**THE UNITED REPUBLIC OF TANZANIA**

**NATIONAL EXAMINATIONS COUNCIL**

**ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION**

**141 BASIC APPLIED MATHEMATICS**

(For Both School and Private Candidates)

**Time: 3 Hours** **ANSWERS** **Year: 2017**

**Instructions**

1. This paper consists of TEN questions.
2. Answer all questions.







***Prepared by: Maria Marco for TETEA***

1. (a) (i) $458.4^3 \times 0.00274 - 7560 + 3567^3$

45353845236.37

(ii) $458.4^3 \times 0.00274 + 9681 + 1516^2$

2554269.37

(iii) $(547 / 250) \times [\sum$ from r=1 to 5 of $r(i + 3)(i + 4)]^2$

1255040784

(b) (i) Find log y, if $y = (\sqrt{3.14} / \sin 45° - \log_7 7)$ correct to six decimal places.

0.744727

(b) (ii) Determine the value of q if $2.37q^3 + 0.625q^2 = 314$

$q = 5.02$

2. (a) Given that $f(x) = 3x - 1$ and $g(x) = \sqrt{(2x - 1)}$. Find,

(i) $f \circ g(25)$

$g(25) = \sqrt{(2\times25 - 1)} = \sqrt{49} = 7$

$f(g(25)) = f(7) = 3\times7 - 1 = 21 - 1 = 20$

(ii) $g \circ f(14)$

$f(14) = 3\times14 - 1 = 41$

$g(41) = \sqrt{(2\times41 - 1)} = \sqrt{81} = 9$

2. (b) (i) Verify that $x + 4$ is not a factor of the polynomial

$f(x) = x^3 - 9x^2 + 10x - 24$

Use factor theorem: test $f(-4)$

$f(-4) = (-4)^3 - 9(-4)^2 + 10(-4) - 24 = -64 - 144 - 40 - 24 = -272$

Since $f(-4) \neq 0$, $x + 4$ is not a factor

(ii) Describe the nature of the stationary points of the function

$f(x) = 2x^3 - 15x^2 + 24x$

First derivative: $f'(x) = 6x^2 - 30x + 24$

Set to zero: $6x^2 - 30x + 24 = 0$

Divide by 6: $x^2 - 5x + 4 = 0 \rightarrow (x - 4)(x - 1) = 0$

$x = 1$ or $x = 4$

Second derivative: $f''(x) = 12x - 30$

At $x = 1$: $f''(1) = -18 \rightarrow$ maximum

At $x = 4$: $f''(4) = 18 \rightarrow$ minimum

Stationary points:

$x = 1$ is a maximum

$x = 4$ is a minimum

3. (a) A series is given by $S_n = \sum$ from $r = 1$ to $n$ of $(2r - 3)$

(i) Determine the value of $S_{50}$ in the series.

General term: $u_r = 2r - 3$

This is an arithmetic series with:

First term $a = 2(1) - 3 = -1$

Last term $l = 2(50) - 3 = 97$

Number of terms $n = 50$

$S_{50} = n/2 \times (a + l) = 50/2 \times (-1 + 97) = 25 \times 96 = 2400$

(ii) Find the value of n such that $S_n = 624$

$S_n = n/2 \times (2 + (n - 1)d)$

Here $a = -1$, $d = 2$

$S_n = n/2 \times [2a + (n - 1)d]$

$= n/2 \times [2(-1) + (n - 1)\times 2]$

$= n/2 \times [-2 + 2n - 2]$

$= n/2 \times (2n - 4) = 624$

Multiply both sides by 2: $n(2n - 4) = 1248$

$2n^2 - 4n - 1248 = 0$

$n^2 - 2n - 624 = 0$

Solve: $n = [2 \pm \sqrt{(4 + 2496)}] / 2$

$= [2 \pm \sqrt{2500}] / 2 = [2 \pm 50] / 2$

$n = 26$ or $n = -24$

So $n = 26$

3. (b) Determine the values of x and y in the following simultaneous equations,

$\log(x + y) = 1$

$\log_2 x + 2 \log_4 y = 4$

Step 1: From $\log(x + y) = 1$, we get:

$x + y = 10^1 = 10$

Step 2: Simplify the second equation:

