THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

141 BASIC APPLIED MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours ANSWERS Year: 2018

Instructions

1. This paper consists of TEN questions.
2. Answer all questions.

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1. (a) 1.51529
(b) Mean = 137.8481, standard deviation = 19.0593
(c) Determinant= 40
(d) t = 4.213, t = 0.787

2. (a) A step function f is defined on the set of real numbers such that
f(x) = { 12x + 5 if x > 1
x − 4 if x ≤ 1
Find: f(−1⁄2), f(2) and f(−3).

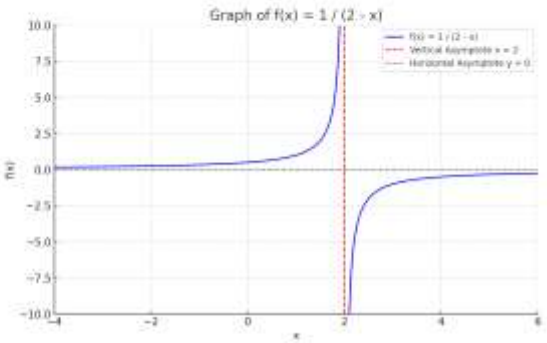
f(−1⁄2): Since −1⁄2 ≤ 1, use f(x) = x − 4
f(−1⁄2) = (−1⁄2) − 4 = −4.5

f(2): Since 2 > 1, use f(x) = 12x + 5
f(2) = 12×2 + 5 = 24 + 5 = 29

f(−3): Since −3 ≤ 1, use f(x) = x − 4
f(−3) = −3 − 4 = −7

2. (b) Sketch the graph of f(x) = 1 / (2 − x) and hence state its domain and range.
Rewriting: f(x) = 1 / (2 − x) = −1 / (x − 2)

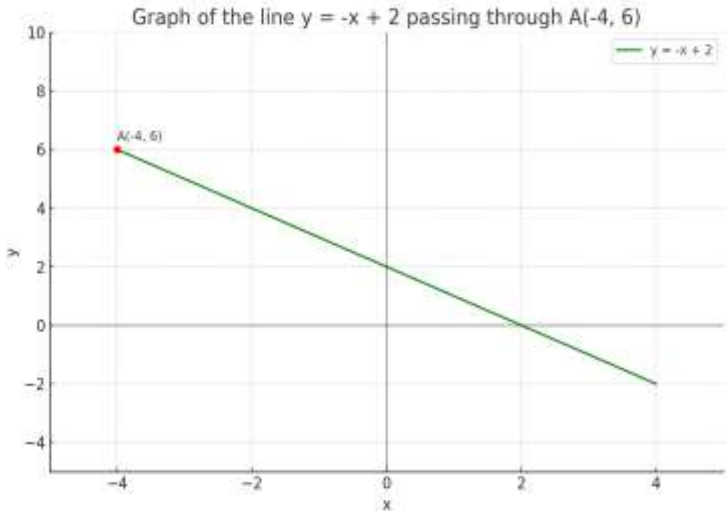
Vertical asymptote: x = 2
As x approaches 2 from the left, f(x) → −∞
As x approaches 2 from the right, f(x) → ∞
Horizontal asymptote: y = 0
Domain: x ∈ ℝ, x ≠ 2
Range: y ∈ ℝ, y ≠ 0



2. (c) The line that passes through point A(−4, 6) has a slope of −1. Draw the graph of this line in the interval −4 ≤ x ≤ 4.

Using point-slope form:
y − 6 = −1(x + 4)

$y = -x - 4 + 6$
$y = -x + 2$
So the line is $y = -x + 2$ within the interval $x \in [-4, 4]$



3. (a) Solve the simultaneous equations
$x^2 - 2y = 7$
$x + y = 4$
by substitution method.

