THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

141

BASIC APPLIED MATHEMATICS
(For Both School and Private Candidates)

Time: 3 Hours

Friday 03 May 2002 a.m.

Instructions

1. This paper consists of sections A and B.
2. Answer **ALL** questions in section A and any **FOUR (4)** questions from section B.
3. All answers must be written in the answer booklet provided.
4. All work done in answering each question must be shown clearly.
5. Mathematical tables, mathematical formulae, slide rules and unprogrammable pocket calculators may be used.
6. Cellular phones are not allowed in the examination room.
7. Write your Examination Number on every page of your answer booklet.

This paper consists of 4 printed pages.

SECTION A (60 marks)

Answer **ALL** questions in this section showing all necessary steps and answers.

1. Use common logarithms to find the value of the following:

   (a) $\frac{(7.04)^2}{(31.7)\sqrt{1.09}}$ (03 marks)

   (b) $(115)^{1/2} + (35.2)^{2/3}$ (03 marks)

2. The population of a sample is given by $P(t) = 10\,000e^{0.4t}$, where t is in years. Use a non-programmable calculator to find the time to the nearest whole number on which the population of the sample will double.

   (06 marks)

3. Find the equations of the straight lines which pass through the centre of the circle $x^2 + y^2 - 4x - 6y - 5 = 0$ and at the points where the given circle cuts the x axis (each line at one point respectively). (06 marks)

4. Let g be the function which is the set of all ordered pairs (x,y) such that $g(x) = \sqrt{x(x-2)}$.

   (a) Find the domain and range of g.
   (b) Draw a sketch of the graph of g. (06 marks)

5. (a) Show that $2x^2 - 3x + 4 = 0$ has no real roots. (02 marks)
   (b) If $3x^2 - 6x + 8 = 0$ has roots α and β, find the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. (04 marks)

6. The variables x and y are related by $y = ax^n$, where a and n are constants. Determine a linear relationship between variables related to y and x. By using a graph and the data given below, estimate the values of a and n.

x	1	2	3	4	5	6
y	3	4.2	5.2	6	6.7	7.3

   (06 marks)

7. (a) Eliminate θ from
   $x = \cos 2\theta + 1$
   $y = \sin\theta + 1$

   (02½ marks)

   (b) Solve the equation $\sin 2\theta - \sin\theta = 0$ for θ between 0° and 180° inclusive. (03½ marks)

8. Solve the following equations:

   (a) $\log_x 3 + \log_x 27 = 2$

   (b) $\log_3 x + 3\log_x 3 = 4$ (03 marks)

   (03 marks)

9. (a) Find the value of $\frac{dy}{dx}$ at the point (1, -1) if $x^2 - 3xy + 2y^2 - 2x = 4$. (02½ marks)

   (b) If $x = \frac{2t}{t+2}$ and $y = \frac{3t}{t+3}$, find the value of $\frac{dy}{dx}$ at the point $(\frac{2}{3}, \frac{3}{4})$ (03½ marks)

. Evaluate:

(a) $\int_2^3 x\sqrt{x-2}\,dx$ (03 marks)

(b) $\int_0^2 \frac{x}{(x^2+1)^2}\,dx$ (03 marks)

SECTION B (40 marks)

Answer any **FOUR (4)** questions from this section showing all necessary steps and answers.

1. Given the vectors $\underline{a} = 2\underline{i} + \underline{j} - 2\underline{k}$ and $\underline{b} = \underline{i} - \underline{k}$,

(a) Find the vector $\underline{c}$ such that $\underline{a} + 2\underline{b} + \underline{c} = \underline{0}$ (02 marks)

(b) What is the sine of the acute angle enclosed by the vectors $\underline{a}$ and $\underline{b}$? (04 marks)

(c) Find the unit vector perpendicular to the vectors $\underline{a}$ and $\underline{b}$. (04 marks)

2. The following table summarises the masses measured to the nearest kg, of 200 animals of the same species:

Mass (kg)	Number of animals
75 – 79	7
80 – 84	30
85 – 89	66
90 – 94	57
95 – 99	27
100 – 104	13

Calculate

(a) the mean using a deviation approach

(b) the standard deviation of masses correct to two decimal places. (10 marks)

13. In a certain school, there is an equal number of boys and girls students. ¼ of boys and $\frac{1}{10}$ of the girls go to school on foot. $\frac{1}{3}$ of the boys and ½ of the girls go to school by school buses, and the rest go by their parents vehicles. Find the proportion of the students:

(a) that are girls who go by their parents' vehicles

(b) that go by their parents' vehicles. (10 marks)

14. (a) A transformation M is given by the matrix M where M = $\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$. Find

(i) the image of point (-2, 5) under M
(ii) the inverse of M. (05 marks)

(b) Solve the following system of equations by the matrix method.

$$2x - 3y + z = 3$$
$$-x + 4y + 3z = 16$$
$$3x + 2y - 2z = 1$$

(05 marks)

15. Students are about to take a test that contains questions of type A worth 10 points and questions of type B worth 25 points. They must do at least 3 questions of type A but not more than 12. They must do at least 4 questions of type B but not more than 15. In total they cannot do more than 20 questions. How many of each type of question must a student do to maximize the score? What is the maximum score? **(10 marks)**

16. (a) Find an expression for the area under the curve $y = x^2(2x^3 + 3)^5$. **(04 marks)**

(b) A rectangular block has a square base whose length is x centimetres. Its total surface area is $150\ cm^2$.

(i) Show that the volume of the block is $\frac{1}{2}(75x - x^3)\ cm^3$.

(ii) Calculate the dimensions of the block when its volume is maximum. **(06 marks)**