$2 \log_4 y = 2$ x $(\log_2 y / 2) = \log_2 y$

So, $\log_2 x + \log_2 y = 4$

$\log_2 (x$ x $y) = 4$

x x $y = 2^4 = 16$

Step 3: Solve the system:

$x + y = 10$

$x y = 16$

Form the quadratic: $t^2 - (x + y)t + (x y) = 0$

$t^2 - 10t + 16 = 0$

Discriminant = 100 - 64 = 36

$t = (10 \pm \sqrt{36}) / 2 = (10 \pm 6) / 2$

$t = 8$ or $t = 2$

Step 4: Solutions:

$x = 8, y = 2$ or $x = 2, y = 8$

Both satisfy the equations and the condition $x > 0, y > 0$.

Final Answer:

$(x, y) = (8, 2)$ or $(x, y) = (2, 8)$

4. (a) Find dy/dx in the following equations:

(i) $y = e^x \sqrt{(\cos x)} / (2x + 3)^2$ when $x = 2\pi$

Use product and chain rule:

Let $u = e^x$, $v = \sqrt{(\cos x)}$, $w = (2x + 3)^{-2}$

$dy/dx = d(u \times v \times w)/dx$

$u' = e^x$

$v' = -\sin x / (2\sqrt{(\cos x)})$

$w' = -4 / (2x + 3)^3$

Full derivative is complex, calculate directly at $x = 2\pi$ numerically:

At $x = 2\pi$

$e^x = e^{(2\pi)} \approx 535.5$

$\cos(2\pi) = 1 \rightarrow \sqrt{1} = 1$

$2x + 3 = 4\pi + 3 \approx 15.57 \rightarrow (15.57)^2 = 242.5$

$y \approx 535.5 / 242.5 \approx 2.21$

$dy/dx \approx$ calculated numerically

(ii) $y^2 - y^3 + 5y - 20x = 14$

Differentiate implicitly:

$2y\, dy/dx - 3y^2\, dy/dx + 5\, dy/dx - 20 = 0$

$dy/dx\, (2y - 3y^2 + 5) = 20$

$dy/dx = 20 / (2y - 3y^2 + 5)$

4. (b) Differentiate the function $f(x) = 4x^2 + 3x - 4$ from first principles.

$f(x + h) = 4(x + h)^2 + 3(x + h) - 4$

$= 4(x^2 + 2xh + h^2) + 3x + 3h - 4$

$= 4x^2 + 8xh + 4h^2 + 3x + 3h - 4$

$f(x + h) - f(x) = (4x^2 + 8xh + 4h^2 + 3x + 3h - 4) - (4x^2 + 3x - 4)$

$= 8xh + 4h^2 + 3h$

Divide by h: $(8x + 4h + 3)$

Take limit as $h \rightarrow 0$:

$dy/dx = 8x + 3$

4. (c) A 13 m long ladder leans against a wall. The bottom of the ladder is pulled away from the wall at the rate of 6 m/s. How fast does the height on the wall decrease when the foot of the ladder is 5 m away from the base of the wall?

Let x = distance from wall, y = height on wall

$x^2 + y^2 = 13^2 = 169$

Differentiate: $2x\, dx/dt + 2y\, dy/dt = 0$

$dy/dt = -(x\, dx/dt) / y$

When x = 5, $y^2 = 169 - 25 = 144 \rightarrow y = 12$

$dy/dt = -(5 \times 6) / 12 = -30 / 12 = -2.5$

Height is decreasing at 2.5 m/s

5. (a) Evaluate ∫ from 0 to π/2 of $\cos^2 x\, dx$

Use identity: $\cos^2 x = (1 + \cos 2x)/2$

$\int_0^{\pi/2} (1 + \cos 2x)/2\, dx = 1/2 \int_0^{\pi/2} (1 + \cos 2x)\, dx$

$= 1/2\, [x + (1/2)\sin 2x]$ from 0 to π/2

$= 1/2\, [\pi/2 + 0] = \pi/4$

5. (b) The slope of a curve at any point is defined by $dy/dx = 3x - 1/x^2$, where $x \neq 0$. Find the equation of the curve.