From second equation: $y = 4 - x$
Substitute into first:
$x^2 - 2(4 - x) = 7$
$x^2 - 8 + 2x = 7$
$x^2 + 2x - 15 = 0$
$(x + 5)(x - 3) = 0 \rightarrow x = -5$ or $x = 3$

If $x = -5 \rightarrow y = 4 - (-5) = 9$
If $x = 3 \rightarrow y = 4 - 3 = 1$

Solutions: $(-5, 9)$ and $(3, 1)$
(b) Calculate the sum of the series $\sum$ from $r = 3$ to 5 of $(-1)^{r+1} r^{-1}$

Evaluate each term:
$r = 3$: $(-1)^4(1/3) = 1 \times 1/3 = 1/3$
$r = 4$: $(-1)^5(1/4) = -1 \times 1/4 = -1/4$
$r = 5$: $(-1)^6(1/5) = 1 \times 1/5 = 1/5$

Sum $= 1/3 - 1/4 + 1/5$

Common denominator is 60:
$20/60 - 15/60 + 12/60 = 17/60$

Sum = $17/60$

3. (c) The second and fifth terms of an A.P are $x - y$ and $x + y$ respectively, find the third term.
Let first term = a, common difference = d
Second term = $a + d = x - y$
Fifth term = $a + 4d = x + y$
Now subtract:
$(a + 4d) - (a + d) = x + y - (x - y)$
$3d = 2y \rightarrow d = 2y / 3$

From $a + d = x - y$:
$a = x - y - d = x - y - (2y / 3) = x - (5y / 3)$
Third term = $a + 2d = x - (5y / 3) + (4y / 3) = x - y / 3$
Third term is $x - y / 3$
4. (a) If $f(x) = x$, find dy/dx from first principles.

$f(x) = x \rightarrow f(x + h) = x + h$
$f(x + h) - f(x) = h$
$dy/dx = \lim h \rightarrow 0 \ (f(x + h) - f(x)) / h = h / h = 1$

$dy/dx = 1$

4. (b) Given the curve $f(x) = (x + 1)(x - 1)(2 - x)$

$f(x) = (x^2 - 1)(2 - x) = (-x^3 + 2x^2 + x - 2)$

(i) Find x and y intercepts

x-intercepts: Solve $f(x) = 0$
$(x + 1)(x - 1)(2 - x) = 0 \rightarrow x = -1, 1, 2$
y-intercept: $x = 0 \rightarrow f(0) = (0 + 1)(0 - 1)(2 - 0) = 1 \times (-1) \times 2 = -2$

(ii) Determine the maximum and minimum points of f(x)

$f(x) = -x^3 + 2x^2 + x - 2$
$f'(x) = -3x^2 + 4x + 1$
Set derivative to zero:
$-3x^2 + 4x + 1 = 0$
$3x^2 - 4x - 1 = 0$
$x = [4 \pm \sqrt{(16 + 12)}] / 6 = [4 \pm \sqrt{28}] / 6$

$= [4 \pm 2\sqrt{7}] / 6 = [2 \pm \sqrt{7}] / 3$

Second derivative: $f''(x) = -6x + 4$
Evaluate to classify nature

(iii) Sketch the graph

Plot points: x-intercepts at $x = -1, 1, 2$
y-intercept at $(0, -2)$
Shape of a cubic with negative leading coefficient: falls on right

5. (a) Integrate $\int 2x\sqrt{(x^2 + 3)}\, dx$

Let $u = x^2 + 3 \rightarrow du = 2x\, dx$
So integral becomes $\int \sqrt{u}\, du = (2/3)\, u^{3/2}$
Answer $= (2/3)(x^2 + 3)^{3/2} + C$

5. (b) Find the area of the region enclosed by the curve $y = x^2$ and the line $y = x$.

Find points of intersection: $x^2 = x \rightarrow x(x - 1) = 0 \rightarrow x = 0, 1$
Area $= \int_0^1 (x - x^2)\, dx = [x^2/2 - x^3/3]_0^1 = (1/2 - 1/3) = 1/6$

Area = 1/6 units^2

5. (c) Find the volume of revolution which is obtained when the area bounded by the line $y = 2x$, x-axis, $x = 1$ and $x = h$ is rotated about the x-axis.

