

THE UNITED REPUBLIC OF TANZANIA  
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA  
ADVANCED CERTIFICATE OF SECONDARY EDUCATION  
EXAMINATION

141

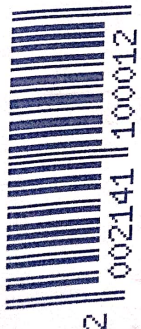
BASIC APPLIED MATHEMATICS  
(For Both School and Private Candidates)

Time: 3 Hours

Year: 2020

**Instructions**

1. This paper consists of **ten (10)** questions. Each question carries **10** marks.
2. Answer **all** questions.
3. All work done in answering each question must be shown clearly.
4. Non-programmable calculators and NECTA mathematical tables may be used.
5. Cellular phones and any unauthorized materials are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s)



1. Use a non-programmable scientific calculator to compute:

(a) the value of  $\frac{3+3(\sqrt[3]{0.65})}{3-3(\sqrt[3]{0.65})}$  correct to 4 significant figures.

(b) the mean and standard deviation of 33, 28, 26, 35 and 38 correct to 2 decimal places.

(c) the value of  $\frac{{}^5C_2 + {}^9P_6}{11!}$  correct to 4 decimal places.

Handwritten:  $5.5987$   
 $13.95$

Handwritten:  $1.51544 \times 10^{-3}$   
 $= 0.00151544$

2. (a) The function  $f$  is defined as  $f(x) = \begin{cases} 2x-1 & \text{if } -2 < x \leq 1 \\ x^2 & \text{if } 1 < x \leq 2 \\ 10-3x & \text{if } 2 < x < 3 \end{cases}$

- (i) Sketch the graph of  $f(x)$ .
- (ii) State the domain and range of  $f(x)$ .

(b) Given that  $f(x) = 3x + 3$  and  $g(x) = x + 3$ , find:

- (i)  $(f \circ g)(x)$ .
- (ii)  $(f \circ g)^{-1}(x)$ .

Handwritten notes for Q2:  
 $x_1 = 2$   
 $x_2 = 0.2$   
 $x \leq \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$   
 $\frac{a^{10}}{b^{10}} = a^{10} \div b^{10}$   
 $\frac{dx}{dt} = \dots$

3. (a) Use the substitution method to solve the following system of equations:

$$\begin{cases} 3x - y = 9 \\ x^2 + xy + 2 = 0 \end{cases}$$

(b) Find the value(s) of  $x$  satisfying the equation  $4^x - 6(2^x) - 16 = 0$ .

(c) Find the sum of the first  $n$  terms of the series  $1 + 3 + 5 + \dots$

4. (a) Find the first derivative of  $f(x) = x^2$  from first principles.

(b) Find the slope of the tangent to the curve  $8x^3 + xy^3 - 5y^2 = 0$  at  $(1, -1)$ .

(c) Use second derivative test to classify the stationary point(s) of the curve  $f(x) = 2x^3 + 3x^2 - 12x - 5$ .

5. (a) Integrate  $\sin^2 2x \cos 2x$  with respect to  $x$ .

(b) Evaluate  $\int_0^{\sqrt{a}} \frac{x}{x^2 + a} dx$  (express your answer in the form  $m\sqrt{n}$ ).

(c) Find the area enclosed between the curves  $y = x^2 + 2$  and  $y = 10 - x^2$ .

Handwritten notes for Q5:  
 $\frac{dx}{dt} = \dots$   
 $2x + b$

6. The following table shows litres of milk produced by 131 cows each day.

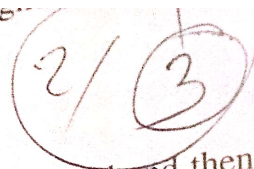
Litres of milk	5-10	11-16	17-22	23-28	29-34	35-40
Number of cows	15	28	37	26	18	7

- (a) Estimate the mode.
- (b) Draw the cumulative frequency curve and use it to estimate the median.

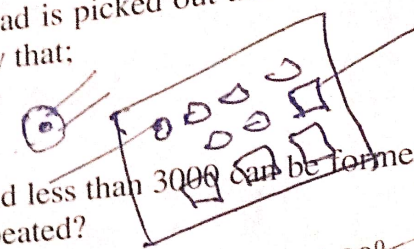
Handwritten:  $\frac{1}{3} \times \frac{2^x - 2^x}{-x} = 4 \times \frac{1}{3}$

Handwritten:  $4^x - 2^x = \frac{4}{6}$   
 $2^x - 2^x = \dots$

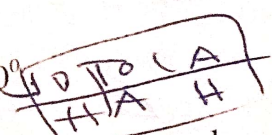
7. (a) A family has three children. Assuming equal chances for boys and girls.
- (i) use tree diagram to show all possible outcomes.
  - (ii) find the probability that two are girls and one is a boy.