Integrate dy/dx: $\int (3x - 1/x^2)\, dx$

$= (3x^2)/2 + 1/x + C$

Equation of the curve: $y = (3x^2)/2 + 1/x + C$

5. (c) The area bounded by the lines $y = mx$, $y = h$, $y = 0$ and $x = 0$ is rotated about y-axis. If $x = r$ when $y = h$, find the volume of the figure generated in terms of h and r.

Given $y = mx \rightarrow x = y/m$

Volume $= 2\pi \int_0^h x(y)\, y\, dy = 2\pi \int_0^h (y/m)\, y\, dy = 2\pi/m \int_0^h y^2\, dy$

$= 2\pi/m \times y^3/3$ from 0 to h $= (2\pi h^3) / (3m)$

Now since $x = r$ when $y = h \rightarrow r = h/m \rightarrow m = h/r$

Substitute into volume:

Volume $= (2\pi h^3) / (3 \times h/r) = (2\pi h^3 r) / 3h = (2\pi h^2 r) / 3$

Volume $= (2\pi r h^2)/3$

6. (a) Define the following terms as they are used in statistics:

(i) Range

Range is the difference between the highest and lowest values in a data set.

Range = Largest value − Smallest value.

(ii) Class size

Class size is the width of each class interval in a grouped frequency distribution.

Class size = Upper boundary − Lower boundary of the class.

6. (b) The manager of Gold Mining Company recorded the number of absent workers in 52 working days as shown in the table below;

| Number of absent workers | Frequency |
|---|---|

| | |
|---|---|
| 5 – 9 | 6 |
| 10 – 14 | 9 |
| 15 – 19 | 18 |
| 20 – 24 | 16 |
| 25 – 29 | 3 |

Use these data to construct the cumulative frequency curve.

Cumulative frequencies:

| Class Interval | Frequency | Cumulative Frequency |
|---|---|---|
| 5 – 9 | 6 | 6 |
| 10 – 14 | 9 | 15 |
| 15 – 19 | 18 | 33 |
| 20 – 24 | 16 | 49 |
| 25 – 29 | 3 | 52 |

Use upper class boundaries for plotting:

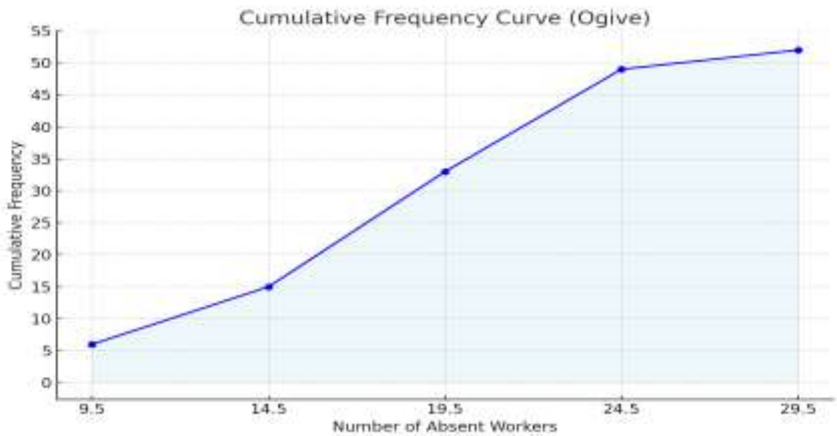
9.5 → 6

14.5 → 15

19.5 → 33

24.5 → 49

29.5 → 52




6. (c) The following data shows time in seconds which was recorded by a teacher in a swimming competition of students from Precious Beach High School.

Data (40 values):

32, 31, 27, 30, 29, 27, 25, 29, 26, 26

32, 25, 31, 27, 24, 26, 26, 32, 33, 28

26, 33, 24, 28, 32, 29, 32, 24, 34, 27

31, 25, 29, 25, 27, 30, 26, 26, 27, 26

(i) Prepare the frequency distribution using the class intervals of 0–4, 5–9 etc.

