**THE UNITED REPUBLIC OF TANZANIA**

**NATIONAL EXAMINATIONS COUNCIL**

**ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION**

**141 BASIC APPLIED MATHEMATICS**

(For Both School and Private Candidates)

**Time: 3 Hours** **ANSWERS** **Year: 2019**

**Instructions**

1. This paper consists of TEN questions.
2. Answer all questions.

1. Use a non-programmable calculator to:

(a) Compute the value of

$\sin^{-1}(2/3) / [7.4(\ln\sqrt{87}) + 2817 \log 6289]$

correct to 4 decimal places.

$\sin^{-1}(2/3) \approx 0.7297$

$\ln\sqrt{87} = \ln(87^{1/2}) = ½ \ln 87 \approx 2.2287$

So $7.4 \times 2.2287 \approx 16.4924$

$\log 6289 \approx 3.7985$

$2817 \times 3.7985 \approx 10691.8845$

Denominator: $16.4924 + 10691.8845 \approx 10708.3769$

So full expression = $0.7297 / 10708.3769 \approx 0.0000681$

(b) Evaluate $\int_0^1 (3x - 2^5)\, dx$

Note: $2^5 = 32$

So expression = $\int_0^1 (3x - 32)\, dx$

$= [3x^2/2 - 32x]$ from 0 to 1

$= (3/2 \times 1^2 - 32\times1) - 0 = 1.5 - 32 = -30.5$

(c) Solve the equation $x^2 + 6x - 8 = 0$ correct to 3 decimal places.

Use quadratic formula:

$x = [-6 \pm \sqrt{(36 + 32)}] / 2 = [-6 \pm \sqrt{68}]/2$

$\sqrt{68} \approx 8.2462$

$x_1 = (-6 + 8.2462)/2 = 2.2462/2 \approx 1.123$

$x_2 = (-6 - 8.2462)/2 = -14.2462/2 \approx -7.123$

Roots: $x \approx 1.123$ and $x \approx -7.123$

(d) Find $2h(4) + f(4.5)$ correct to 4 decimal places, given that

$h(x) = \sqrt{[(x + 4 + (3 + e^x)) / (x + \sqrt{x})]}$

and

$f(x) = \sqrt{[((x - 3)^{1/3} + (x + 1)^6) / (1 + x)]}$

First compute h(4):

$e^4 \approx 54.5981$

Numerator: $4 + 4 + (3 + 54.5981) = 8 + 57.5981 = 65.5981$

Denominator: $4 + \sqrt{4} = 4 + 2 = 6$

$h(4) = \sqrt{(65.5981 / 6)} \approx \sqrt{10.933} = 3.305$

$2h(4) = 2 \times 3.305 = 6.610$

Now f(4.5):

$x - 3 = 1.5 \rightarrow (1.5)^{1/3} \approx 1.145$

$x + 1 = 5.5 \rightarrow (5.5)^6 \approx 25000$

Numerator ≈ 1.145 + 25000 = 25001.145

Denominator = 1 + 4.5 = 5.5

$f(4.5) = \sqrt{(25001.145 / 5.5)} = \sqrt{4545.66} \approx 67.39$

Total = 6.610 + 67.39 = 74.00

2. (a) The function is defined by

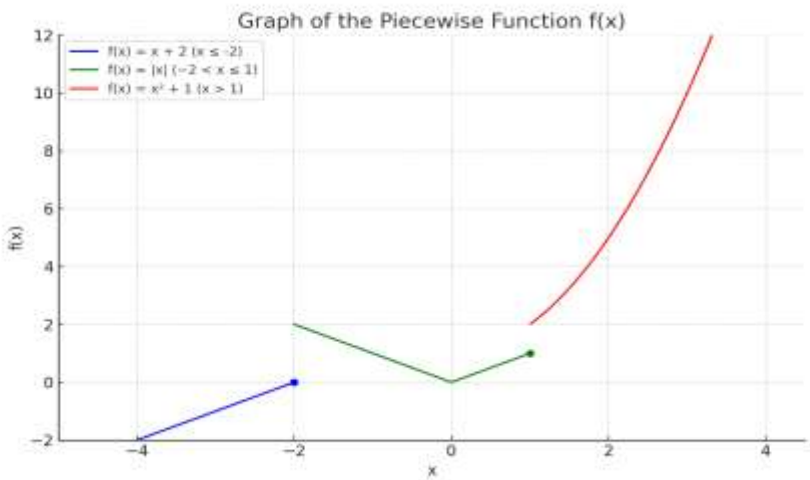
$f(x) =$

| | |
|---|---|
| $x^2 + 1$ | for $x > 1$ |
| $\lvert x \rvert$ | for $-2 < x \leq 1$ |
| $x + 2$ | for $x \leq -2$ |

(i) Sketch the graph of f(x)

