THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

141 BASIC APPLIED MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2015**

Instructions

1. This paper consists of **Ten (10)** questions.
2. Answer all questions.
3. **All** work done and answers of each question must be shown clearly.
4. NECTA'S Mathematical tables and Non-programmable calculations may be used
5. All writing must be in **black** or **blue** ink, **except** drawing which must be in pencil.

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1. Evaluate the following expressions with the help of a calculator (write your answers correct to 2 decimal places).

(a) $\cos(1/2) + \sin^2(1/3)$

$\cos(1/2) \approx 0.8776$ (using radians, since no degree symbol is specified)

$\sin(1/3) \approx 0.3272$, so $\sin^2(1/3) = (0.3272)^2 \approx 0.1071$

Total: $0.8776 + 0.1071 = 0.9847$

Answer: 0.98

(b) $\sqrt{8\sin(25°)\cos(55°)}$

$\sin(25°) \approx 0.4226$, $\cos(55°) \approx 0.5736$

$\sin(25°)\cos(55°) \approx 0.4226 \times 0.5736 = 0.2424$

$8 \times 0.2424 = 1.9392$

$\sqrt{1.9392} \approx 1.3925$

Answer: 1.39

(c) $\log_e 17 - \ln(5)$

$\log_e(17) = \ln(17) \approx 2.8332$

$\ln(5) \approx 1.6094$

$2.8332 - 1.6094 = 1.2238$

Answer: 1.22

(d) $T(t) = 280 + 920e^{(-0.018t)}$ at $t = 10$ given that $e \approx 2.72$

$e^{(-0.018 \times 10)} = e^{(-0.18)} \approx 2.72^{(-0.18)} \approx 0.8353$

$920 \times 0.8353 = 768.476$

$280 + 768.476 = 1048.476$

Answer: 1048.48

(e) The number of ways for 20 people to be seated on a bench if only 5 seats are available

This is a permutation problem: $P(20, 5) = 20! / (20-5)! = 20 \times 19 \times 18 \times 17 \times 16$

$20 \times 19 = 380$

$380 \times 18 = 6840$

$6840 \times 17 = 116280$

$116280 \times 16 = 1860480$

Answer: 1,860,480

(f) The value of the function $f(x) = (1 + 1/x)^x$ when x = 10, 100, 1000, 10,000 and hence comment on the value of f(x) when x gets very large

At $x = 10$: $(1 + 1/10)^{10} = (1.1)^{10} \approx 2.5937 \approx 2.59$

At $x = 100$: $(1 + 1/100)^{100} = (1.01)^{100} \approx 2.7048 \approx 2.70$

At $x = 1000$: $(1 + 1/1000)^{1000} = (1.001)^{1000} \approx 2.7169 \approx 2.72$

At $x = 10,000$: $(1 + 1/10000)^{10000} = (1.0001)^{10000} \approx 2.7181 \approx 2.72$

Comment: As x gets very large, f(x) approaches $e \approx 2.72$ (the limit of $(1 + 1/x)^x$ as $x \to \infty$ is e).

Answer: 2.59, 2.70, 2.72, 2.72; as x gets very large, f(x) approaches 2.72.

2. (a) Find the coordinates of the points where the line $y = 2x + 5$ meets the curve $3x^2 - 4y^2 + 10 = y$

Substitute $y = 2x + 5$ into the second equation:

$3x^2 - 4(2x + 5)^2 + 10 = 2x + 5$

$(2x + 5)^2 = 4x^2 + 20x + 25$

$4(2x + 5)^2 = 16x^2 + 80x + 100$

So: $3x^2 - (16x^2 + 80x + 100) + 10 = 2x + 5$

Simplify: $3x^2 - 16x^2 - 80x - 100 + 10 = 2x + 5$

$-13x^2 - 80x - 90 = 2x + 5$

$-13x^2 - 82x - 95 = 0$

$13x^2 + 82x + 95 = 0$

Discriminant: $82^2 - 4 \times 13 \times 95 = 6724 - 4940 = 1784$

$x = (-82 \pm \sqrt{1784}) / (2 \times 13)$

$\sqrt{1784} \approx 42.24$

$x = (-82 \pm 42.24) / 26$

$x1 = (-82 + 42.24) / 26 \approx -1.53$

$x2 = (-82 - 42.24) / 26 \approx -4.78$

For $x \approx -1.53$, $y = 2(-1.53) + 5 \approx 1.94$

For $x \approx -4.78$, $y = 2(-4.78) + 5 \approx -4.56$

Answer: Coordinates: (-1.53, 1.94) and (-4.78, -4.56)

