# Basic Applied Maths Paper 1 - May 1991
# Answers

1. a) $\log(x^4+3x^3) - \log(x+3) + \log 2 - \log 6 = 2\log x$

$\log\left(\frac{x^4+3x^3}{x+3}\right) + \log\frac{1}{3} = \log x^2$

$x^3 + \frac{1}{3} = x^2$

b) $y^2 = 3^2 \cdot 2^4 = 9 \cdot 16 = 144$

$y = \pm 12$

2. a) Given $N(K \cup L) = N(K) + N(L) - N(K \cap L)$

Let $A, B, C$ be 3 non-empty sets and $D = B \cup C$

Then $A \cup B \cup C = A \cup D$ and.

$N(A \cup D) = N(A) + N(D) - N(A \cap D)$

But $N(D) = N(B) + N(C) - N(B \cap C)$

So, $N(A \cup D) = N(A) + N(B) + N(C) - N(B \cap C) - N(A \cap D)$

$= N(A) + N(B) + N(C) - N(B \cap C) - N(A \cap B) - N(A \cap C) + N(A \cap B \cap C)$

b) $N(D \cup S \cup T) = 28 + 24 + 23 - 18 - 12 - 14 + 9 = 40$

3. a) $a_1x + b_1y + c_1 = 0$ $\quad a_2x + b_2y + c_2 = 0$

$\Rightarrow y_1 = -\frac{a_1}{b_1}x - \frac{c_1}{b_1}$ $\quad \Rightarrow y_2 = -\frac{a_2}{b_2}x - \frac{c_2}{b_2}$

if $y_1 \perp y_2 \Rightarrow m_1 = -\frac{1}{m_2} \Rightarrow -\frac{a_1}{b_1} = -\frac{1}{-\frac{a_2}{b_2}}$

$\Rightarrow -\frac{a_1}{b_1} = \frac{b_2}{a_2}$

$\Rightarrow -a_1a_2 = b_1b_2$

$\Rightarrow a_1a_2 + b_1b_2 = 0$ qed.

b) $y = 2x + 3$

$m = 2$

$m_\perp = -\frac{1}{2}$

$y - 1 = -\frac{1}{2}(x - 3)$

$y = -\frac{1}{2}x + \frac{5}{2}$

4. a) $C(n,2) = 10$

$$\binom{n}{2} = \frac{n!}{(n-2)!\,2!} = 10$$

$$\frac{n(n-1)}{2} = 10$$

$$n^2 - n - 20 = 0 \qquad n = 5$$

b)

| | paint | no paint | |
|---|---|---|---|
| heads | 10 | 24 | 34 |
| no head | 10 | 6 | 16 |
| | 20 | 30 | 50 |

i) $P(\text{no paint \& no head}) = \frac{6}{50}$

ii) $P(\text{head or paint}) = P(\text{no head}) + P(\text{paint}) - P(\text{no head + paint})$

$$= \frac{16}{50} + \frac{20}{50} - \frac{10}{50} = \frac{26}{50}$$

5. $\left[\begin{array}{cc|cc} 2 & -4 & 1 & 0 \\ 6 & 9 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & -2 & 1/2 & 0 \\ 0 & 21 & -3 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 0 & 3/14 & 2/21 \\ 0 & 1 & -1/7 & 1/21 \end{array}\right]$ $C^{-1} = \begin{bmatrix} 3/14 & 2/21 \\ -1/7 & 1/21 \end{bmatrix}$

$\left[\begin{array}{cc|cc} 2 & 4 & 1 & 0 \\ 0 & 3 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 2 & 0 & 1 & -4/3 \\ 0 & 1 & -3 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 0 & 1/2 & -2/3 \\ 0 & 1 & 0 & 1/3 \end{array}\right]$ $D^{-1} = \begin{bmatrix} 1/2 & -2/3 \\ 0 & 1/3 \end{bmatrix}$

$$CD = \begin{bmatrix} 2 & -4 \\ 6 & 9 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 12 & 51 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 4 & -4 & 1 & 0 \\ 12 & 51 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & -1 & 1/4 & 0 \\ 0 & 63 & 3 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 0 & 17/84 & 1/63 \\ 0 & 1 & -1/21 & 1/63 \end{array}\right]$$

$$(CD)^{-1} = \begin{bmatrix} \frac{17}{84} & 1/63 \\ -1/21 & 1/63 \end{bmatrix}$$

$$D^{-1}C^{-1} = \begin{bmatrix} 1/2 & -2/3 \\ 0 & 1/3 \end{bmatrix}\begin{bmatrix} 3/14 & 2/21 \\ -1/7 & 1/21 \end{bmatrix} = \begin{bmatrix} 17/84 & 1/63 \\ -1/21 & 1/63 \end{bmatrix}$$

6. a) $\vec{v} = 2i + 3j - k$

$\vec{u} = -2i + 2j + 2k$

if $\vec{v} \perp \vec{u}$ then $\vec{v} \cdot \vec{u} = 0$

$$\vec{v} \cdot \vec{u} = 2(-2) + (3)(2) + (-1)(2) = 0$$

b) $\vec{v} = 2a+b = 2(i+2j-2k)+(3i+2j+6k)$

$= 5i+6j+2k$

$|\vec{v}| = (25+36+4)^{1/2} = (65)^{1/2} \approx 8.06$

c) $\vec{F} = i-2j+5k$

$(-1,1,0)$ to $(2,4,1)$ $\quad \vec{r} = (2+1)i+(4-1)j+(1-0)k$

$= 3i+3j+k$

$W = \vec{F}\cdot\vec{r} = 3(1)+3(-2)+1(5) = 2\ J$

7.