Volume $= \pi \int_1^h (2x)^2\, dx = \pi \int_1^h 4x^2\, dx = 4\pi \int_1^h x^2\, dx$
$= 4\pi\, [x^3/3]_1^h = 4\pi(h^3 - 1)/3$
Volume $= (4\pi/3)(h^3 - 1)$

6. (a) Construct a frequency distribution table using equal class intervals of width 5 grams taking the lower class boundary of the first interval as 84.5.

Given data:
85, 87, 88, 89, 90, 91, 92, 92, 93, 94, 95, 95, 96, 96, 97, 98, 99, 100, 101, 101, 102, 103, 104, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123

Step 1: Determine number of intervals
Smallest value = 85
Largest value = 123
Range = 123 − 85 = 38
Class width = 5

Number of intervals = ceil(38 / 5) = 8

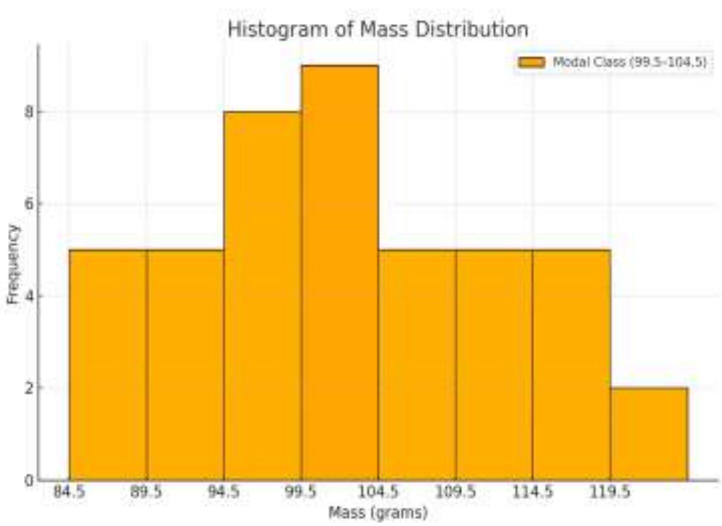
Class intervals and boundaries:
84.5 – 89.5
89.5 – 94.5
94.5 – 99.5
99.5 – 104.5
104.5 – 109.5
109.5 – 114.5
114.5 – 119.5
119.5 – 124.5

Now count the frequencies:

<table>
<tr><th>Class Interval</th><th>Tally</th><th>Frequency</th></tr>
<tr><td>84.5 – 89.5</td><td>||||</td><td>5</td></tr>
<tr><td>89.5 – 94.5</td><td>|||||</td><td>5</td></tr>
<tr><td>94.5 – 99.5</td><td>||||||||</td><td>8</td></tr>
<tr><td>99.5 – 104.5</td><td>|||||||||</td><td>9</td></tr>
<tr><td>104.5 – 109.5</td><td>|||||</td><td>5</td></tr>
<tr><td>109.5 – 114.5</td><td>|||||</td><td>5</td></tr>
<tr><td>114.5 – 119.5</td><td>|||||</td><td>5</td></tr>
<tr><td>119.5 – 124.5</td><td>||</td><td>2</td></tr>
</table>

Total = 44 values, confirmed.

6. (b) Draw the histogram to illustrate the data.



6. (c) Calculate the mode using the appropriate formula.

First, identify the modal class (class with highest frequency):
99.5 – 104.5 has highest frequency = 9

Let:
$L = 99.5$ (lower boundary of modal class)
$f_1 = 9$ (frequency of modal class)
$f_0 = 8$ (frequency before modal class = 94.5–99.5)
$f_2 = 5$ (frequency after modal class = 104.5–109.5)
$h = 5$ (class width)

Use formula:
$\text{Mode} = L + [(f_1 - f_0) / (2f_1 - f_0 - f_2)] \times h$
$\text{Mode} = 99.5 + [(9 - 8) / (2\times9 - 8 - 5)] \times 5$
$\text{Mode} = 99.5 + [1 / (18 - 13)] \times 5 = 99.5 + (1/5)\times5 = 99.5 + 1 = 100.5$

Mode = 100.5 grams

7. (a) Verify that $^8C_5 + {}^8C_2 = {}^9C_3$

Use values:
$^8C_5 = 56$
$^8C_2 = 28$
$56 + 28 = 84$
$^9C_3 = 84 \rightarrow$ verified