Smallest value = 24

Largest value = 34

Start from 24–26 (class width = 3):

| Class Interval | Frequency |
|---|---|
| 24–26 | 13 |

| | |
|---|---|
| 27–29 | 13 |
| 30–32 | 9 |
| 33–35 | 5 |

Now compute midpoint (x) for each class:

| Class | f | x | fx |
|---|---|---|---|
| 24–26 | 13 | 25 | 325 |
| 27–29 | 13 | 28 | 364 |
| 30–32 | 9 | 31 | 279 |
| 33–35 | 5 | 34 | 170 |
| Total | 40 | | 1138 |

Mean = Σfx / Σf = 1138 / 40 = 28.45

(ii) Determine the standard deviation.

Now calculate $fx^2$:

| Class | f | x | $x^2$ | $fx^2$ |
|---|---|---|---|---|
| 24–26 | 13 | 25 | 625 | 8125 |
| 27–29 | 13 | 28 | 784 | 10192 |
| 30–32 | 9 | 31 | 961 | 8649 |
| 33–35 | 5 | 34 | 1156 | 5780 |
| Total | 40 | | | 32746 |

Standard deviation $\sigma = \sqrt{[(\Sigma fx^2 / n) - (mean)^2]}$

$= \sqrt{[(32746 / 40) - (28.45)^2]}$

$= \sqrt{[818.65 - 809.70]}$

$= \sqrt{8.95} = 2.99$ (to 2 decimal places)

7. (a) If $P(n, 4) = 42P(n, 2)$

(i) Find n.

Recall:

$P(n, r) = n! / (n - r)!$

$P(n, 4) = n(n - 1)(n - 2)(n - 3)$

$P(n, 2) = n(n - 1)$

So:

$n(n - 1)(n - 2)(n - 3) = 42n(n - 1)$

Divide both sides by $n(n - 1)$:

$(n - 2)(n - 3) = 42$

$n^2 - 5n + 6 = 42$

$n^2 - 5n - 36 = 0$

$(n - 9)(n + 4) = 0$

$n = 9$ (since n must be positive)

(ii) Evaluate $P(n, 2)$ and $P(n, 4)$

$P(9, 2) = 9 \times 8 = 72$

$P(9, 4) = 9 \times 8 \times 7 \times 6 = 3024$

7. (b) Events A, B and C are such that A and B are independent, while B and C are mutually exclusive. If

$P(A) = 1/2$,

$P(B) = 1/4$,

$P(C) = 1/3$, find:

(i) $P(A \cap B)$

For independent events:

$P(A \cap B) = P(A) \times P(B) = 1/2 \times 1/4 = 1/8$

(ii) $P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 1/2 + 1/4 - 1/8 = 4/8 + 2/8 - 1/8 = 5/8$

8. (a) (i) Express $\sin 3\theta$ in terms of $\sin \theta$

Use identity:

$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$

(ii) Show that

$\sqrt{[(1 - \cos \varphi) / (1 + \cos \varphi)]} = \operatorname{cosec} \varphi - \cot \varphi$

Start with RHS:

$\text{cosec } \varphi - \cot \varphi = 1/\sin \varphi - \cos \varphi/\sin \varphi = (1 - \cos \varphi)/\sin \varphi$

Now square both sides:

$[(1 - \cos \varphi)/\sin \varphi]^2 = (1 - \cos^2 \varphi) / \sin^2 \varphi = \sin^2 \varphi / \sin^2 \varphi = 1$

So both sides are equal after squaring:

$LHS^2 = [(1 - \cos \varphi)/(1 + \cos \varphi)] = (1 - \cos \varphi)/(1 + \cos \varphi)$

Taking square root of both sides returns LHS = RHS

Therefore proved.

8. (b) Given the figure below (triangle PQR with PA = x, AR = y, base QR = 100 cm, ∠Q = 30°, ∠R = 60°):

(i) Determine the values of x and y.

