THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2 **ADVANCED MATHEMATICS 2**

(For Both School and Private Candidates)

Duration: 3 Hours **Year : 2025**

Instructions

1. This paper consists of sections A and B with a total of **eight (8)** questions.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. All work done in answering each question must be shown clearly.
4. NECTA's mathematical tables and non-programmable calculators may be used.
5. All writing must be in **black** or **blue** ink, **except** drawings which must be in pencil.
6. Communication devices and any unauthorised materials are **not** allowed in the examination room.
7. Write your **Examination Number** on every page of your answer booklet(s).

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SECTION A (60 Marks)

Answer **all** questions in this section.

1. (a) A function is defined as $f(x)=\frac{3}{26}x^2$;

   (i) Show that this function is a probability density function of a random variable X for the interval $1 \leq x \leq 3$.

   (ii) Find the mean and standard deviation of X for the interval given in part (a) (i).

   (b) The mean weight and standard deviation for 2500 students at Kabarimu Primary School as recorded in 1989 were 35 kilograms and 6.3 kilograms respectively. Assuming that their weights are normally distributed, determine;

   (i) the probability that a student picked at random will weigh more than 32 kilograms.

   (ii) the number of overweight students at this school, if an overweight is considered for all students weighing above 55 kilograms.

2. (a) For each of the following statements, write down the corresponding contrapositive statement:

   (i) If the graph of $y = mx + b$ is an oblique, then $m \neq 0$.

   (ii) If a quadrilateral has two sides of equal lengths, then the quadrilateral has two equal angles.

   (iii) If it rains today, then it will rain tomorrow.

   (iv) If Ubaya committed a crime, then he will be at the crime scene.

   (b) Draw an electrical network corresponding to the proposition $(P \wedge \sim Q) \vee ((\sim P \vee R) \wedge Q)$.

   (c) Eliminate the logical connective symbol "$\rightarrow$" from the statement $(\sim P \wedge \sim Q) \rightarrow (Q \rightarrow R)$ and then simplify the statement as much as possible.

   (d) Test the validity of the argument "If I like Mathematics, then I will study hard. Either I study hard or I fail. Therefore, if I fail then I do not like Mathematics."

3. (a) Suppose $\underset{\sim}{p} = t\underset{\sim}{i} + t^2\underset{\sim}{j} + 2t\underset{\sim}{k}$ and $\underset{\sim}{q} = (1+t^2)\underset{\sim}{i} + (2-t)\underset{\sim}{j} + 3\underset{\sim}{k}$, find the derivative of $\underset{\sim}{p} \bullet \underset{\sim}{q}$ with respect to t.

   (b) The velocity of a body at time t is given by $\underset{\sim}{v} = 3t^2\underset{\sim}{i} - 2t\underset{\sim}{j} + 4\underset{\sim}{k}$. Find the expression for the acceleration $\underset{\sim}{a}$ and the displacement $\underset{\sim}{s}$ of the body at time t, given that $\underset{\sim}{s} = 3\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ when $t = 1$.

   (c) Determine the area of the parallelogram ABCD with vertices $A(1,-2,3)$, $B(4,3,1)$, $C(2,2,1)$ and $D(5,7,-3)$.

4. (a) Use the De Movre's theorem to find the roots of the equation $x^6 - 2x^3 + 4 = 0$.

(b) Find the Cartesian equation of the locus of $Z = x + iy$, when $|Z-1| = 2|Z|$.

(c) Solve for $Z^3 = 8i$.

SECTION B (40 Marks)

Answer **two (2)** questions from this section.

5. (a) Solve the equation $3 + 2\sin 2\theta = 2\sin\theta + 3\cos^2\theta$ for $0 \le \theta \le 180$.

(b) Find the expression of $\sin 3x$ in terms of $\sin x$.

(c) Prove that $\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$.

(d) Solve the equation $5\cos x - 2\sin x = 2$ for $-360^\circ \le \theta \le 360^\circ$ using the substitution $t = \tan\left(\frac{x}{2}\right)$.

6. (a) Show that $\log_{16} xy = \frac{1}{2}\log_4 x + \frac{1}{2}\log_4 y$, hence use it to solve simultaneously the system of the following equations $\begin{cases} \log_{16}(xy) = 3.5 \\ \dfrac{\log_4 x}{\log_4 y} = -8 \end{cases}$.

(b) Use the principles of mathematical induction to prove that $\sum_{r=1}^{n} r^3 = \left[\frac{1}{2}n(n+1)\right]^2$.

(c) Use Cramer's rule to solve $\begin{cases} 7x + y + z = -1 \\ x - 3y + 2z = 0 \\ x + 4y - 3z = 4 \end{cases}$

(d) In the expansion of $\left(x^2 + \frac{1}{x}\right)^n$, the coefficients of the fourth and ninth term are equal. Find the value of n and the sixth term of the expansion.

7. (a) Find the general solution of the following differential equations;

(i) $\frac{dy}{dx} - \frac{y}{x} = x^2$

(ii) $\frac{dy}{dx} - x\tan(y - x) = 1$

(b) Eliminate the constants A and B in the equation $x = A\cos(2t + B)$.

(c) A particle P falls vertically under gravity. The air resistance of the falling particle is taken to be proportional to its speed at any instant. The distance x at which the particle has fallen after t seconds, is given by the differential equation

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$\frac{d^2x}{dt^2} = g - k\frac{dx}{dt}$, where k and g are positive constants. Find the distance x as a function of t given $\frac{dx}{dt} = 0$ and $x = 0$ when $t = 0$.

(d) Determine the solution of the differential equation $(2x-1)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$ given that $x = 0$, $y = 2$ and $\frac{dy}{dx} = 3$.

8. (a) An equilateral triangle is inscribed in a parabola $y^2 = 4ax$, whose vertex is at the vertex of the parabola. Find the length of the side of the triangle.

(b) If two tangent lines to the ellipse $9x^2 + 4y^2 = 36$ intersect the y-axis at point $P(0,6)$, find the point of tangency.

(c) Find the equation of a circle which passes through points $A(1,2)$, $B(2,5)$ and $C(-3,4)$. Write the equation in the form $x^2 + y^2 + 2fx + 2fy + c = 0$ where f, g and c are constants.

(d) What is the length of the tangents from point $(6,2)$ to a circle with centre $(2,1)$ and radius 2.

(e) Write the equation of ellipse $18x^2 + 12y^2 - 144x + 48y + 120 = 0$ in a standard form.

(f) Sketch the ellipse having the these characteristics: a central ellipse, foci at $(\pm 4,0)$ and vertices at $(\pm 5,0)$.