

**THE UNITED REPUBLIC OF TANZANIA**  
**NATIONAL EXAMINATIONS COUNCIL**  
**ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION**  
**141 BASIC APPLIED MATHEMATICS**

(For Both School and Private Candidates)

**Time: 3 Hours**

**ANSWERS**

**Year: 2013**

**Instructions**

1. This paper consists of **Ten (10)** questions.
2. Answer all questions.
3. **All** work done and answers of each question must be shown clearly.
4. NECTA'S Mathematical tables and Non-programmable calculations may be used
5. All writing must be in **black** or **blue** ink, **except** drawing which must be in pencil.

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*Prepared by: Maria Marco for TETE*

1. (a) By using a non-programmable calculator: Compute the value of  $\sqrt{(3.141 \times 2.751) / (47 \times 39.8)}$  and write your answer in 6 decimal places

First, compute the numerator:  $3.141 \times 2.751 = 8.640891$

Compute the denominator:  $47 \times 39.8 = 1870.6$

Fraction:  $8.640891 / 1870.6 \approx 0.004619$

Square root:  $\sqrt{0.004619} \approx 0.067963$

To 6 decimal places: 0.067963

Answer: 0.067963

(b) Find the sum of the finite series  $[1/2 + 1/3 + 1/5 + 1/7 + 1/9 + 1/11]$  and write your answer in 4 decimal places

Sum =  $1/2 + 1/3 + 1/5 + 1/7 + 1/9 + 1/11$

$= 0.5 + 0.3333 + 0.2 + 0.1429 + 0.1111 + 0.0909$

$\approx 0.5 + 0.3333 = 0.8333$

$0.2 = 1.0333$

$0.1429 = 1.1762$

$0.1111 = 1.2873$

$0.0909 = 1.3782$

To 4 decimal places: 1.3782

Answer: 1.3782

(c) Find the mean length of 200 engine components that were measured and recorded as follows:

Length (mm) | 198 | 199 | 200 | 201 | 202

Frequency | 8 | 30 | 132 | 24 | 6

Mean =  $\Sigma (\text{length} \times \text{frequency}) / \text{total frequency}$

Total frequency =  $8 + 30 + 132 + 24 + 6 = 200$

Sum =  $(198 \times 8) + (199 \times 30) + (200 \times 132) + (201 \times 24) + (202 \times 6)$

$= 1584 + 5970 + 26400 + 4824 + 1212 = 45990$

$$\text{Mean} = 45990 / 200 = 229.95$$

Answer: Mean = 229.95 mm

(d) Find the value of  ${}^{15}P_6 \times {}^{10}P_2$

${}^{15}P_6$  (Permutations of 15 items taken 6 at a time):

$${}^{15}P_6 = 15! / (15 - 6)! = 15! / 9!$$

$$= 15 \times 14 \times 13 \times 12 \times 11 \times 10$$

$$15 \times 14 = 210$$

$$210 \times 13 = 2730$$

$$2730 \times 12 = 32760$$

$$32760 \times 11 = 360360$$

$$360360 \times 10 = 3603600$$

$$\text{So, } {}^{15}P_6 = 3603600$$

${}^{10}P_2$  (Permutations of 10 items taken 2 at a time):

$${}^{10}P_2 = 10! / (10 - 2)! = 10! / 8!$$

$$= 10 \times 9$$

$$= 90$$

Multiply:

$${}^{15}P_6 \times {}^{10}P_2 = 3603600 \times 90$$

$$3603600 \times 90 = 324324000$$

Answer: 324324000

(e) Compute the value of  $C_{5,1} \times C_{5,0} \times 91$

$$C_{5,1} = 5! / (1! \times 4!) = 5$$

$$C_{5,0} = 5! / (0! \times 5!) = 1$$

$$C_{5,1} \times C_{5,0} \times 91 = 5 \times 1 \times 91 = 455$$

2. (a) The functions f and g are defined by f:  $x \rightarrow \ln x$  and g:  $x \rightarrow e^x$

(i) Sketch the graphs of  $f$  on  $0.5 < x \leq 3$  and  $g$  on  $-3.5 < x \leq 3$  on the same  $x$  and  $y$  plane

For  $f(x) = \ln x$ :

Domain:  $0.5 < x \leq 3$

At  $x = 0.5$ ,  $f(0.5) = \ln 0.5 \approx -0.693$

At  $x = 1$ ,  $f(1) = \ln 1 = 0$

At  $x = 3$ ,  $f(3) = \ln 3 \approx 1.099$

Graph: Starts at  $(0.5, -0.693)$ , crosses  $(1, 0)$ , ends at  $(3, 1.099)$ , increasing logarithmically.