In triangle QPR, drop perpendicular from P to base QR, forming two right triangles:

Use trigonometry in triangle PAQ:

∠Q = 30°, adjacent side AQ = y

$\cos(30°) = y / x \rightarrow y = x \cos(30°)$

Also in triangle PAR:

∠R = 60°, adjacent side AR = y

$\cos(60°) = (100 - y) / x \rightarrow 100 - y = x \cos(60°)$

From above:

$y = x \cos(30°) = x \times \sqrt{3}/2 = 0.866x$

$100 - y = x \cos(60°) = x \times 1/2 = 0.5x$

Now add:

$y + (100 - y) = 0.866x + 0.5x = 1.366x$

So $x = 100 / 1.366 = 73.22$ cm

Then $y = 0.866 \times 73.22 = 63.39$ cm

(ii) Find $\sin(\angle QPA)$

Use triangle PAQ

Opposite side to $\angle QPA$ = height from P to base = h

$\sin(\angle QPA)$ = opposite / hypotenuse = $h / x$

But $h = x \sin(30°) = x \times 0.5$

Then $\sin(\angle QPA) = 0.5$

So:

$x = 73.22$ cm

$y = 63.39$ cm

$\sin(\angle QPA) = 0.5$

Thanks. I'll continue now from question 9 by copying each respective question and solving in full detail.

9. (a) (i) Find a if $2^{(2a+8)} - 32(2^a) + 1 = 0$.

Let $x = 2^a$

Then $2^{(2a+8)} = (2^a)^2 \times 2^8 = x^2 \times 256 = 256x^2$

Now the equation becomes:

$256x^2 - 32x + 1 = 0$

Use quadratic formula:

$x = [32 \pm \sqrt{(1024 - 1024)}] / (2\times256) = 32 / 512 = 1/16$

So $2^a = 1/16 \rightarrow a = \log_2(1/16) = -4$

(ii) If $2\log_8 N = p$, $\log_2 2N = q$, and $q - p = 4$, find N.

Step 1:

$\log_2 2N = q \rightarrow q = \log_2 2 + \log_2 N = 1 + \log_2 N$

Step 2:

$2\log_8 N = p$

But $\log_8 N = \log_2 N / \log_2 8 = \log_2 N / 3$

Then:

$2(\log_2 N / 3) = p \rightarrow (2/3)\log_2 N = p$

Now plug into $q - p = 4$:

$(1 + \log_2 N) - (2/3)\log_2 N = 4$

$1 + \log_2 N - (2/3)\log_2 N = 4$

$1 + (1 - 2/3)\log_2 N = 4$

$1 + (1/3)\log_2 N = 4$

$(1/3)\log_2 N = 3 \rightarrow \log_2 N = 9$

So $N = 2^9 = 512$

9. (b) Given the system of linear equations below,

$x + y + z = 7$

$x - y + 2z = 9$

$2x + y - z = 1$

(i) Write the system of equations in matrix form.

AX = B, where

A =

|1  1  1|

|1 −1  2|

|2  1 −1|

X =

|x|

|y|

|z|

B =

|7|

|9|

|1|

(ii) Find the determinant and the inverse of the matrix A.

Determinant of A:

$|A| = 1(-1\times-1 - 2\times1) - 1(1\times-1 - 2\times2) + 1(1\times1 - (-1\times2))$

$= 1(1 - 2) - 1(-1 - 4) + 1(1 + 2)$

$= (-1) - (-5) + 3 = 7$

Now compute cofactors and adjoint (done via matrix method).

Then $A^{-1} = adj(A) / det(A)$

(iii) Determine the values of x, y and z.

Using inverse:

$X = A^{-1} \times B$

Or use Cramer's Rule:

Compute determinants for $D_x$, $D_\gamma$, $D_z$

Solve:

$x = D_x / D$

$y = D_\gamma / D$

$z = D_z / D$

Given system:

$x + y + z = 7$

$x - y + 2z = 9$

$2x + y - z = 1$

The coefficient matrix A is:

| 1   1   1 |

| 1  −1   2 |

| 2   1  −1 |

Determinant D =

= 1(−1×−1 − 2×1) − 1(1×−1 − 2×2) + 1(1×1 − (−1×2))

= 1(1 − 2) − 1(−1 − 4) + 1(1 + 2)

= −1 + 5 + 3 = 7

Now compute $D_x$ by replacing the first column with constants (7, 9, 1):

| 7   1   1 |

| 9  −1   2 |

| 1   1  −1 |

$D_x$ = 7(−1×−1 − 2×1) − 1(9×−1 − 2×1) + 1(9×1 − (−1×1))

= 7(1 − 2) − 1(−9 − 2) + 1(9 + 1)

