

(b) (i) Sketch the graph of $r=1+2\cos(t)$ for $0\le t\le 2\pi$.

(ii) Prove that the equation of a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point (x_1, y_1) is $\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$.

(c) Find the points where curves of the polar equations $r = 1 + \cos\theta$ and $\sqrt{3} = r \operatorname{cosec}\theta$ meet.

(d) The axis of an arch is vertical. If the arch is 10 m high and 5 m wide at a base, how wide is it at 2 m from the vertex?