For  $g(x) = e^x$ :

Domain:  $-3.5 < x \leq 3$

At  $x = -3.5$ ,  $g(-3.5) = e^{-3.5} \approx 0.030$

At  $x = 0$ ,  $g(0) = e^0 = 1$

At  $x = 3$ ,  $g(3) = e^3 \approx 20.085$

Graph: Starts at  $(-3.5, 0.030)$ , passes through  $(0, 1)$ , ends at  $(3, 20.085)$ , exponential growth.

(ii) State the domain and range of  $f$  and  $g$

$f(x) = \ln x$ :

Domain:  $0.5 < x \leq 3$  (given)

Range:  $f(0.5)$  to  $f(3) \rightarrow (-0.693, 1.099]$

$g(x) = e^x$ :

Domain:  $-3.5 < x \leq 3$  (given)

Range:  $g(-3.5)$  to  $g(3) \rightarrow (0.030, 20.085]$

Answer:  $f$ : Domain  $(0.5, 3]$ , Range  $(-0.693, 1.099]$ ;  $g$ : Domain  $(-3.5, 3]$ , Range  $(0.030, 20.085]$

(iii) Identify the asymptotes for  $f$ ,  $g$  and describe briefly how  $f$  and  $g$  behave near the asymptotes

$f(x) = \ln x$ :

Asymptote: As  $x \rightarrow 0^+$ ,  $\ln x \rightarrow -\infty \rightarrow$  vertical asymptote at  $x = 0$  (just outside the domain).

Behavior: As  $x$  approaches  $0.5$  from the right,  $f(x)$  decreases rapidly toward  $-\infty$ .

$g(x) = e^x$ :

Asymptote: As  $x \rightarrow -\infty$ ,  $e^x \rightarrow 0 \rightarrow$  horizontal asymptote at  $y = 0$ .

Behavior: As  $x$  approaches  $-3.5$  from the right,  $g(x)$  gets closer to  $0$  but never reaches it.

Answer:  $f$ : Vertical asymptote at  $x = 0$ ,  $f$  decreases rapidly as  $x \rightarrow 0.5^+$ ;  $g$ : Horizontal asymptote at  $y = 0$ ,  $g$  approaches  $0$  as  $x \rightarrow -3.5$ .

(b) Given  $f(a) = (a - 1)/(a + 1)$ , show that  $f(a) = -f(-a)$

The equation  $f(a) = -f(-a)$  seems to imply a typo. Let's assume the intended problem is to show  $f(a) = -f(-a)$  (a common property to verify).

$$f(a) = (a - 1)/(a + 1)$$

$$f(-a) = (-a - 1)/(-a + 1) = -(a + 1)/(1 - a) = (a + 1)/(a - 1)$$

$$-f(-a) = -(a + 1)/(a - 1)$$

3. arithmetic series, one is a geometric series and the remaining is neither

A:  $4 + 128 + 64 + 32 + \dots$

B:  $4 + 8 + 12 + 24 + \dots$

C:  $38 + 35 + 32 + 29 + \dots$

(i) State the value of the common difference of the arithmetic series and calculate the 21st term of the series

A:  $4, 128, 64, 32 \rightarrow$  Differences:  $124, -64, -32$  (not constant); Ratios:  $32, 1/2, 1/2$  (geometric).

B:  $4, 8, 12, 24 \rightarrow$  Differences:  $4, 4, 12$  (not constant); Ratios:  $2, 1.5, 2$  (not geometric).

C:  $38, 35, 32, 29 \rightarrow$  Differences:  $-3, -3, -3$  (arithmetic).

Common difference of C:  $-3$

21st term of C:  $a_n = a_1 + (n-1)d$

$$a_1 = 38, d = -3, n = 21$$

$$a_{21} = 38 + (21-1)(-3) = 38 + 20(-3) = 38 - 60 = -22$$

Answer: Common difference =  $-3$ , 21st term =  $-22$

(ii) Find  $S_{10}$  of the geometric series

Geometric series A: 4, 128, 64, 32  $\rightarrow a = 4, r = 128/4 = 32$  (but  $64/128 = 1/2$ , so correct  $r = 1/2$  from the 2nd term onward; first term seems off). Let's correct: 128, 64, 32  $\rightarrow a = 128, r = 1/2$ .

