THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

141 BASIC APPLIED MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2014**

Instructions

1. This paper consists of **Ten (10)** questions.
2. Answer all questions.
3. **All** work done and answers of each question must be shown clearly.
4. NECTA'S Mathematical tables and Non-programmable calculations may be used
5. All writing must be in **black** or **blue** ink, **except** drawing which must be in pencil.

Prepared by: Maria Marco for TETEA

1. (a) Solve the equation ln (2x + 3) - 3 = ln (x - 5) and hence write your answer correct to 2 decimal places

$\ln(2x + 3) - 3 = \ln(x - 5)$

$\ln(2x + 3) - \ln(x - 5) = 3$

$\ln((2x + 3)/(x - 5)) = 3$

$(2x + 3)/(x - 5) = e^3 \approx 20.0855$ (since $e \approx 2.718282$)

$2x + 3 = 20.0855 (x - 5)$

$2x + 3 = 20.0855x - 100.4275$

$2x - 20.0855x = -100.4275 - 3$

$-18.0855x = -103.4275$

$x = 103.4275 / 18.0855 \approx 5.719$

To 2 decimal places: $x \approx 5.72$

Answer: $x \approx 5.72$

(b) Find the value of t correct to 4 decimal places given that 3^t = 5^t

3^t = 5^t

This implies 3^t - 5^t = 0, but let's interpret correctly: if 3^t = 5^t, then t must be 0 (since 3^0 = 5^0 = 1), but that's trivial. The equation might be a typo. Let's assume the intended equation is 3^t = 5 (a common form).

3^t = 5

$t \ln 3 = \ln 5$

$t = \ln 5 / \ln 3$

$\ln 5 \approx 1.6094$, $\ln 3 \approx 1.0986$

$t \approx 1.6094 / 1.0986 \approx 1.4650$

Answer: $t \approx 1.4650$

(c) Solve the quadratic equation $x^2 + 9x - 2.718282 = 0$

$x^2 + 9x - 2.718282 = 0$

Use quadratic formula: $x = (-b \pm \sqrt{(b^2 - 4ac)}) / (2a)$, where $a = 1$, $b = 9$, $c = -2.718282$

Discriminant: $9^2 - 4(1)(-2.718282) = 81 + 10.873128 = 91.873128$

$\sqrt{91.873128} \approx 9.585$

$x = (-9 \pm 9.585) / 2$

$x_1 = (-9 + 9.585) / 2 = 0.585 / 2 = 0.2925$

$x_2 = (-9 - 9.585) / 2 = -18.585 / 2 = -9.2925$

Answer: $x = 0.2925$ or $x = -9.2925$

2. (a) Let $f(x) = \begin{cases} 2 & \text{for } x < -2 \\ x^2 + 2x & \text{for } -2 \leq x < 1 \\ 4 - x & \text{for } x \geq 1 \end{cases}$

Find: (i) f(1/8), (ii) f(13) and (iii) f(-32)

(i) x = 1/8: $-2 \leq 1/8 < 1$, so $f(1/8) = (1/8)^2 + 2(1/8) = 1/64 + 2/8 = 1/64 + 16/64 = 17/64$

(ii) x = 13: $13 \geq 1$, so $f(13) = 4 - 13 = -9$

(iii) x = -32: $-32 < -2$, so $f(-32) = 2$

Answer: (i) f(1/8) = 17/64, (ii) f(13) = -9, (iii) f(-32) = 2

(b) A function is defined by $f(x) = (2x - 1)/(x + 1)$

(i) Find its asymptotes and hence sketch the graph of this rational function

Vertical asymptote: $x + 1 = 0 \rightarrow x = -1$

Horizontal asymptote: As $x \rightarrow \pm\infty$, $f(x) \approx 2x/x = 2$, so $y = 2$

x-intercept: $2x - 1 = 0 \rightarrow x = 1/2 \rightarrow (1/2, 0)$

y-intercept: $f(0) = (2(0) - 1)/(0 + 1) = -1 \rightarrow (0, -1)$

Sketch:

Vertical asymptote at $x = -1$

Horizontal asymptote at $y = 2$

Passes through (1/2, 0) and (0, -1)

(ii) State the domain and range of this function

Domain: All real numbers except $x = -1$ (where the denominator is zero), so $x \in \mathbb{R}, x \neq -1$

Range: $f(x) = (2x - 1)/(x + 1) = 2 - 3/(x + 1)$. As $x \to -1^-$, $f(x) \to +\infty$; as $x \to -1^+$, $f(x) \to -\infty$; as $x \to \pm\infty$, $f(x) \to 2$. The function takes all values except 2 (solve $f(x) = 2$: $2 - 3/(x + 1) = 2 \to -3/(x + 1) = 0$, no solution). So, range is all real numbers except 2.

Answer: Domain: $x \in \mathbb{R}, x \neq -1$; Range: $y \in \mathbb{R}, y \neq 2$

(c) Find the set of values of (x, y) that satisfies the equations $x + y = 3$ and $xy = 2$

$x + y = 3 \to y = 3 - x$

$xy = 2 \to x(3 - x) = 2$

$3x - x^2 = 2$

$x^2 - 3x + 2 = 0$

$(x - 1)(x - 2) = 0$

$x = 1 \to y = 3 - 1 = 2 \to (1, 2)$

$x = 2 \to y = 3 - 2 = 1 \to (2, 1)$

Answer: (x, y) = (1, 2) or (2, 1)

3.(a) Given that the first term of a geometric progression is 2 and its common ratio is 1/2;

(i) Write down the first four terms of the progression

First term $a = 2$, common ratio $r = 1/2$

Terms: $2, 2\times(1/2), 2\times(1/2)^2, 2\times(1/2)^3 = 2, 1, 1/2, 1/4$

Answer: 2, 1, 1/2, 1/4

(ii) Find the 20th term

nth term = $a\,r^{(n-1)}$

20th term = $2 \times (1/2)^{(20-1)} = 2 \times (1/2)^{19} = 2 / 2^{19} = 2 / 524288 = 1 / 262144$

Answer: 1/262144

(iii) Find the sum to infinity of the series

Sum to infinity = $a / (1 - r)$

$= 2 / (1 - 1/2) = 2 / (1/2) = 4$

Answer: 4

(b) The ages of a certain singers group form an arithmetic progression whose common difference is 4. If the youngest singer is 8 years old and the sum of the ages of all singers in the group is 168 years, find the number of singers in the group

First term $a = 8$, common difference $d = 4$

Sum of n terms: $S_n = (n/2) [2a + (n-1)d]$

$168 = (n/2) [2(8) + (n-1)(4)]$

$168 = (n/2) [16 + 4n - 4] = (n/2) [4n + 12] = n(2n + 6)$

$2n^2 + 6n = 168$

$n^2 + 3n - 84 = 0$

$(n + 12)(n - 7) = 0$

$n = 7$ (since n must be positive)

Answer: 7 singers

4. (a) Differentiate with respect to x the following functions:

(i) $y = (2e^x)/(3\sin x)$

Use quotient rule: $(u/v)' = (u'v - uv')/v^2$, where $u = 2e^x$, $v = 3\sin x$

$u' = 2e^x$, $v' = 3\cos x$

$dy/dx = (2e^x \times 3\sin x - 2e^x \times 3\cos x) / (3\sin x)^2$

$= (6e^x \sin x - 6e^x \cos x) / (9\sin^2 x)$

$= (6e^x (\sin x - \cos x)) / (9\sin^2 x)$

$= (2e^x (\sin x - \cos x)) / (3\sin^2 x)$

Answer: $dy/dx = (2e^x (\sin x - \cos x)) / (3\sin^2 x)$

(ii) $y = x^2 \sin 3x$

Use product rule: $(uv)' = u'v + uv'$, where $u = x^2$, $v = \sin 3x$

$u' = 2x$, $v' = 3\cos 3x$

$dy/dx = 2x \sin 3x + x^2 (3\cos 3x)$

$= 2x \sin 3x + 3x^2 \cos 3x$

Answer: $dy/dx = 2x \sin 3x + 3x^2 \cos 3x$

(b) Find the slope of the curves:

(i) $f(x) = x^3 - 5x^2$ at the point $x = 2$

$f'(x) = 3x^2 - 10x$

At $x = 2$: $f'(2) = 3(2)^2 - 10(2) = 12 - 20 = -8$

Answer: Slope = -8

(ii) $x^2 - 3xy + 2y^2 - 2x = 4$ at the point (1, 3)

Implicit differentiation:

$2x - 3(x\, dy/dx + y) + 4y\, dy/dx - 2 = 0$

$2x - 3x\, dy/dx - 3y + 4y\, dy/dx - 2 = 0$

$(4y - 3x)\, dy/dx = -2x + 3y + 2$

$dy/dx = (-2x + 3y + 2) / (4y - 3x)$

At (1, 3): $dy/dx = (-2(1) + 3(3) + 2) / (4(3) - 3(1)) = (-2 + 9 + 2) / (12 - 3) = 9/9 = 1$

Answer: Slope = 1

(c) The volume of air which is pumped into a rubber ball every second is 4 cm^3. Given that the volume of the ball is $v = (4/3)\pi r^3$ and that its radius (r) changes with the increase of air, find the rate of change of the radius when the radius is 6 cm

$dv/dt = 4\ cm^3/s$

$v = (4/3)\pi r^3$

$dv/dr = 4\pi r^2$

Chain rule: $dv/dt = (dv/dr)(dr/dt)$

$4 = 4\pi(6)^2$ dr/dt

$4 = 4\pi(36)$ dr/dt

$4 = 144\pi$ dr/dt

dr/dt = 4 / (144π) = 1 / (36π) cm/s

Answer: dr/dt = 1/(36π) cm/s

5. (a) Evaluate the following integrals:

(i) ∫ (from 2 to 5) $(3x^2 - 5x)$ dx

$\int (3x^2 - 5x)\,dx = x^3 - (5/2)x^2$

From 2 to 5: $[5^3 - (5/2)(5)^2] - [2^3 - (5/2)(2)^2]$

= [125 - (5/2)(25)] - [8 - (5/2)(4)]

= [125 - 62.5] - [8 - 10]

= 62.5 - (-2) = 64.5

Answer: 64.5

5. (a) Evaluate the following integrals:

(ii) $\int x^4 \sqrt{(x^5 + 3)}\,dx$

Notice the integrand: $x^4 \sqrt{(x^5 + 3)}$. The exponents suggest a substitution involving x^5.

Let's substitute $u = x^5 + 3$

Then, $du = 5x^4\,dx \rightarrow x^4\,dx = (1/5)\,du$

The integral becomes: $\int x^4 \sqrt{(x^5 + 3)}\,dx = \int x^4 \sqrt{u}\,(1/5)\,du$

= (1/5) ∫ u^(1/2) du

= (1/5) × (u^(3/2) / (3/2))

= (1/5) × (2/3) u^(3/2)

= (2/15) u^(3/2) + C

Substitute back $u = x^5 + 3$:

$= (2/15)\,(x^5 + 3)$^(3/2) + C

Answer: $(2/15) (x^5 + 3)^{(3/2)} + C$

(b) Given that ∫ (from 1 to 1) f(x) dx = 4, evaluate ∫ (from 1 to 2) (f(x) + 3) dx

∫ (from 1 to 1) f(x) dx = 0 (since the limits are the same), so the given condition seems incorrect. Let's assume the problem meant ∫ (from 1 to 2) f(x) dx = 4.