(b) The graph of a function f(x) is given below

(Note: The graph shows a piecewise function: horizontal at y = 2 from x = -5 to x = 0, then a line from (0, 2) to (5, 5).)

(i) Use the graph to determine: The function f(x)

From x = -5 to x = 0, f(x) = 2 (horizontal line).

From x = 0 to x = 5, f(x) increases from 2 to 5, slope = $(5-2)/(5-0) = 3/5$.

So, $f(x) = (3/5)x + 2$ for $0 \leq x \leq 5$.

Answer: $f(x) = \{ 2 \text{ if } -5 \leq x < 0, (3/5)x + 2 \text{ if } 0 \leq x \leq 5 \}$

(ii) Use the graph to determine: The domain and range of f(x)

Domain: x from -5 to 5, so [-5, 5].

Range: y from 2 to 5, so [2, 5].

Answer: Domain: [-5, 5], Range: [2, 5]

(c) Find the asymptotes and the intercepts of the function $f(x) = 3x/(x - 3)$ and then sketch its graph

Vertical asymptote: $x - 3 = 0$, so $x = 3$.

Horizontal asymptote: As $x \to \pm\infty$, $f(x) \approx 3x/x = 3$, so $y = 3$.

x-intercept: $3x = 0$, so $x = 0$, $f(0) = 0$, point (0, 0).

y-intercept: At $x = 0$, $f(0) = 0$, same as x-intercept.

Sketch:

At $x = 0$, $(0, 0)$.

As $x \to 3^-$, $f(x) \to -\infty$; as $x \to 3^+$, $f(x) \to +\infty$.

As $x \to \pm\infty$, $f(x) \to 3$.

Answer: Vertical asymptote: $x = 3$, Horizontal asymptote: $y = 3$, Intercepts: $(0, 0)$.

3. (a) Given the series -1 + 1 - 3, …

Terms: -1, 1, -3, …

Pattern: $a1 = -1$, $a2 = 1$, $a3 = -3$

Observing: Signs alternate, magnitudes are 1, 1, 3, … (odd numbers).

So: -1, 1, -3, 5, -7, …

$a_n = (-1)^n \times (2n - 1)$.

Answer: $a_n = (-1)^n \times (2n - 1)$

(b) Express it in the form $S_n = \Sigma f(r)$

From (a), $f(r) = (-1)^r \times (2r - 1)$.

$S_n = \Sigma$ (from r=1 to n) $[(-1)^r \times (2r - 1)]$.

Answer: $S_n = \Sigma$ (from r=1 to n) $[(-1)^r \times (2r - 1)]$

(c) Give one reason as to whether the series is an arithmetic or a geometric progression

Differences: $1 - (-1) = 2$, $-3 - 1 = -4$ (not constant, not arithmetic).

Ratios: $1/(-1) = -1$, $(-3)/1 = -3$ (not constant, not geometric).

Reason: The series alternates signs and magnitudes increase by odd numbers, fitting neither progression.

Answer: The series is neither arithmetic nor geometric because the differences and ratios between consecutive terms are not constant.