Assume A starts at 128:  $S_n = a(1 - r^n) / (1 - r)$

$$S_{10} = 128(1 - (1/2)^{10}) / (1 - 1/2)$$

$$(1/2)^{10} = 1/1024$$

$$1 - 1/1024 = 1023/1024$$

$$S_{10} = 128(1023/1024) / (1/2) = 128 \times 1023/1024 \times 2 = 255.75$$

$$\text{Adjust for A starting at 4: } r = 1/2 \rightarrow S_{10} = 4(1 - (1/2)^{10}) / (1 - 1/2) = 4(1023/1024) / (1/2) = 7.992$$

Answer:  $S_{10} = 7.992$

(b) If the length of a paper in a roll of given dimensions varies inversely as the thickness of the paper, find the increase in length when the thickness of a paper in a 100 m roll is decreased from 0.25 mm to 0.20 mm

Length L varies inversely with thickness t:  $L = k/t$

$$\text{At } L = 100 \text{ m, } t = 0.25 \text{ mm: } 100 = k/0.25 \rightarrow k = 100 \times 0.25 = 25$$

$$\text{New thickness } t = 0.20 \text{ mm: } L = 25/0.20 = 125 \text{ m}$$

$$\text{Increase} = 125 - 100 = 25 \text{ m}$$

Answer: Increase = 25 m

4. (a) Differentiate with respect to x the functions:

$$(i) f(x) = x^{(\ln x)}$$

$$\text{Let } y = x^{(\ln x)}$$

$$\text{Take ln: } \ln y = \ln x \times \ln x = (\ln x)^2$$

$$\text{Differentiate: } (1/y) dy/dx = 2 \ln x \times (1/x)$$

$$dy/dx = y \times 2 \ln x / x = x^{(\ln x)} \times (2 \ln x / x)$$

$$\text{Answer: } dy/dx = x^{(\ln x)} \times (2 \ln x / x)$$

$$(ii) y = (x^3 - 1)/(x^2 - 2x)$$

$$\text{Quotient rule: } (u/v)' = (u'v - uv')/v^2$$

$$u = x^3 - 1, v = x^2 - 2x$$

$$u' = 3x^2, v' = 2x - 2$$

$$dy/dx = [(3x^2)(x^2 - 2x) - (x^3 - 1)(2x - 2)] / (x^2 - 2x)^2$$

$$= [3x^4 - 6x^3 - (2x^4 - 2x^3 - 2x + 2)] / (x^2 - 2x)^2$$

$$= [3x^4 - 6x^3 - 2x^4 + 2x^3 + 2x - 2] / (x^2 - 2x)^2$$

$$= [x^4 - 4x^3 + 2x - 2] / (x^2 - 2x)^2$$

$$\text{Answer: } dy/dx = (x^4 - 4x^3 + 2x - 2) / (x^2 - 2x)^2$$

$$(iii) x^2 + y^2 + x^2y - 2x + 3y = 0$$

Implicit differentiation:

$$2x + 2y dy/dx + (2xy + x^2 dy/dx) - 2 + 3 dy/dx = 0$$

$$(2y + x^2 + 3) dy/dx = -2x - 2xy + 2$$

$$dy/dx = (-2x - 2xy + 2) / (2y + x^2 + 3)$$

$$\text{Answer: } dy/dx = (-2x - 2xy + 2) / (2y + x^2 + 3)$$

(b) Find the turning points of the polynomial function  $y = x^3 - x^2$  and hence sketch the graph of this function

$$y = x^3 - x^2$$

$$dy/dx = 3x^2 - 2x = x(3x - 2)$$

$$\text{Set } dy/dx = 0: x = 0 \text{ or } 3x - 2 = 0 \rightarrow x = 2/3$$

$$\text{Second derivative: } d^2y/dx^2 = 6x - 2$$

$$\text{At } x = 0: d^2y/dx^2 = -2 < 0 \rightarrow \text{local maximum}$$

$$\text{At } x = 2/3: d^2y/dx^2 = 6(2/3) - 2 = 4 - 2 = 2 > 0 \rightarrow \text{local minimum}$$

Turning points:

$$x = 0: y = 0 \rightarrow (0, 0)$$

$$x = 2/3: y = (2/3)^3 - (2/3)^2 = 8/27 - 4/9 = 8/27 - 12/27 = -4/27 \rightarrow (2/3, -4/27)$$

Sketch:

$$x\text{-intercepts: } x^2(x - 1) = 0 \rightarrow x = 0, x = 1$$

Passes through (0, 0) (max), (2/3, -4/27) (min), (1, 0).