∫ (from 1 to 2) (f(x) + 3) dx = ∫ (from 1 to 2) f(x) dx + ∫ (from 1 to 2) 3 dx

= 4 + 3[x] (from 1 to 2)

= 4 + 3(2 - 1) = 4 + 3 = 7

Answer: 7

(c) Sketch the graph of the curve f(x) = x(x - 4) and hence find the area between the x-axis and the curve

$f(x) = x(x - 4) = x^2 - 4x$

x-intercepts: x = 0, x = 4

Vertex: f'(x) = 2x - 4 = 0 → x = 2 → f(2) = 2(2 - 4) = -4

Graph: Parabola opening upwards, crosses x-axis at (0, 0) and (4, 0), vertex at (2, -4).

Area (below x-axis from x = 0 to 4):

∫ (from 0 to 4) $|x^2 - 4x|$ dx = ∫ (from 0 to 4) $-(x^2 - 4x)$ dx (since $x^2 - 4x < 0$ in this interval)

$= -[x^3/3 - 2x^2]$ (from 0 to 4)

= -[(64/3 - 32) - 0] = -[64/3 - 96/3] = -[-32/3] = 32/3

Answer: Area = 32/3

6. During a biology practical, a random sample of 20 grasshoppers was selected and the length of each grasshopper recorded in centimeters as follows: 1.0, 1.0, 5.0, 4.0, 5.0, 4.0, 2.0, 4.0, 2.0, 4.0, 2.0, 3.0, 2.0, 3.0, 3.0, 2.0, 3.0, 3.0, 3.0, 3.0

(a) Without grouping the data:

(i) Prepare a frequency table and a histogram for the length distribution

Frequency Table:

1.0: 2

2.0: 5

3.0: 7

4.0: 4

5.0: 2

Histogram:

X-axis: Lengths (1.0, 2.0, 3.0, 4.0, 5.0)

Y-axis: Frequency (2, 5, 7, 4, 2)

Bars at:

1.0: height 2

2.0: height 5

3.0: height 7

4.0: height 4

5.0: height 2

(ii) Find the range, mode, median, mean and standard deviation

Range:

Max = 5.0, Min = 1.0

Range = 5.0 - 1.0 = 4.0

Mode:

Most frequent value = 3.0 (frequency 7)

Median:

Ordered data: 1.0, 1.0, 2.0, 2.0, 2.0, 2.0, 2.0, 3.0, 3.0, 3.0, 3.0, 3.0, 3.0, 3.0, 4.0, 4.0, 4.0, 4.0, 5.0, 5.0

n = 20, median = average of 10th and 11th values

10th and 11th values = 3.0, 3.0

Median = (3.0 + 3.0)/2 = 3.0

Mean:

Sum = (1.0×2) + (2.0×5) + (3.0×7) + (4.0×4) + (5.0×2)

$= 2.0 + 10.0 + 21.0 + 16.0 + 10.0 = 59.0$

Mean = 59.0 / 20 = 2.95

Standard Deviation:

Variance = $\Sigma(x - \text{mean})^2 / n$

Mean = 2.95

$(1.0 - 2.95)^2 = 1.95^2 = 3.8025$ (×2)

$(2.0 - 2.95)^2 = 0.95^2 = 0.9025$ (×5)

$(3.0 - 2.95)^2 = 0.05^2 = 0.0025$ (×7)

$(4.0 - 2.95)^2 = 1.05^2 = 1.1025$ (×4)

$(5.0 - 2.95)^2 = 2.05^2 = 4.2025$ (×2)

Sum = (3.8025×2) + (0.9025×5) + (0.0025×7) + (1.1025×4) + (4.2025×2)

$= 7.605 + 4.5125 + 0.0175 + 4.41 + 8.405 = 24.95$

Variance = 24.95 / 20 = 1.2475

Standard deviation = $\sqrt{1.2475} \approx 1.117$

Answer: Range = 4.0, Mode = 3.0, Median = 3.0, Mean = 2.95, Standard deviation ≈ 1.117

(b) From part (a) (ii), indicate the measures of central tendency and the measures of dispersion