(d) Determine the value of n for which $S_n = 575$

$S_n = (-1)^1(1) + (-1)^2(3) + (-1)^3(5) + \ldots + (-1)^n(2n-1)$

This is complex to compute directly without a closed form. Let's try a pattern:

n = 1: -1

n = 2: -1 + 1 = 0

n = 3: 0 - 3 = -3

n = 4: -3 + 5 = 2

n = 5: 2 - 7 = -5

n = 6: -5 + 9 = 4

4. (a) Find dy/dx from first principle given $y = 2x^2$

First principle: $dy/dx = \lim(h\to 0) [(f(x+h) - f(x))/h]$

$y = 2x^2$, so $f(x) = 2x^2$

$f(x+h) = 2(x+h)^2 = 2(x^2 + 2xh + h^2) = 2x^2 + 4xh + 2h^2$

$f(x+h) - f(x) = (2x^2 + 4xh + 2h^2) - 2x^2 = 4xh + 2h^2$

$(f(x+h) - f(x))/h = (4xh + 2h^2)/h = 4x + 2h$

$\lim(h\to 0) (4x + 2h) = 4x$

Answer: $dy/dx = 4x$

(b) If $x = 2t + 9$ and $y = (t + 1)^{11}$, find dy/dx and d^2y/dx^2 in terms of t

$x = 2t + 9$, so $dx/dt = 2$

$y = (t + 1)^{11}$, so $dy/dt = 11(t + 1)^{10}$

$dy/dx = (dy/dt) / (dx/dt) = 11(t + 1)^{10} / 2$

Now, $d^2y/dx^2 = d/dx (dy/dx) = d/dt (dy/dx) / (dx/dt)$

$dy/dx = (11/2)(t + 1)^{10}$

$d/dt (dy/dx) = (11/2) \times 10(t + 1)^9 = 55(t + 1)^9$

$d^2y/dx^2 = [55(t + 1)^9] / 2$

Answer: $dy/dx = (11/2)(t + 1)^{10}$, $d^2y/dx^2 = 55(t + 1)^9 / 2$

(c) Given $f(x) = x^3 - 2x^2 + x - 7$

(i) Find the stationary values of the function

$f(x) = x^3 - 2x^2 + x - 7$

$f'(x) = 3x^2 - 4x + 1$

Set $f'(x) = 0$: $3x^2 - 4x + 1 = 0$

Discriminant: $(-4)^2 - 4 \times 3 \times 1 = 16 - 12 = 4$

$x = (4 \pm \sqrt{4}) / (2 \times 3) = (4 \pm 2) / 6$

$x = 1$ or $x = 1/3$

$f(1) = 1^3 - 2(1)^2 + 1 - 7 = 1 - 2 + 1 - 7 = -7$

$f(1/3) = (1/3)^3 - 2(1/3)^2 + 1/3 - 7 = 1/27 - 2/9 + 1/3 - 7 = 1/27 - 6/27 + 9/27 - 189/27 = -185/27 \approx -6.85$

Answer: Stationary values: -7 at $x = 1$, $-185/27 \approx -6.85$ at $x = 1/3$

(ii) Find the equation of the tangent line to the curve at the point (0, -7)

At $x = 0$, $f'(x) = 3(0)^2 - 4(0) + 1 = 1$ (slope of tangent)

Point: (0, -7)

Equation: $y - (-7) = 1(x - 0)$

$y + 7 = x$

$y = x - 7$

Answer: $y = x - 7$

(iii) Draw the graph of this function for $-2 \leq x \leq 3$ and indicate on the graph the stationary points and the equation of the tangent line obtained in part (c) (ii)

Key points:

$x = 0$: $f(0) = -7$

$x = 1$: $f(1) = -7$ (stationary)

$x = 1/3$: $f(1/3) = -185/27 \approx -6.85$ (stationary)

$x = -2$: $f(-2) = (-2)^3 - 2(-2)^2 + (-2) - 7 = -8 - 8 - 2 - 7 = -25$

$x = 3$: $f(3) = 3^3 - 2(3)^2 + 3 - 7 = 27 - 18 + 3 - 7 = 5$

Tangent at (0, -7): $y = x - 7$ (e.g., at $x = 1$, $y = 1 - 7 = -6$).

Graph:

Curve passes through (-2, -25), (0, -7), (1/3, -185/27), (1, -7), (3, 5).

Stationary points at (1, -7) and (1/3, -185/27).

Tangent line $y = x - 7$.

Answer: (I can confirm if you'd like to generate a sketch.)