Break into three parts:

- $x > 1$: $f(x) = x^2 + 1 \rightarrow$ starts at open point (1, 2), rises upward as parabola
- $-2 < x \leq 1$: $f(x) = \lvert x \rvert \rightarrow$ V-shaped graph, turning at (0, 0), defined up to (1, 1)
- $x \leq -2$: $f(x) = x + 2 \rightarrow$ line with slope 1, passes through (−2, 0)




Key points:

- (−3, −1), (−2, 0) closed, from $f(x) = x + 2$
- (−1, 1), (0, 0), (1, 1) from $f(x) = \lvert x \rvert$ with (1, 1) closed
- Open circle at $x = 1$ for $x^2 + 1$: (1, 2) open
- e.g., (2, 5), (3, 10) for $f(x) = x^2 + 1$

(ii) Determine the domain and range of f(x)

Domain: $x \in \mathbb{R}$ (all x are covered by the three pieces)

Range:
$x + 2$ for $x \leq -2 \rightarrow$ values $\leq 0$
$|x|$ for $-2 < x \leq 1 \rightarrow$ values from 0 to 1
$x^2 + 1$ for $x > 1 \rightarrow$ values $> 2$

So:
Range = $(-\infty, 0] \cup [0, 1] \cup (2, \infty)$

(iii) Find the value of f(−3), f(0.5) and f(2)

$f(-3) = (-3) + 2 = -1$
$f(0.5) = |0.5| = 0.5$
$f(2) = 2^2 + 1 = 5$

(b) If $f(x) = x + 1/x$, show that $[f(x)]^3 = f(x^3) + 3f(x)$

LHS: $[f(x)]^3 = (x + 1/x)^3$
Use identity: $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
$= x^3 + 1/x^3 + 3x(1/x)(x + 1/x)$
$= x^3 + 1/x^3 + 3(x + 1/x)$
RHS: $f(x^3) + 3f(x) = (x^3 + 1/x^3) + 3(x + 1/x)$

So LHS = RHS → proven
3. (a) The difference of two numbers is 1 and the difference of their squares is 7. Find the two numbers.

Let the two numbers be x and y such that:
$x - y = 1$ …(1)
$x^2 - y^2 = 7$ …(2)

From identity: $x^2 - y^2 = (x - y)(x + y)$
Substitute (1) into (2):
$1(x + y) = 7 \rightarrow x + y = 7$ …(3)

Now solve equations (1) and (3) simultaneously:
$x - y = 1$
$x + y = 7$

Add the equations:
$2x = 8 \rightarrow x = 4$
Then from (1): $4 - y = 1 \rightarrow y = 3$

The two numbers are 4 and 3.

3. (b) If the first term of a geometric progression exceeds the second term by 4 and the sum of the second and third terms is $2\frac{2}{3}$, find the first three terms of the progression.

Let the first term be a and the common ratio be r.
Second term = $ar$
Third term = $ar^2$

Given:
$a - ar = 4 \rightarrow a(1 - r) = 4$ …(1)
$ar + ar^2 = 8/3$ …(2)

Factor (2):
$ar(1 + r) = 8/3$

From (1): $a = 4 / (1 - r)$
Substitute into (2):
$(4 / (1 - r)) \times r(1 + r) = 8/3$
$4r(1 + r) = 8(1 - r)/3$
Multiply both sides by 3:
$12r(1 + r) = 8(1 - r)$
$12r + 12r^2 = 8 - 8r$
$12r^2 + 20r - 8 = 0$
Divide by 4: $3r^2 + 5r - 2 = 0$

Use quadratic formula:
$r = [-5 \pm \sqrt{(25 + 24)}] / 6 = [-5 \pm \sqrt{49}] / 6$
$r = (-5 + 7)/6 = 2/6 = 1/3$
or $r = (-5 - 7)/6 = -12/6 = -2$

Use $r = 1/3$:
$a = 4 / (1 - 1/3) = 4 / (2/3) = 6$
First term = 6
Second term = $6 \times 1/3 = 2$
Third term = $2 \times 1/3 = 2/3$
The first three terms are 6, 2, and 2/3.

4. (a) Differentiate $f(x) = e^{x^2-3x+2}$
Let $u = x^2 - 3x + 2$
Then $f(x) = e^u$
So $df/dx = e^u \times du/dx$
$du/dx = 2x - 3$
Therefore, $df/dx = e^{(x^2 - 3x + 2)} \times (2x - 3)$

4. (b) Use implicit differentiation to find dy/dx from
$x^2 + y^2 - 6xy + 3x - 2y + 5 = 0$
Differentiate both sides with respect to x:
$d/dx[x^2] + d/dx[y^2] - d/dx[6xy] + d/dx[3x] - d/dx[2y] + d/dx[5] = 0$
$2x + 2y(dy/dx) - [6(dy/dx \cdot x + y)] + 3 - 2(dy/dx) = 0$
$2x + 2y\, dy/dx - 6x\, dy/dx - 6y + 3 - 2\, dy/dx = 0$

Group dy/dx terms:
$(2y - 6x - 2)\, dy/dx = 6y - 2x - 3$
$dy/dx = (6y - 2x - 3) / (2y - 6x - 2)$

4. (c) Find stationary points of $f(x) = 2x^3 - 3x^2 - 36x + 14$ and determine the nature of each stationary point.