Measures of central tendency: Mode, Median, Mean

Mode = 3.0, Median = 3.0, Mean = 2.95

Measures of dispersion: Range, Standard deviation

Range = 4.0, Standard deviation ≈ 1.117

Answer: Measures of central tendency: Mode = 3.0, Median = 3.0, Mean = 2.95; Measures of dispersion: Range = 4.0, Standard deviation ≈ 1.117

7. (a) Evaluate:

(i) ${}^{9}P_{4}$

$^9P_4 = 9! / (9-4)! = 9 \times 8 \times 7 \times 6 = 3024$

Answer: 3024

(ii) $^9C_4$

$^9C_4 = 9! / (4! \times 5!) = (9 \times 8 \times 7 \times 6) / (4 \times 3 \times 2 \times 1) = 3024 / 24 = 126$

Answer: 126

(b) In how many different ways can the letters in the word STATISTICS be arranged?

Letters: S, T, A, T, I, S, T, I, C, S

Total letters = 10

S: 3, T: 3, I: 2, A: 1, C: 1

Arrangements = $10! / (3! \times 3! \times 2! \times 1! \times 1!)$

$10! = 3628800$

$3! = 6$, $2! = 2$

$= 3628800 / (6 \times 6 \times 2) = 3628800 / 72 = 50400$

Answer: 50400

(c) In a pack of 52 playing cards, two cards which are not hearts are removed and not replaced. If the remaining cards are well shuffled, what is the probability that the next card drawn is a heart?

Total cards initially = 52, hearts = 13

Two non-heart cards removed: Total cards = 50, hearts = 13, non-hearts = 37

Probability of drawing a heart = 13/50

Answer: 13/50

(d) Given that $P(A) = 1/4$, $P(B) = 1/8$ and $P(C) = 1/6$, find:

(i) $P(A \cap B)$ if A and B are independent

If A and B are independent, $P(A \cap B) = P(A) \times P(B) = (1/4) \times (1/8) = 1/32$

Answer: 1/32

(ii) $P(A \cup C)$ if A and C are mutually exclusive events

If A and C are mutually exclusive, $P(A \cup C) = P(A) + P(C) = 1/4 + 1/6 = 3/12 + 2/12 = 5/12$

Answer: 5/12

8. (a) Find the value of $\sin(A + B)$, given that $\sin A = 3/5$, $\cos B = 12/13$ and that A and B are both acute angles

$\sin A = 3/5 \rightarrow \cos A = \sqrt{1 - (3/5)^2} = \sqrt{1 - 9/25} = \sqrt{16/25} = 4/5$

$\cos B = 12/13 \rightarrow \sin B = \sqrt{1 - (12/13)^2} = \sqrt{1 - 144/169} = \sqrt{25/169} = 5/13$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$= (3/5)(12/13) + (4/5)(5/13)$

$= 36/65 + 20/65 = 56/65$

Answer: $\sin(A + B) = 56/65$

(b) Show that $(\sec\theta + \operatorname{cosec}\theta)/(1 + \tan\theta) = \operatorname{cosec}\theta$

Left: $(\sec\theta + \csc\theta)/(1 + \tan\theta)$

$\sec\theta = 1/\cos\theta$, $\csc\theta = 1/\sin\theta$, $\tan\theta = \sin\theta/\cos\theta$

Numerator: $(1/\cos\theta) + (1/\sin\theta) = (\sin\theta + \cos\theta)/(\sin\theta\cos\theta)$

Denominator: $1 + \tan\theta = 1 + (\sin\theta/\cos\theta) = (\cos\theta + \sin\theta)/\cos\theta$

$(\sec\theta + \csc\theta)/(1 + \tan\theta) = [(\sin\theta + \cos\theta)/(\sin\theta\cos\theta)] / [(\cos\theta + \sin\theta)/\cos\theta]$

$= (\sin\theta + \cos\theta)/(\sin\theta\cos\theta) \times (\cos\theta)/(\cos\theta + \sin\theta)$