5. (a) Evaluate the following integrals:

(i) $\int (x - 9)^{11}\, dx$

$\int (x - 9)^{11}\, dx$

Use substitution: $u = x - 9$, $du = dx$

$\int u^{11}\, du = u^{12}/12 + C$

Substitute back: $(x - 9)^{12}/12 + C$

Answer: $(x - 9)^{12}/12 + C$

(ii) $\int x \cos(5x + 9)\, dx$

Use integration by parts: $\int u\, dv = uv - \int v\, du$

Let $u = x$, $dv = \cos(5x + 9)\, dx$

$du = dx$, $v = \int \cos(5x + 9)\, dx = (1/5)\sin(5x + 9)$

$\int x \cos(5x + 9)\, dx = x \times (1/5)\sin(5x + 9) - \int (1/5)\sin(5x + 9)\, dx$

$= (x/5)\sin(5x + 9) - (1/5) \int \sin(5x + 9)\, dx$

$= (x/5)\sin(5x + 9) + (1/25)\cos(5x + 9) + C$

Answer: $(x/5)\sin(5x + 9) + (1/25)\cos(5x + 9) + C$

(b) Given that $\int (3x^2 - 3x - 16)\, dx = 40$, find the value of the constant a

First, compute the definite integral: $\int$ (from 2 to 4) $(3x^2 - ax - 16/x^2)\, dx$

Split the integral: $\int$ (from 2 to 4) $3x^2\, dx - \int$ (from 2 to 4) $ax\, dx - \int$ (from 2 to 4) $(16/x^2)\, dx$

$\int 3x^2\, dx = 3(x^3/3) = x^3$

From 2 to 4: $[4^3 - 2^3] = 64 - 8 = 56$

$\int ax\, dx = a(x^2/2)$

From 2 to 4: $a[(4^2/2) - (2^2/2)] = a[8 - 2] = 6a$

$\int (16/x^2)\,dx = \int 16x^{-2}\,dx = 16(-1/x) = -16/x$

From 2 to 4: [-16/4 - (-16/2)] = [-4 - (-8)] = -4 + 8 = 4

Total integral: 56 - 6a + 4 = 60 - 6a

Set equal to 40: 60 - 6a = 40

60 - 40 = 6a

20 = 6a

a = 20/6 = 10/3

Answer: a = 10/3

(c) Sketch the graph of the curve $y = x^3 - 3x^2 + 2x$ and hence find the area bound by the curve and the x-axis

$y = x^3 - 3x^2 + 2x = x(x^2 - 3x + 2) = x(x - 1)(x - 2)$

x-intercepts: x = 0, x = 1, x = 2

y-intercept: (0, 0)

Stationary points: $dy/dx = 3x^2 - 6x + 2 = 0$

Discriminant: $(-6)^2 - 4 \times 3 \times 2 = 36 - 24 = 12$

$x = (6 \pm \sqrt{12}) / (2 \times 3) = (6 \pm 2\sqrt{3}) / 6 = 1 \pm \sqrt{3}/3$

$x \approx 1.58, 0.42$

At $x \approx 0.42$, $y \approx 0.42(0.42 - 1)(0.42 - 2) \approx 0.42(-0.58)(-1.58) \approx 0.38$

At $x \approx 1.58$, $y \approx 1.58(1.58 - 1)(1.58 - 2) \approx 1.58(0.58)(-0.42) \approx -0.38$

Graph: Crosses x-axis at (0, 0), (1, 0), (2, 0); local max at (0.42, 0.38), local min at (1.58, -0.38).