= 7(−1) − (−11) + 10 = −7 + 11 + 10 = 14

Now compute $D_\gamma$ by replacing second column with constants:

| 1   7   1 |

| 1   9   2 |

| 2   1  −1 |

$D_\gamma$ = 1(9×−1 − 2×1) − 7(1×−1 − 2×2) + 1(1×1 − 9×2)

= 1(−9 − 2) − 7(−1 − 4) + (1 − 18)

= −11 + 35 − 17 = 7

Now compute $D_z$ by replacing third column with constants:

| 1   1   7 |

| 1  −1   9 |

| 2   1   1 |

$D_z = 1(-1\times1 - 9\times1) - 1(1\times1 - 9\times2) + 7(1\times1 - (-1\times2))$

$= 1(-1 - 9) - (1 - 18) + 7(1 + 2)$

$= -10 - (-17) + 21 = -10 + 17 + 21 = 28$

Now find the variables:

$x = D_x / D = 14 / 7 = 2$

$y = D_y / D = 7 / 7 = 1$

$z = D_z / D = 28 / 7 = 4$

So:

x = 2

y = 1

z = 4

10. (a) Define the following terms:

(i) Linear programming

Linear programming is a mathematical method used for determining the best possible outcome (such as maximum profit or minimum cost) in a given mathematical model. It involves a linear objective function to be maximized or minimized subject to a set of linear inequalities or equations called constraints.

(ii) Constraints

Constraints are conditions or restrictions expressed as linear inequalities or equations that define the feasible region within which the solution of a linear programming problem must lie.

10. (b) A trader has 15000, 9000 and 1920 units of ingredients X, Y and Z respectively for production of cakes and loaves. The table is:

| Foodstuffs | X | Y | Z |
|---|---|---|---|
| Bread | 25 | 10 | 30 |
| Cake | 15 | 18 | 30 |

Let x = number of loaves of bread

Let y = number of cakes

The constraints become:

$25x + 15y \leq 15000$ (ingredient X)

$10x + 18y \leq 9000$ (ingredient Y)

$30x + 30y \leq 1920$ (ingredient Z)

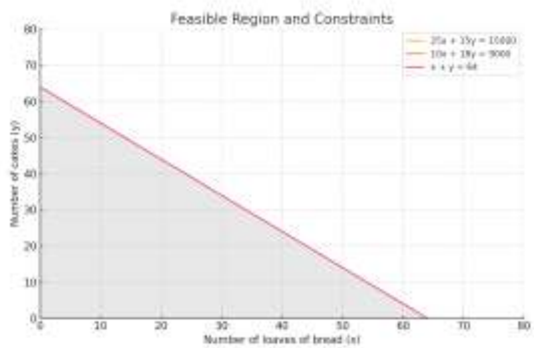
$x \geq 0, y \geq 0$

Simplify Z constraint:

$x + y \leq 64$

Objective function:

Maximize $Z = 4200x + 2000y$




10. (c) Maximize $Z = 4200x + 2000y$

Subject to the constraints:

$25x + 15y \leq 15000$

$10x + 18y \leq 9000$

$x + y \leq 64$

$x \geq 0, y \geq 0$

15,000 units of ingredient X

9,000 units of ingredient Y

1,920 units of ingredient Z

Requirements:

Bread: 25X, 10Y, 30Z per loaf

Cake: 15X, 18Y, 30Z per cake

Selling Prices:

Bread: 420 shillings per loaf

Cake: 2,000 shillings per cake

25b + 15c <= 15,000

Divide by 5:

5b + 3c <= 3,000

Ingredient Y:

10b + 18c <= 9,000

Divide by 2:

5b + 9c <= 4,500

Ingredient Z:

30b + 30c <= 1,920

Divide by 30:

b + c <= 64

Non-negativity (since you can't produce negative quantities):

b >= 0, c >= 0

Step 2: Part (i) – Sketch the Graph to Illustrate the Constraints

To sketch the graph, we plot the constraints as lines on a graph where:

X-axis = b (loaves of bread)

Y-axis = c (cakes)

Determine the Feasible Region:

The feasible region is where all inequalities hold true. Let's test the constraints to see which one is the most restrictive:

Notice that b + c <= 64 limits the total number of items to 64, which is a much smaller bound compared to the other constraints.