$= 1/(\sin\theta) = \csc\theta$

Answer: $(\sec\theta + \csc\theta)/(1 + \tan\theta) = \csc\theta$ (proven)

(c) A triangular flower garden ABC has the angle $A\angle BC = 110°$. If AB is 50 m and BC is 40 m, find the length CA

Use the Law of Cosines in triangle ABC:

$CA^2 = AB^2 + BC^2 - 2(AB)(BC)\cos(\angle ABC)$

$CA^2 = 50^2 + 40^2 - 2(50)(40)\cos 110°$

$\cos 110° = \cos(180° - 70°) = -\cos 70° \approx -0.342$

$CA^2 = 2500 + 1600 - 4000(-0.342)$

$= 4100 + 1368 = 5468$

CA = √5468 ≈ 73.95 m

Answer: CA ≈ 73.95 m

I apologize if my previous response was unclear or frustrating. Let's proceed by copying the questions exactly as they appear in the image and providing the answers with the requested formatting (replacing * with ×, y^2 with y², x^2 with x², and so forth).

9.

(a) Given: A = [1 0]

[0 1]

[1 2], B = [-1 2 1]

[3 0 1], C = [2 1]

[1 1]

Find: (i) AB (ii) BA (iii) CA and comment on the results

(i) AB

A is 3×2, B is 2×3 → AB is 3×3

AB = [1 0] [-1 2 1] = [(1×-1 + 0×3) (1×2 + 0×0) (1×1 + 0×1)]

[0 1] [3 0 1] [(0×-1 + 1×3) (0×2 + 1×0) (0×1 + 1×1)]

[1 2] [(1×-1 + 2×3) (1×2 + 2×0) (1×1 + 2×1)]

= [-1 2 1]

[3 0 1]

[5 2 3]

(ii) BA

B is 2×3, A is 3×2 → BA is 2×2

BA = [-1 2 1] [1 0] = [(-1×1 + 2×0 + 1×1) (-1×0 + 2×1 + 1×2)]

[3 0 1] [0 1] [(3×1 + 0×0 + 1×1) (3×0 + 0×1 + 1×2)]

[1 2]

= [0 4]

[4 2]

(iii) CA

C is 2×2, A is 3×2 → CA is not defined (inner dimensions 2 and 3 don't match).

Comment:

AB and BA are defined because the inner dimensions match (3×2 × 2×3 and 2×3 × 3×2). CA is not defined because the inner dimensions (2×2 × 3×2) do not match. Matrix multiplication is not commutative, as AB ≠ BA (different dimensions and values).

Answer: (i) AB = [-1 2 1]

[3 0 1]

[5 2 3], (ii) BA = [0 4]

[4 2], (iii) CA is not defined. Comment: AB and BA are defined, but CA is not due to dimension mismatch; matrix multiplication is not commutative.

(b) Solve the following system of equations by the inverse method: x + y + z = 3, 3x - y - 3z = 1, x + y + 2z = 0

Write in matrix form: AX = B

A = [1 1 1]

[3 -1 -3]

[1 1 2], X = [x]

[y]

[z], B = [3]

[1]

[0]

Find A^{-1}:

Det(A) = 1((-1)×2 - (-3)×1) - 1(3×2 - (-3)×1) + 1(3×1 - (-1)×1)

= 1(-2 + 3) - 1(6 + 3) + 1(3 + 1) = 1 - 9 + 4 = -4

Adjoint of A:

Cofactors:

$C_{11} = (-1)^{1+1} ((-1)\times2 - (-3)\times1) = 1$

$C_{12} = (-1)^{1+2} (3\times2 - (-3)\times1) = -9$

$C_{13} = (-1)^{1+3} (3\times1 - (-1)\times1) = 4$

$C_{21} = (-1)^{2+1} (1\times2 - 1\times1) = -1$

$C_{22} = (-1)^{2+2} (1\times2 - 1\times1) = 1$

$C_{23} = (-1)^{2+3} (1\times1 - 1\times1) = 0$

$C_{31} = (-1)^{3+1} (1\times-3 - (-1)\times1) = -2$

$C_{32} = (-1)^{3+2} (1\times-3 - 3\times1) = 6$

$C_{33} = (-1)^{3+3} (1\times-1 - 3\times1) = -4$

Adjoint = [1 -1 -2]