Area:

From x = 0 to 1 (above x-axis): $\int$ (from 0 to 1) $(x^3 - 3x^2 + 2x)\,dx = [x^4/4 - x^3 + x^2]$ from 0 to 1 = (1/4 - 1 + 1) - 0 = 1/4

From x = 1 to 2 (below x-axis): $\int$ (from 1 to 2) $(x^3 - 3x^2 + 2x)\,dx = [x^4/4 - x^3 + x^2]$ from 1 to 2 = (4 - 8 + 4) - (1/4 - 1 + 1) = 0 - 1/4 = -1/4

Total area = 1/4 + | -1/4 | = 1/4 + 1/4 = 1/2

Answer: Area = 1/2

6. The following were the scores obtained by 22 students from Sarawak Secondary School in a mathematics classroom test: 49, 64, 38, 60, 46, 64, 68, 42, 38, 68, 57, 63, 76, 51, 54, 66, 62, 63, 58, 59, 47, 55

(a) Summarize the scores in a frequency table with equal class intervals of size 5. Take the lowest limit to be 35

Scores: 38, 38, 42, 46, 47, 49, 51, 54, 55, 57, 58, 59, 60, 62, 63, 63, 64, 64, 66, 68, 68, 76

Class intervals: 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69, 70-74, 75-79

35-39: 38, 38 (2)

40-44: 42 (1)

45-49: 46, 47, 49 (3)

50-54: 51, 54 (2)

55-59: 55, 57, 58, 59 (4)

60-64: 60, 62, 63, 63, 64, 64 (6)

65-69: 66, 68, 68 (3)

70-74: (0)

75-79: 76 (1)

Answer: Frequency table:

35-39: 2, 40-44: 1, 45-49: 3, 50-54: 2, 55-59: 4, 60-64: 6, 65-69: 3, 70-74: 0, 75-79: 1

(b) Find the mean score by using the data in part (a)

Midpoints: 37, 42, 47, 52, 57, 62, 67, 72, 77

Mean = Σ (frequency $\times$ midpoint) / total frequency

$= (2 \times 37 + 1 \times 42 + 3 \times 47 + 2 \times 52 + 4 \times 57 + 6 \times 62 + 3 \times 67 + 0 \times 72 + 1 \times 77) / 22$

$= (74 + 42 + 141 + 104 + 228 + 372 + 201 + 0 + 77) / 22$

$= 1239 / 22 \approx 56.32$

Answer: Mean $\approx$ 56.32

(c) Find the interquartile range

Ordered scores: 38, 38, 42, 46, 47, 49, 51, 54, 55, 57, 58, 59, 60, 62, 63, 63, 64, 64, 66, 68, 68, 76

n = 22, Q1 position = (22+1)/4 = 5.75 → between 5th and 6th: (47 + 49)/2 = 48

Q3 position = 3(22+1)/4 = 17.25 → between 17th and 18th: (64 + 64)/2 = 64

IQR = Q3 - Q1 = 64 - 48 = 16

Answer: IQR = 16

(d) How many students scored above the mean score?

Mean ≈ 56.32

Scores above 56.32: 57, 58, 59, 60, 62, 63, 63, 64, 64, 66, 68, 68, 76 (13 students)

Answer: 13 students

7. (a) If A and B are two events such that P(A) = 1/4, P(B) = 1/2 and P(A ∩ B) = 1/8, find P(A ∪ B) and P(A' ∩ B')

P(A ∪ B) = P(A) + P(B) - P(A ∩ B) = 1/4 + 1/2 - 1/8 = 2/8 + 4/8 - 1/8 = 5/8

P(A' ∩ B') = 1 - P(A ∪ B) = 1 - 5/8 = 3/8

Answer: P(A ∪ B) = 5/8, P(A' ∩ B') = 3/8

(b) A fair die was rolled and the events A and B were recorded as follows: A = {1, 3, 5} and B = {2, 3, 4, 5}. Find P(A ∪ B)

A = {1, 3, 5}, P(A) = 3/6

B = {2, 3, 4, 5}, P(B) = 4/6

A ∩ B = {3, 5}, P(A ∩ B) = 2/6

P(A ∪ B) = P(A) + P(B) - P(A ∩ B) = 3/6 + 4/6 - 2/6 = 5/6

Answer: P(A ∪ B) = 5/6

(c) In Section B of CSEE Basic Mathematics Examination each candidate has to choose and answer four out of six questions. How many choices are there for each candidate?