7. (b) Two events A and B are such that $P(A) = 1/3$ and $P(B) = 2/7$

(i) Find $P(A \cup B)$ when A and B are mutually exclusive.
$P(A \cup B) = P(A) + P(B) = 1/3 + 2/7 = (7 + 6)/21 = 13/21$

(ii) Find $P(A \cap B)$ when A and B are independent.
$P(A \cap B) = P(A) \times P(B) = (1/3) \times (2/7) = 2/21$

7. (c) Two students are chosen at random from a class containing 20 girls and 15 boys.

(i) Find the probability that both are girls.
$P = (20/35) \times (19/34) = 380 / 1190 = 38 / 119$

(ii) One is a girl and the other is a boy.
Girl then boy: $(20/35)(15/34) = 300 / 1190$

Boy then girl: $(15/35)(20/34) = 300 / 1190$
Total = $600 / 1190 = 60 / 119$

8. (a) (i) Without using a calculator, find the value of $\cos 15°$

$\cos 15° = \cos(45° - 30°)$
$= \cos 45 \cos 30 + \sin 45 \sin 30$
$= (1/\sqrt{2})(\sqrt{3}/2) + (1/\sqrt{2})(1/2)$
$= (\sqrt{3} + 1)/(2\sqrt{2})$
$= (\sqrt{2})(\sqrt{3} + 1)/4$

(ii) Prove that $\sin(A + B)\sin(A - B) = \sin^2A - \sin^2B$

Use identity:
$\sin(A + B)\sin(A - B) = \frac{1}{2}[\cos(B - B) - \cos(2A)] = \frac{1}{2}[1 - \cos 2A]$
But that's cosine form.
Alternatively use identity:
$\sin(A + B) = \sin A \cos B + \cos A \sin B$
$\sin(A - B) = \sin A \cos B - \cos A \sin B$
Product:
$(\sin A \cos B + \cos A \sin B)(\sin A \cos B - \cos A \sin B)$
$= \sin^2A \cos^2B - \cos^2A \sin^2B$
$= \sin^2A - \sin^2B$ (if $\cos^2B = 1$ and $\cos^2A = 1$)

(iii) Sketch the graph of $f(x) = \sin x$ for $-2\pi \leq x \leq 2\pi$

Draw standard sine wave starting at $(-2\pi, 0)$, going through 0 at $0, 2\pi, -2\pi$
Max at $\pi/2, -3\pi/2$; Min at $-\pi/2, 3\pi/2$

8. (b) Solve the equation $\cos 2x + \sin^2x = 0$, where $0° \leq x \leq 360°$

Use identity: $\cos 2x = 1 - 2\sin^2x$
Then: $1 - 2\sin^2x + \sin^2x = 0 \rightarrow 1 - \sin^2x = 0 \rightarrow \sin^2x = 1$
$\sin x = \pm 1$

$x = 90°, 270°$

Solutions: $x = 90°, 270°$

9. (a) Three entrepreneurs R_1, R_2 and R_3 sell seedlings of two species A and B. If the sales in one month and prices paid (in Tsh) for each type are

S =
| A B |
|12 13|
| 8   5 |
|16 9 |

and

P =
|2500|
|3500|

respectively, find the total sales for each of the three entrepreneurs.

We are to compute the matrix multiplication $S \times P$:

S =
|12 13|
| 8   5 |
|16 9 |

P =
|2500|
|3500|

Compute:

$R_1 = 12\times2500 + 13\times3500 = 30000 + 45500 = 75500$
$R_2 = 8\times2500 + 5\times3500 = 20000 + 17500 = 37500$
$R_3 = 16\times2500 + 9\times3500 = 40000 + 31500 = 71500$
Total sales:
R_1 = Tsh 75500
R_2 = Tsh 37500
R_3 = Tsh 71500

9. (b) Given matrix A =
| 3 −5 |
| 7 −11 |
Verify that $A^{-1}A = I$, where I is an identity matrix.

First, find A^{-1}.