Answer: Turning points: (0, 0) maximum, (2/3, -4/27) minimum.

5. (a) Evaluate the integrals:

(i)  $\int \sin(4x + 6) \, dx$

$\int \sin(4x + 6) \, dx$

Substitute  $u = 4x + 6 \rightarrow du = 4 \, dx \rightarrow dx = du/4$

$$\int \sin u \, (du/4) = (1/4) \int \sin u \, du = (1/4)(-\cos u) + C$$

$$= -(1/4) \cos(4x + 6) + C$$

Answer:  $-(1/4) \cos(4x + 6) + C$

(ii)  $\int (x^2 - 1/x) \, dx$

$$\int (x^2 - 1/x) \, dx = \int x^2 \, dx - \int (1/x) \, dx$$

$$= (x^3/3) - \ln|x| + C$$

Answer:  $(x^3/3) - \ln|x| + C$

(b) The graph of  $y = 4/(x - 1)$  is shown in the sketch below together with the line  $y = 4$

Graph shows  $y = 4/(x - 1)$  with a vertical asymptote at  $x = 1$ , and the line  $y = 4$ . Shaded area is between  $x = 2$  and  $x = 3$ .

Find the volume generated when the shaded area is rotated completely about the x-axis, leaving your answer in terms of  $\pi$

Shaded area: Between  $y = 4/(x - 1)$  and  $y = 4$  from  $x = 2$  to  $x = 3$ .

At  $x = 2$ ,  $y = 4/(2 - 1) = 4$ ; at  $x = 3$ ,  $y = 4/(3 - 1) = 2$ .

$y = 4/(x - 1)$  is below  $y = 4$  in this interval.

Volume by washers:  $V = \pi \int (\text{from } 2 \text{ to } 3) [(outer \, radius)^2 - (inner \, radius)^2] \, dx$

Outer radius = 4, inner radius =  $4/(x - 1)$

$$V = \pi \int (\text{from } 2 \text{ to } 3) [4^2 - (4/(x - 1))^2] \, dx$$

$$= \pi \int (\text{from } 2 \text{ to } 3) [16 - 16/(x - 1)^2] \, dx$$



$$= \pi [16x + 16/(x - 1)] \text{ (from 2 to 3)}$$

$$\text{At } x = 3: 16(3) + 16/(3 - 1) = 48 + 8 = 56$$

$$\text{At } x = 2: 16(2) + 16/(2 - 1) = 32 + 16 = 48$$

$$V = \pi (56 - 48) = 8\pi$$

Answer: Volume =  $8\pi$

6. The scores of 40 students in a mathematics test are given below

Scores | 21-30 | 31-40 | 41-50 | 51-60 | 61-70 | 71-80 | 81-90

Number of students | 3 | 6 | 10 | 8 | 6 | 4 | 3

(a) Represent the students' scores in a histogram and then use it to calculate the mode

Histogram:

Class widths: 10 (e.g., 21-30, 31-40, etc.)

Heights = frequencies: 3, 6, 10, 8, 6, 4, 3

Mode:

Modal class: 41-50 (highest frequency = 10)

$$\text{Mode} = L + [(f_m - f_{(m-1)}) / (2f_m - f_{(m-1)} - f_{(m+1)})] \times w$$

$$L = 41, f_m = 10, f_{(m-1)} = 6, f_{(m+1)} = 8, w = 10$$

$$\text{Mode} = 41 + [(10 - 6) / (2 \times 10 - 6 - 8)] \times 10$$

$$= 41 + (4 / (20 - 14)) \times 10 = 41 + (4/6) \times 10 = 41 + 20/3 \approx 47.67$$

Answer: Mode  $\approx 47.67$  (Histogram: bars at 21-30: 3, 31-40: 6, 41-50: 10, 51-60: 8, 61-70: 6, 71-80: 4, 81-90: 3)

(b) Find:

(i) Variance of the scores

Midpoints: 25.5, 35.5, 45.5, 55.5, 65.5, 75.5, 85.5

Mean =  $\Sigma (\text{midpoint} \times \text{frequency}) / \text{total frequency}$

$$\text{Total frequency} = 3 + 6 + 10 + 8 + 6 + 4 + 3 = 40$$

$$\begin{aligned}\text{Sum} &= (25.5 \times 3) + (35.5 \times 6) + (45.5 \times 10) + (55.5 \times 8) + (65.5 \times 6) + (75.5 \times 4) + (85.5 \times 3) \\ &= 76.5 + 213 + 455 + 444 + 393 + 302 + 256.5 = 2140\end{aligned}$$

$$\text{Mean} = 2140 / 40 = 53.5$$

$$\text{Variance} = \Sigma (\text{frequency} \times (\text{midpoint} - \text{mean})^2) / n$$

$$(25.5 - 53.5)^2 = 28^2 = 784$$

$$(35.5 - 53.5)^2 = 18^2 = 324$$

$$(45.5 - 53.5)^2 = 8^2 = 64$$

$$(55.5 - 53.5)^2 = 2^2 = 4$$

$$(65.5 - 53.5)^2 = 12^2 = 144$$

$$(75.5 - 53.5)^2 = 22^2 = 484$$

$$(85.5 - 53.5)^2 = 32^2 = 1024$$

$$\begin{aligned}\text{Sum} &= (784 \times 3) + (324 \times 6) + (64 \times 10) + (4 \times 8) + (144 \times 6) + (484 \times 4) + (1024 \times 3) \\ &= 2352 + 1944 + 640 + 32 + 864 + 1936 + 3072 = 10840\end{aligned}$$

$$\text{Variance} = 10840 / 40 = 271$$

Answer: Variance = 271

(ii) The standard deviation of the scores

$$\text{Standard deviation} = \sqrt{\text{Variance}} = \sqrt{271} \approx 16.462$$

Answer: Standard deviation  $\approx 16.462$

7. (a) Draw 2 possible venn diagrams representing two events A and B in a sample space S

Venn Diagram 1 (A and B overlapping):

Sample space S is a rectangle.

A and B are two circles inside S, with some overlap ( $A \cap B$ ).

Regions: A only, B only,  $A \cap B$ , and outside both ( $S - (A \cup B)$ ).

Venn Diagram 2 (A and B disjoint):

Sample space S is a rectangle.

A and B are two non-overlapping circles inside S.

Regions: A only, B only, and outside both ( $S - (A \cup B)$ ).

Answer: (I can confirm if you'd like to generate sketches for the Venn diagrams.)

(i) and then write down the formulae corresponding to  $P(A \cap B)$

If A and B overlap:

$P(A \cap B)$  is the probability of the overlapping region.

General formula:  $P(A \cap B) = P(A)P(B)$  if A and B are independent, or use  $P(A \cap B) = P(A) + P(B) - P(A \cup B)$  if not independent.

If A and B are disjoint (mutually exclusive):

$P(A \cap B) = 0$  (since they don't overlap).

Answer: For overlapping:  $P(A \cap B) = P(A)P(B)$  (if independent) or  $P(A) + P(B) - P(A \cup B)$ ; for disjoint:  $P(A \cap B) = 0$

(ii) Given that A and B are mutually exclusive events with probabilities  $P(A) = 1/4$  and  $P(B) = 2/3$ , find  $P(A \cap B)$

Since A and B are mutually exclusive,  $P(A \cap B) = 0$ .

Answer:  $P(A \cap B) = 0$

(b) Ntibagomba is going on holiday. He has 6 different shirts and has decided that he only needs to take 3 shirts. Find the number of different selections that he can make

Number of ways to choose 3 shirts from 6:  $C(6, 3)$

$$C(6, 3) = 6! / (3! \times 3!) = (6 \times 5 \times 4) / (3 \times 2 \times 1) = 20$$

Answer: 20 selections

(c) Find the number of ways that six children Kauki, John, Tito, Ben, Kato and Sara can stand in a line outside the canteen waiting for lunch

Number of ways to arrange 6 distinct children:  $6!$

$$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$$

Answer: 720 ways

8. (a) If A is an acute angle such that  $\tan A = 3/4$ , find without using tables or calculator, the values of (i)  $\cos 2A$  and (ii)  $\sin A/2$

Given  $\tan A = 3/4$ , A is acute.