Number of ways to choose 4 questions out of 6: C(6, 4) = 6! / (4! × 2!) = (6 × 5) / (2 × 1) = 15

Answer: 15 choices

(d) A box contains 4 ripe mangoes and 9 non ripe mangoes. If two mangoes are randomly chosen from the box, find the probability that both will be ripe mangoes

Total mangoes = 4 + 9 = 13

Probability both are ripe = (4/13) × (3/12) = 12/156 = 1/13

Answer: 1/13

8. (a) Without using a mathematical table or a calculator, evaluate:

(i) cos(165°)

cos(165°) = cos(180° - 15°) = -cos(15°) (since cos(180° - θ) = -cos(θ))

cos(15°) = cos(45° - 30°)

Use identity: cos(A - B) = cosA cosB + sinA sinB

cos(15°) = cos(45°)cos(30°) + sin(45°)sin(30°)

= (√2/2)(√3/2) + (√2/2)(1/2) = (√6/4) + (√2/4) = (√6 + √2)/4

So, cos(165°) = -(√6 + √2)/4

Answer: cos(165°) = -(√6 + √2)/4

(ii) tan(A + B) given that A and B are acute angles having sin(A) = 7/25 and cos(B) = 5/13

sin(A) = 7/25, so cos(A) = $\sqrt{(1 - (7/25)^2)}$ = √(1 - 49/625) = √(576/625) = 24/25 (A is acute)

cos(B) = 5/13, so sin(B) = $\sqrt{(1 - (5/13)^2)}$ = √(1 - 25/169) = √(144/169) = 12/13 (B is acute)

tan(A + B) = (tanA + tanB)/(1 - tanA tanB)

tanA = sinA/cosA = (7/25)/(24/25) = 7/24

tanB = sinB/cosB = (12/13)/(5/13) = 12/5

tanA + tanB = 7/24 + 12/5 = (7×5 + 12×24)/(24×5) = (35 + 288)/120 = 323/120

tanA tanB = (7/24)×(12/5) = 84/120 = 7/10

1 - tanA tanB = 1 - 7/10 = 3/10

tan(A + B) = (323/120)/(3/10) = (323/120)×(10/3) = 323/36

Answer: tan(A + B) = 323/36

(b) Find the values of x that satisfy the equation sin 2x + cos x = 0 for 0° ≤ x ≤ 360°

Use identity: sin 2x = 2 sin x cos x

Equation: 2 sin x cos x + cos x = 0

Factor: cos x (2 sin x + 1) = 0

cos x = 0 → x = 90°, 270°

2 sin x + 1 = 0 → sin x = -1/2 → x = 210°, 330° (since sin x is negative in 3rd and 4th quadrants)

Answer: x = 90°, 210°, 270°, 330°

(ii) Verify the solution of the equation in part (b)(i) can be obtained graphically by plotting the graph of y = sin 2x + cos x for 0° ≤ x ≤ 360°

To verify, we'd plot y = sin 2x + cos x and find where y = 0.

From (b)(i), solutions are x = 90°, 210°, 270°, 330°.

Graphically:

At x = 90°, sin(180°) + cos(90°) = 0 + 0 = 0

At x = 210°, sin(420°) + cos(210°) = sin(60°) + cos(30° - 210°) = (√3/2) + (-√3/2) = 0

At x = 270°, sin(540°) + cos(270°) = 0 + 0 = 0

At x = 330°, sin(660°) + cos(330°) = sin(300°) + cos(30°) = (-√3/2) + (√3/2) = 0

The graph crosses the x-axis at these points, confirming the solutions.

Answer: The solutions x = 90°, 210°, 270°, 330° can be verified graphically as the points where y = sin 2x + cos x crosses the x-axis.

9. (a) Given:

A = [1 2]

[3 4]

[-1 5],

B = [-2 3 4]

[3 2 1],

C = [3 5]

[1 2]

(i) State with one reason as to whether the matrix operations AB, BA and BC are defined or not

AB: A is 3×2, B is 2×3 → 3×3 matrix (defined).

BA: B is 2×3, A is 3×2 → 2×2 matrix (defined).