[-9 1 6]

[4 0 -4]

A^{-1} = (1/Det(A)) × Adj(A) = (-1/4) [1 -1 -2]

[-9 1 6]

[4 0 -4]

= [-1/4 1/4 1/2]

[9/4 -1/4 -3/2]

[-1   0   1]

$X = A^{-1} B$

= [-1/4 1/4 1/2] [3]

[9/4 -1/4 -3/2] [1]

[-1   0   1] [0]

$x = (-1/4)\times3 + (1/4)\times1 + (1/2)\times0 = -3/4 + 1/4 = -1/2$

$y = (9/4)\times3 + (-1/4)\times1 + (-3/2)\times0 = 27/4 - 1/4 = 26/4 = 13/2$

$z = (-1)\times3 + 0\times1 + 1\times0 = -3$

Answer: x = -1/2, y = 13/2, z = -3

10. (a) What is a linear programming problem?

A linear programming problem involves optimizing (maximizing or minimizing) a linear objective function, subject to a set of linear constraints (inequalities or equations), often represented graphically or algebraically.

Answer: A linear programming problem involves optimizing a linear objective function subject to linear constraints.

(b) Write 6 steps one would undertake in solving a linear programming problem graphically

Identify the decision variables (e.g., x and y).

Formulate the objective function to maximize or minimize (e.g., $f(x, y) = 10x + 15y$).

Write down the constraints as linear inequalities (e.g., $3x + 12y \leq 36$).

Graph the constraints on a coordinate plane, shading the feasible region where all inequalities overlap.

Identify the vertices (corner points) of the feasible region by solving the systems of equations at the intersection points.

Evaluate the objective function at each vertex to determine the optimal solution (maximum or minimum value).

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Evaluate the objective function at each vertex to determine the optimal solution (maximum or minimum value).

(c) Maximize the objective function $f(x, y) = 10x + 15y$ subject to the constraints: $3x + 12y \leq 36$, $9x + 6y \leq 30$, $x \geq 0$, $y \geq 0$

Variables: x, y

Objective: Maximize $f(x, y) = 10x + 15y$

Constraints:

$3x + 12y \leq 36 \rightarrow x + 4y \leq 12$

$9x + 6y \leq 30 \rightarrow 3x + 2y \leq 10$

$x \geq 0, y \geq 0$

Graph:

$x + 4y \leq 12 \rightarrow$ Intercepts: (12, 0), (0, 3)

$3x + 2y \leq 10 \rightarrow$ Intercepts: (10/3, 0), (0, 5)

Vertices of feasible region:

(0, 0)

(0, 3) (from $x + 4y = 12$, $x = 0$)

(10/3, 0) (from $3x + 2y = 10$, $y = 0$)

Intersection of $x + 4y = 12$ and $3x + 2y = 10$:

Solve: Multiply first by 3: $3x + 12y = 36$

Subtract second: $(3x + 12y) - (3x + 2y) = 36 - 10 \rightarrow 10y = 26 \rightarrow y = 2.6$

$x + 4(2.6) = 12 \rightarrow x + 10.4 = 12 \rightarrow x = 1.6$

Vertex: (1.6, 2.6)

Evaluate f(x, y):

(0, 0): $f = 10(0) + 15(0) = 0$

(0, 3): $f = 10(0) + 15(3) = 45$

(10/3, 0): $f = 10(10/3) + 15(0) = 100/3 \approx 33.33$

(1.6, 2.6): $f = 10(1.6) + 15(2.6) = 16 + 39 = 55$

Maximum at (1.6, 2.6).

Answer: Maximum f(x, y) = 55 at (x, y) = (1.6, 2.6)