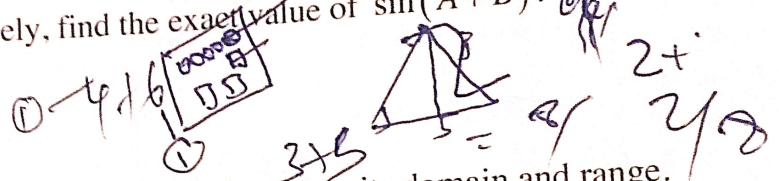
- (b) A bag contains 6 red and 4 blue beads. A bead is picked out and retained and then a second bead is picked out. Find the probability that:
- (i) both beads are red.
  - (ii) the beads are of different colours.
- (c) How many odd numbers greater than 2000 and less than 3000 can be formed using the digits 1, 2, 3, 4, 5 and 6 if each digit is not repeated?



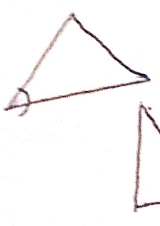
8. (a) (i) Evaluate the value of  $\tan 15^\circ$  from sine and cosine of  $45^\circ$  and  $30^\circ$ .
- (ii) Given that  $\sin A = \frac{3}{5}$  and  $\cos B = \frac{15}{17}$  where  $A$  and  $B$  are angles in the first and second quadrants respectively, find the exact value of  $\sin(A+B)$ .



(b) Prove that  $\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$ .



9. (a) Sketch the graphs of  $f(x) = \ln|x|$  for  $x \in \mathbb{R}$  and hence state its domain and range.
- (b) The amount ( $A$ ) of the radioactive isotope Carbon-14 at any time  $t$  is given by the formula  $A(t) = A_0 e^{kt}$  where  $A_0$  is the initial amount of the element. If the half-life of the radioactive isotope Carbon-14 is about 5730 years:
- (i) express the amount of Carbon-14 left from an initial  $N$  milligrams as a function of time  $t$  in years.
  - (ii) what percentage of the original amount of Carbon-14 left after 20,000 years?



10. An aircraft has  $600 \text{ m}^2$  of cabin space and can carry  $5,000 \text{ kg}$  of luggage. An economy class passenger gets  $3 \text{ m}^2$  of space and is allowed to travel with  $20 \text{ kg}$  of luggage. The first class passenger gets  $4 \text{ m}^2$  of space and is allowed to have  $50 \text{ kg}$  of luggage in the aircraft. In the aircraft, there is space for at least 50 economy class passengers. The profit per flight for the economy and first class passengers are  $40,000/-$  and  $100,000/-$  respectively.
- (a) Write down all the constraints.
  - (b) Use graphical method to find the number of economy passengers and first class passengers which will give the maximum profit per flight.

Handwritten work for Question 10(b):

Let  $x$  be the number of economy class passengers and  $y$  be the number of first class passengers.

Constraints:

$$3x + 4y \leq 600$$

$$20x + 50y \leq 5000$$

$$x \geq 50$$

$$x \geq 0, y \geq 0$$

Profit function:

$$P = 40000x + 100000y$$

Graphical solution:

$$3x + 4y = 600 \implies y = \frac{600 - 3x}{4}$$

$$20x + 50y = 5000 \implies 2x + 5y = 500$$

Intersection point:

$$3x + 4\left(\frac{2x + 500 - 2x}{5}\right) = 600$$

$$3x + \frac{4(2x + 500 - 2x)}{5} = 600$$

$$3x + \frac{8x + 2000 - 4x}{5} = 600$$

$$\frac{15x + 8x + 2000 - 4x}{5} = 600$$

$$\frac{9x + 2000}{5} = 600$$

$$9x + 2000 = 3000$$

$$9x = 1000$$

$$x = \frac{1000}{9} \approx 111.11$$

Substituting  $x \approx 111.11$  into  $y = \frac{600 - 3x}{4}$ :

$$y = \frac{600 - 3(111.11)}{4} = \frac{600 - 333.33}{4} = \frac{266.67}{4} \approx 66.67$$

Maximum profit:

$$P = 40000(111.11) + 100000(66.67) = 4,444,444 + 6,666,666 = 11,111,110$$

Handwritten notes include:  $\sin A = \frac{op}{hyp}$ ,  $\sin(A+B) = \sin A \cos B + \cos A \sin B$ ,  $\sin 2A = 2 \sin A \cos A$ ,  $\cos 2A = \cos^2 A - \sin^2 A$ ,  $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$ .