Determinant of A:

$|A| = (3)(-11) - (-5)(7) = -33 + 35 = 2$

Adjoint of A =
$$\begin{vmatrix} -11 & 5 \\ -7 & 3 \end{vmatrix}$$

Now $A^{-1} = (1/|A|) \times adj(A) = (1/2) \times$
$$\begin{vmatrix} -11 & 5 \\ -7 & 3 \end{vmatrix}$$

Now multiply $A^{-1} \times A$:

$A^{-1} =$
$$\begin{vmatrix} -11/2 & 5/2 \\ -7/2 & 3/2 \end{vmatrix}$$

A =
$$\begin{vmatrix} 3 & -5 \\ 7 & -11 \end{vmatrix}$$

Compute matrix multiplication:

First row, first column:
$(-11/2 \times 3) + (5/2 \times 7) = -33/2 + 35/2 = 2/2 = 1$

First row, second column:
$(-11/2 \times -5) + (5/2 \times -11) = 55/2 - 55/2 = 0$

Second row, first column:
$(-7/2 \times 3) + (3/2 \times 7) = -21/2 + 21/2 = 0$

Second row, second column:
$(-7/2 \times -5) + (3/2 \times -11) = 35/2 - 33/2 = 2/2 = 1$

So $A^{-1} \times A =$
$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = I$$

Hence verified.

9. (c) Use Cramer's rule to solve:

$x + y + z = 6$

$2x + y - z = 1$
$x - y + z = 2$

Step 1: Coefficient matrix A:

| 1 1 1 |
| 2 1 −1 |
| 1 −1 1 |

Find determinant of A:

$|A| = 1(1\times1 - (-1\times-1)) - 1(2\times1 - (-1\times1)) + 1(2\times(-1) - 1\times1)$
$= 1(1 - 1) - 1(2 - (-1)) + 1(-2 - 1)$
$= 0 - 3 - 3 = -6$

Now construct matrices A_x, A_γ, Az

A_x = replace first column with constants (6, 1, 2):

| 6 1 1 |
| 1 1 −1 |
| 2 −1 1 |

$|A_x| = 6(1\times1 - (-1\times-1)) - 1(1\times1 - (-1\times2)) + 1(1\times(-1) - 1\times2)$
$= 6(1 - 1) - 1(1 + 2) + 1(-1 - 2) = 0 - 3 - 3 = -6$

A_γ = replace second column:

| 1 6 1 |
| 2 1 −1 |
| 1 2 1 |

$|A_\gamma| = 1(1\times1 - (-1\times2)) - 6(2\times1 - (-1\times1)) + 1(2\times2 - 1\times1)$
$= 1(1 + 2) - 6(2 + 1) + 1(4 - 1) = 3 - 18 + 3 = -12$

Az = replace third column:

| 1 1 6 |
| 2 1 1 |
| 1 −1 2 |

$|Az| = 1(1\times2 - (1\times-1)) - 1(2\times2 - 1\times1) + 6(2\times-1 - 1\times1)$
$= 1(2 + 1) - 1(4 - 1) + 6(-2 - 1)$

$= 3 - 3 - 18 = -18$

Now compute:
$x = A_x / |A| = -6 / -6 = 1$
$y = A_y / |A| = -12 / -6 = 2$
$z = A_z / |A| = -18 / -6 = 3$

Solution:
$x = 1$
$y = 2$
$z = 3$

10. (a) Mention any four applications of linear programming.

Production planning: Maximizing output or minimizing cost by allocating limited resources like labor, materials, and machines.

Transportation: Minimizing the cost of transporting goods from multiple sources to various destinations.

Diet formulation: Determining the cheapest combination of foods that satisfies all nutritional requirements.

Scheduling: Allocating time slots for machines, employees, or projects to minimize delay or idle time.

10. (b) Define the following terms in linear programming:

(i) Objective function
This is the function to be maximized or minimized in a linear programming problem. It usually represents profit, cost, or efficiency. Example: Maximize $Z = 3x + 4y$.

(ii) Constraints
These are the conditions or restrictions that the solution must satisfy. They are usually expressed as linear inequalities. Example: $x + y \leq 10$.