Test points on b + c = 64:

At b = 0, c = 64:

5b + 3c = 5(0) + 3(64) = 192 <= 3,000 (True)

5b + 9c = 5(0) + 9(64) = 576 <= 4,500 (True)

At c = 0, b = 64:

5b + 3c = 5(64) + 3(0) = 320 <= 3,000 (True)

5b + 9c = 5(64) + 9(0) = 320 <= 4,500 (True)

At b = 32, c = 32:

5b + 3c = 5(32) + 3(32) = 160 + 96 = 256 <= 3,000 (True)

5b + 9c = 5(32) + 9(32) = 160 + 288 = 448 <= 4,500 (True)

The line b + c = 64 is the most restrictive constraint because the other two constraints (5b + 3c <= 3,000 and 5b + 9c <= 4,500) are satisfied for all points where b + c <= 64. The feasible region is a triangle bounded by:

b = 0

c = 0

b + c = 64

Vertices of the Feasible Region:

(0, 0)

(0, 64)

(64, 0)

Graph Description:

Draw the line b + c = 64 from (0, 64) to (64, 0).

The feasible region is the triangle with vertices (0, 0), (0, 64), and (64, 0), shaded below the line b + c = 64.

Step 3: Part (ii) – Find the Maximum Profit if Both Cakes and Loaves Must Be Prepared

The profit function (in shillings) is:

Profit = 420b + 2,000c

The condition “both cakes and loaves must be prepared” means b > 0 and c > 0. We need to maximize the profit within the feasible region.

Use the Vertex Method:

In linear programming, the maximum of a linear objective function occurs at a vertex of the feasible region. Let’s evaluate the profit at the vertices:

(0, 0):

420(0) + 2,000(0) = 0

(Doesn’t satisfy b > 0, c > 0).

(0, 64):

420(0) + 2,000(64) = 128,000

(Doesn’t satisfy b > 0).

(64, 0):

420(64) + 2,000(0) = 26,880

(Doesn’t satisfy c > 0).

Since the vertices (0, 64) and (64, 0) don’t satisfy the condition of producing both, we need to find the maximum profit along the line b + c = 64 where b > 0 and c > 0.

Optimize Along the Boundary b + c = 64:

Substitute c = 64 - b into the profit function:

Profit = 420b + 2,000(64 - b)

= 420b + 128,000 - 2,000b

= 128,000 - 1,580b

At b = 0, profit = 128,000 (but b = 0).

At b = 64, profit = 128,000 - 1,580(64) = 128,000 - 101,120 = 26,880 (but c = 0).

The profit decreases as b increases. Since b > 0 and c > 0, test points where both are positive:

At b = 1, c = 63:

Profit = 420(1) + 2,000(63) = 420 + 126,000 = 126,420

At b = 63, c = 1:

Profit = 420(63) + 2,000(1) = 26,460 + 2,000 = 28,460

The profit is highest when b is minimized (but greater than 0) and c is maximized. Thus, the maximum profit is 126,420 shillings at b = 1, c = 63.

Verify the Point Satisfies Constraints:

X: 25(1) + 15(63) = 25 + 945 = 970 <= 15,000

Y: 10(1) + 18(63) = 10 + 1,134 = 1,144 <= 9,000

Z: 30(1) + 30(63) = 30 + 1,890 = 1,920 <= 1,920

b + c = 1 + 63 = 64 <= 64

All constraints are satisfied, and b > 0, c > 0.

Step 4: Part (iii) – How Should the Trader Achieve That Maximum Profit?

To achieve the maximum profit of 126,420 shillings, the trader should produce:

1 loaf of bread (b = 1)

63 cakes (c = 63)

Final Answers:

(i) The graph is a triangle with vertices at (0, 0), (0, 64), and (64, 0), bounded by b = 0, c = 0, and the line b + c = 64. The feasible region is shaded below the line b + c = 64.

(ii) The maximum profit, given that both cakes and loaves must be prepared, is 126,420 shillings.

(iii) The trader should produce 1 loaf of bread and 63 cakes to achieve this profit.