$$\tan A = \text{opposite/adjacent} = 3/4 \rightarrow \text{hypotenuse} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$$\sin A = 3/5, \cos A = 4/5$$

(i)  $\cos 2A$ :

$$\cos 2A = \cos^2 A - \sin^2 A = (4/5)^2 - (3/5)^2 = 16/25 - 9/25 = 7/25$$

(ii)  $\sin A/2$ :

$$\sin A/2 = \pm \sqrt{(1 - \cos A)/2}$$

$$\cos A = 4/5 \rightarrow 1 - \cos A = 1/5$$

$$\sin A/2 = \pm \sqrt{(1/5)/2} = \pm \sqrt{1/10} = \pm \sqrt{10}/10$$

Since A is acute, A/2 is also acute  $\rightarrow \sin A/2 = \sqrt{10}/10$

Answer: (i)  $\cos 2A = 7/25$ , (ii)  $\sin A/2 = \sqrt{10}/10$

(b) Solve the equation  $2 \sin(x + 60^\circ) = \cos(x - 30^\circ)$  in the range  $0^\circ \leq x \leq 360^\circ$  by expanding the sine and the cosine terms

$$\text{Expand: } 2 \sin(x + 60^\circ) = 2 [\sin x \cos 60^\circ + \cos x \sin 60^\circ] = 2 [\sin x (1/2) + \cos x (\sqrt{3}/2)] = \sin x + \sqrt{3} \cos x$$

$$\cos(x - 30^\circ) = \cos x \cos 30^\circ + \sin x \sin 30^\circ = \cos x (\sqrt{3}/2) + \sin x (1/2) = (\sqrt{3}/2) \cos x + (1/2) \sin x$$

$$\text{Equation: } \sin x + \sqrt{3} \cos x = (\sqrt{3}/2) \cos x + (1/2) \sin x$$

$$\text{Rearrange: } \sin x + \sqrt{3} \cos x - (\sqrt{3}/2) \cos x - (1/2) \sin x = 0$$

$$(\sin x - (1/2) \sin x) + (\sqrt{3} \cos x - (\sqrt{3}/2) \cos x) = 0$$

$$(1/2) \sin x + (\sqrt{3}/2) \cos x = 0$$

$$(1/2) \sin x = -(\sqrt{3}/2) \cos x$$

$$\tan x = -\sqrt{3}$$

$$x = 180^\circ - 60^\circ = 120^\circ, 360^\circ - 60^\circ = 300^\circ \text{ (in the range } 0^\circ \leq x \leq 360^\circ)$$

Answer:  $x = 120^\circ, 300^\circ$

9.(a) Given that  $A = \begin{bmatrix} 3 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} -4 & 3 \end{bmatrix}$ , verify that  $(AB)^T = B^T A^T$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \quad \begin{bmatrix} -5 & 2 \end{bmatrix}$$

$AB$ :

$$A \times B = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} -4 & 3 \end{bmatrix} = \begin{bmatrix} (3 \times -4 + 1 \times -5) & (3 \times 3 + 1 \times 2) \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} -5 & 2 \end{bmatrix} \quad \begin{bmatrix} (5 \times -4 + 2 \times -5) & (5 \times 3 + 2 \times 2) \end{bmatrix}$$

$$= \begin{bmatrix} -17 & 11 \end{bmatrix}$$

$$\begin{bmatrix} -30 & 19 \end{bmatrix}$$

$(AB)^T$ :

$$\begin{bmatrix} -17 & -30 \end{bmatrix}$$

$$\begin{bmatrix} 11 & 19 \end{bmatrix}$$

$B^T A^T$ :

$$B^T = \begin{bmatrix} -4 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} -4 & -5 \end{bmatrix} \begin{bmatrix} 3 & 5 \end{bmatrix} = \begin{bmatrix} (-4 \times 3 + -5 \times 1) & (-4 \times 5 + -5 \times 2) \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} \quad \begin{bmatrix} (3 \times 3 + 2 \times 1) & (3 \times 5 + 2 \times 2) \end{bmatrix}$$

$$= \begin{bmatrix} -17 & -30 \end{bmatrix}$$

$$\begin{bmatrix} 11 & 19 \end{bmatrix}$$

$(AB)^T = B^T A^T$ , verified.

$$\text{Answer: } (AB)^T = B^T A^T = \begin{bmatrix} -17 & -30 \end{bmatrix}$$

$$\begin{bmatrix} 11 & 19 \end{bmatrix}$$

(b) Show whether the following system of equations has a common solution or not:  $7x - 3y - 3z = 7$ ,  $2x + 4y + z = 0$ ,  $x - 2y - z = 2$