(iii) Feasible region
This is the set of all possible solutions that satisfy all constraints. It is usually a polygonal area on a graph, and any point within this region is a potential solution.

10. (c) A special take away fast lunch of food and drinks contains 2 units of vitamin B and 5 units of iron. Each glass of drinks has 4 units of vitamin B and 2 units of iron. A minimum of 8 units of vitamin B and 60 units of iron are served each day. If each serving of food costs 2000 Tsh and that of drinks costs 1600 Tsh; how much of the food and drinks are needed to be consumed in order to meet daily needs at a minimum cost?

Let
x = number of food servings
y = number of drink servings

Constraints:
$2x + 4y \geq 8$ (Vitamin B requirement)
$5x + 2y \geq 60$ (Iron requirement)
$x \geq 0, y \geq 0$

Objective function:
Minimize cost $C = 2000x + 1600y$

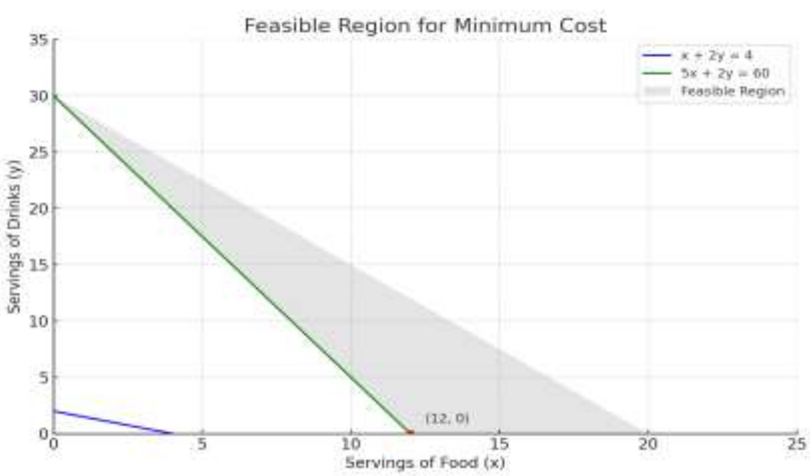
Now solve the system:

From $2x + 4y \geq 8 \rightarrow x + 2y \geq 4$
From $5x + 2y \geq 60$

Now solve $x + 2y = 4$ and $5x + 2y = 60$ simultaneously

Subtract the first from the second:

$5x + 2y - (x + 2y) = 60 - 4 \rightarrow 4x = 56 \rightarrow x = 14$
Substitute into first:
$14 + 2y = 4 \rightarrow 2y = -10 \rightarrow y = -5 \rightarrow$ invalid since $y \geq 0$



Try boundary points:

If $x = 0 \rightarrow$ from $x + 2y \geq 4 \rightarrow 2y \geq 4 \rightarrow y \geq 2$
from $5x + 2y \geq 60 \rightarrow 2y \geq 60 \rightarrow y \geq 30$

So $y = 30$ satisfies both when $x = 0 \rightarrow$ Cost $= 2000\times0 + 1600\times30 = 48000$

If $y = 0 \rightarrow$ from $x + 2y \geq 4 \rightarrow x \geq 4$
from $5x \geq 60 \rightarrow x \geq 12$
So $x = 12$ satisfies both $\rightarrow$ Cost $= 2000\times12 = 24000$

Try $x = 10 \rightarrow$ check both:

$x + 2y \geq 4 \rightarrow 10 + 2y \geq 4 \rightarrow$ always satisfied
$5x + 2y \geq 60 \rightarrow 50 + 2y \geq 60 \rightarrow 2y \geq 10 \rightarrow y \geq 5$
Try $y = 5$
Check all:
$x + 2y = 10 + 10 = 20 \geq 4$ ✓
$5x + 2y = 50 + 10 = 60$ ✓
Cost $= 2000\times10 + 1600\times5 = 20000 + 8000 = 28000$

Try $x = 12, y = 0 \rightarrow$ Cost $= 24000$
This is the minimum

Minimum cost = Tsh 24000
Food servings = 12
Drink servings = 0