$2x+3y+z=3$

$x \quad -z=3$

$x+y+z=4$

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 3 \\ 1 & 0 & -1 & 3 \\ 1 & 1 & 1 & 4 \end{array}\right] \to \left[\begin{array}{ccc|c} 0 & 3 & 3 & -3 \\ 1 & 0 & -1 & 3 \\ 2 & 1 & 0 & 7 \end{array}\right] \to \left[\begin{array}{ccc|c} 0 & 1 & 1 & -1 \\ 1 & 1 & 0 & 2 \\ 2 & 1 & 0 & 7 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -3 \\ 1 & 0 & 0 & 5 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right] \qquad \begin{array}{l} x=5 \\ y=-3 \\ z=2 \end{array}$$

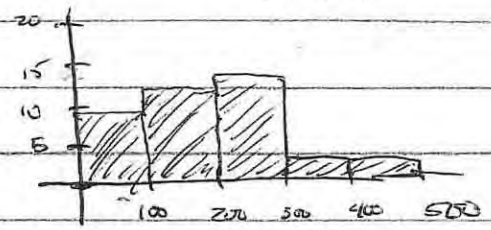
8.

| interval | $x_i$ | $f$ |
|---|---|---|
| 0-99 | 50 | 9 |
| 100-199 | 150 | 11 |
| 200-299 | 250 | 12 |
| 300-399 | 350 | 2 |
| 400-499 | 450 | 2 |
| | | 36 |

a) $\bar{x} = \dfrac{\sum x_i f_i}{\sum f_i} = \dfrac{50\cdot 9+150\cdot 11+250\cdot 12+350\cdot 2+450\cdot 2}{36}$

$= 186$

b)




9. a) $f(x) = \dfrac{1}{x^2-1}$ $\quad D = \mathbb{R} - \{1,-1\}$

$R = \mathbb{R}$

b) $f(x) = \dfrac{1}{x^2+1}$ $\quad D = \mathbb{R}$

$R = \mathbb{R} \geq 0$

2. a) i) $\int (x^2+2)(3x-1)\,dx = \int (3x^3-x^2+6x-2)\,dx = \dfrac{3x^4}{4} - \dfrac{x^3}{3} + 3x^2 - 2x + c$

b) ii) $\int (4x+\frac{3}{x})\,dx = 4\frac{x^2}{2} + 3\ln x + c$ $\quad$ a iii) $\int (2x+3)^2\,dx = \dfrac{1}{6}(2x+3)^3 + c$

iii) $\int_1^4 (x^2+3)\,dx = \dfrac{x^3}{3} + 3x \Big|_1^4 = \dfrac{4^3}{3} + 12 - \dfrac{1}{3} - 3 = 30$

11. a) i) $D(x^2\sin x) = 2x\sin x + x^2\cos x$

ii) $D(4xe^{3x}) = 4e^{3x} + 12xe^{3x}$

iii) $D\left(\frac{x^2-x}{(x+1)^2}\right) = \frac{(2x-1)(x+1)^2-(x^2-x)2(x+1)}{(x+1)^4} = \frac{(2x-1)(x+1)-(x^2-x)\cdot 2}{(x+1)^3}$

b) $\int_1^{\sqrt{3}} \frac{1}{\sqrt{4-2x}}\,dx$ let $u = 4-2x$ $du = -2dx$

$= -\frac{1}{2}\int_2^{4-2\sqrt{3}} u^{-1/2}\,du = -\frac{1}{2}\,u^{1/2}\Big|_2^{4-2\sqrt{3}} = -\frac{1}{2}(4-2x)^{1/2}\Big|_1^{\sqrt{3}}$

$= -\frac{1}{2}\left[(4-2\sqrt{3})^{1/2} - (4-2)^{1/2}\right]$

$= -\frac{1}{2}\left[(4-2\sqrt{3})^{1/2} - \sqrt{2}\right]$

12. a) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

b) $f(x) = x^2-2$ $x_{n+1} = x_n - \frac{x_n^2-2}{2x_n}$

$f'(x) = 2x$

$= \frac{2x_n^2 - x_n^2 + 2}{2x_n} = \frac{x_n^2+2}{2x_n}$

c) $x_0 = 1.5$ $x_1 = \frac{(1.5)^2+2}{2(1.5)} = 1.42$

$x_2 = \frac{(1.42)^2+2}{2(1.42)} = 1.41$

d) $x_0 = 2$ $x_{n+1} = x_n - \frac{x_n^3-9}{3x_n^2}$

$f(x) = x^3-9 = 0$

$f'(x) = 3x^2$ $x_1 = 2 - \frac{2^3-9}{3\cdot 4} = 2.083$

$x_2 = 2.083 - \frac{(2.083)^3-9}{3(2.083)^2} = 2.080$

13. a) i) $V = \int a\,dt = \int -4\,dt = -4t+c$

$v(t) = -4t-2$

ii) $s = \int v\,dt = \int(-4t-2)\,dt = -2t^2-2t+d$

$s(0) = 0 \Rightarrow d = 0$ $s(t) = -2t^2-2t$