Write in matrix form:  $AX = B$

$$A = \begin{bmatrix} 7 & -3 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -1 \end{bmatrix}, X = \begin{bmatrix} x \end{bmatrix}$$

$$\begin{bmatrix} y \end{bmatrix}$$

$$\begin{bmatrix} z \end{bmatrix}, B = \begin{bmatrix} 7 \end{bmatrix}$$

$$\begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 \end{bmatrix}$$

$$\text{Det}(A) = 7(4 \times -1 - 1 \times -2) - (-3)(2 \times -1 - 1 \times 1) + (-3)(2 \times -2 - 4 \times 1)$$

$$= 7(-1 + 2) - (-3)(-2 - 1) + (-3)(-4 - 4)$$

$$= 7(1) - (-3)(-3) + (-3)(-8) = 7 - 9 + 24 = 22$$

Since  $\text{Det}(A) \neq 0$ , A is invertible, and the system has a unique solution.

Answer: The system has a common (unique) solution ( $\text{Det}(A) \neq 0$ ).

(c) The results of three soccer teams Simba (S), Yanga (Y) and Mtiwa Sugar (M) are shown in matrix R and the points awarded for each team in matrix P

$$R = \begin{bmatrix} 6 & 2 & 4 \end{bmatrix} \text{ W D L Points}$$

$$\begin{bmatrix} 4 & 3 & 1 \end{bmatrix} \text{ S P} = \begin{bmatrix} 5 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 1 & 4 \end{bmatrix} \text{ Y D} \begin{bmatrix} 1 \end{bmatrix}$$

$$\text{M L} \begin{bmatrix} 0 \end{bmatrix}$$

Multiply the matrices and hence state which team has many points

$$R \times P = \begin{bmatrix} 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} 5 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 1 & 4 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix}$$

$$S: 6 \times 5 + 2 \times 1 + 4 \times 0 = 30 + 2 = 32$$

$$Y: 4 \times 5 + 3 \times 1 + 1 \times 0 = 20 + 3 = 23$$

$$M: 7 \times 5 + 1 \times 1 + 4 \times 0 = 35 + 1 = 36$$

$$\text{Result:} \begin{bmatrix} 32 \end{bmatrix}$$

$$\begin{bmatrix} 23 \end{bmatrix}$$

[36]

Mtiwa Sugar (M) has the most points (36).

Answer: [32]

[23]

[36]; Mtiwa Sugar has the most points (36).

10.

The data for M & P Company, which manufactures tables and chairs, is given in the table below:

Assembly Department | 16 | 4 | 800

Finishing Department | 4 | 2 | 240

Profit per Item | Shs. 10,000/= | Shs. 40,000/=

(a) How many tables and chairs should be manufactured to realize a maximum profit?

Let  $x$  = number of tables,  $y$  = number of chairs.

Objective: Maximize  $P = 10000x + 40000y$

Constraints:

Assembly:  $16x + 4y \leq 800 \rightarrow 4x + y \leq 200$

Finishing:  $4x + 2y \leq 240 \rightarrow 2x + y \leq 120$

$x \geq 0, y \geq 0$

Graph:

$4x + y \leq 200 \rightarrow (50, 0), (0, 200)$

$2x + y \leq 120 \rightarrow (60, 0), (0, 120)$

Vertices:

$(0, 0)$

$(0, 120)$  (from  $2x + y = 120, x = 0$ )

$(50, 0)$  (from  $4x + y = 200, y = 0$ )

Intersection:  $4x + y = 200, 2x + y = 120 \rightarrow 2x = 80 \rightarrow x = 40, y = 120 - 2(40) = 40 \rightarrow (40, 40)$

Evaluate P:

$$(0, 0): P = 0$$

$$(0, 120): P = 10000(0) + 40000(120) = 4800000$$

$$(50, 0): P = 10000(50) + 40000(0) = 500000$$

$$(40, 40): P = 10000(40) + 40000(40) = 400000 + 1600000 = 2000000$$

Maximum at (0, 120).

Answer: 0 tables, 120 chairs

(b) What is the maximum profit?

From (a), maximum at (0, 120):  $P = 4800000$  Shs.

Answer: Maximum profit = 4800000 Shs.