BC: B is 2×3, C is 2×2 → not defined (inner dimensions 3 and 2 don't match).

Answer: AB and BA are defined because the inner dimensions match; BC is not defined because the inner dimensions (3 and 2) do not match.

(ii) Find $2A + 3B^T$

A = [1 2]

[3 4]

[-1 5]

2A = [2 4]

[6 8]

[-2 10]

B = [-2 3 4]

[3 2 1]

B^T = [-2 3]

[3 2]

[4 1]

$3B^T$ = [-6 9]

[9 6]

[12 3]

$2A + 3B^T$ = [2-6 4+9]

[6+9 8+6]

[-2+12 10+3]

= [-4 13]

[15 14]

[10 13]

Answer: $2A + 3B^T$ = [-4 13]

[15 14]

[10 13]

(b) Verify that | a b c | = (a-b)b¹-c

| a^1 b^1 c^1 |

| a^2 b^2 c^2 |

Determinant: | a b c |

| a^1 b^1 c^1 |

| a^2 b^2 c^2 |

Expand along first row:

$= a(b^1c^2 - c^1b^2) - b(a^1c^2 - c^1a^2) + c(a^1b^2 - b^1a^2)$

$= ab^1c^2 - ac^1b^2 - ba^1c^2 + bc^1a^2 + ca^1b^2 - cb^1a^2$

Group: $(ab^1c^2 - ba^1c^2) + (-ac^1b^2 + ca^1b^2) + (bc^1a^2 - cb^1a^2)$

$= b^1c^2(a - a) + a^1b^2(-c + c) + c^1a^2(b - b) = 0$

(c) If D = [a -4 -6] is the inverse of matrix E = [1 2 -2], find the values of a and b

[-8   5   7]   [3 b   1]

[-5   3   4]   [-1 1 -3]

Since D is the inverse of E, D × E = I (identity matrix).

D × E = [a -4 -6] [1 2 -2] = [1 0 0]

[-8   5   7] [3 b   1]   [0 1 0]

[-5   3   4] [-1 1 -3]   [0 0 1]

First row:

a(1) + (-4)(3) + (-6)(-1) = a - 12 + 6 = 1 → a - 6 = 1 → a = 7

a(2) + (-4)(b) + (-6)(1) = 2a - 4b - 6 = 0 → 2(7) - 4b - 6 = 0 → 14 - 4b - 6 = 0 → 8 - 4b = 0 → b = 2

Verify with third element of first row:

$a(-2) + (-4)(1) + (-6)(-3) = -2a - 4 + 18 = 0 \rightarrow -2(7) - 4 + 18 = -14 - 4 + 18 = 0$ (checks).

Answer: a = 7, b = 2

10. Mr. Taramise owns 480 acres of land on which he grows either maize or beans during the farming period. He normally expects a profit of Tshs 40,000/= per acre on maize and Tshs 30,000/= per acre on beans and he has 800 hours of labour available. If maize requires 2 hours per acre to raise and beans require 1 hour per acre to raise, find how many acres of maize and beans he should plant to get maximum profit

Let x = acres of maize, y = acres of beans.

Objective: Maximize profit $P = 40000x + 30000y$

Constraints:

$x + y \leq 480$ (land)

$2x + y \leq 800$ (labor)

$x \geq 0, y \geq 0$

Solve the system of inequalities:

$x + y = 480 \rightarrow y = 480 - x$

$2x + y = 800 \rightarrow y = 800 - 2x$

Intersection: $480 - x = 800 - 2x \rightarrow 2x - x = 800 - 480 \rightarrow x = 320 \rightarrow y = 480 - 320 = 160$

Vertices of feasible region: (0, 0), (0, 480), (320, 160), (400, 0)

(0, 0): P = 0

(0, 480): $P = 40000(0) + 30000(480) = 14400000$

(320, 160): $P = 40000(320) + 30000(160) = 12800000 + 4800000 = 17600000$

(400, 0): $P = 40000(400) + 30000(0) = 16000000$

Maximum profit at (320, 160).

Answer: 320 acres of maize, 160 acres of beans