First derivative:
$f'(x) = 6x^2 - 6x - 36$
Set $f'(x) = 0$:
$6x^2 - 6x - 36 = 0 \rightarrow x^2 - x - 6 = 0$
$(x - 3)(x + 2) = 0 \rightarrow x = 3$ and $x = -2$

Second derivative: $f''(x) = 12x - 6$
At $x = 3$: $f''(3) = 36 - 6 = 30 > 0 \rightarrow$ minimum point
At $x = -2$: $f''(-2) = -24 - 6 = -30 < 0 \rightarrow$ maximum point
So:
Stationary point at $x = -2$ is a maximum
Stationary point at $x = 3$ is a minimum

5. (a) Use substitution method:

(i) $\int x\sqrt{(x^2 + 1)}\, dx$
Let $u = x^2 + 1 \rightarrow du = 2x\, dx \rightarrow x\, dx = du/2$
So $\int x\sqrt{(x^2 + 1)}\, dx = \int \sqrt{u} \times (du/2) = (1/2) \int u^{1/2}\, du$
$= (1/2) \times (2/3) u^{3/2} = (1/3)(x^2 + 1)^{3/2} + C$

(ii) $\int \tan x\, dx$
$= -\ln|\cos x| + C$

5. (b) Find the area under the curve $y = x^2 - 4x + 3$ between the roots of the equation $x^2 - 4x + 3 = 0$.

Solve $x^2 - 4x + 3 = 0 \rightarrow (x - 1)(x - 3) = 0 \rightarrow x = 1, 3$
Area $= \int_1^3 (x^2 - 4x + 3)\, dx$
$= [x^3/3 - 2x^2 + 3x]_1^3$
$= [(27/3) - 18 + 9] - [(1/3) - 2 + 3]$

$= (9 - 18 + 9) - (1/3 - 2 + 3)$
$= 0 - (1/3 + 1) = -4/3$
Area = 4/3 units²

6. (a) A teacher recorded the height in cm of 22 students as:
155, 160, 163, 165, 166, 168, 168, 169, 170, 172, 172, 172, 173, 174, 174, 175, 175, 176, 176, 176, 178, 180

Form a grouped frequency distribution table with class intervals of 5 starting from 155.

| Class Interval | Frequency |
|---|---|
| 155–159 | 2 |
| 160–164 | 2 |
| 165–169 | 5 |
| 170–174 | 6 |
| 175–179 | 6 |
| 180–184 | 1 |

6. (b) Using the assumed mean method, calculate the mean height.

Class midpoints: 157, 162, 167, 172, 177, 182
Assumed mean A = 172, class width h = 5
Compute $u = (x - A)/h$

| Midpoint (x) | f | u | fu |
|---|---|---|---|
| 157 | 2 | −3 | −6 |
| 162 | 2 | −2 | −4 |
| 167 | 5 | −1 | −5 |
| 172 | 6 | 0 | 0 |
| 177 | 6 | 1 | 6 |
| 182 | 1 | 2 | 2 |

$\Sigma fu = -7$, $\Sigma f = 22$
Mean $= A + (\Sigma fu/\Sigma f) \times h = 172 + (-7/22) \times 5 = 172 - 35/22 \approx 170.41$ cm

6. (c) Estimate the interquartile range.

Cumulative frequency:
$Q_1 = \frac{1}{4}(22) = 5.5^{th}$ value → lies in 165–169
L = 164.5, F = 4, f = 5, h = 5
$Q_1 = 164.5 + [(5.5 - 4)/5] \times 5 = 164.5 + 1.5 = 166$

$Q_3 = \frac{3}{4}(22) = 16.5^{th}$ value → lies in 175–179
L = 174.5, F = 15, f = 6
$Q_3 = 174.5 + [(16.5 - 15)/6] \times 5 = 174.5 + (1.5/6)\times5 = 174.5 + 1.25 = 175.75$

$IQR = Q_3 - Q_1 = 175.75 - 166 = 9.75$ cm

7. (a) Show that $^nC_r = {}^nC_{n-r}$

By definition:
$^nC_r = n! / (r!(n - r)!)$
$^nC_{n-r} = n! / ((n - r)!r!)$ = same
Therefore, $^nC_r = {}^nC_{n-r}$

7. (b) Given P(A) = 0.3, P(B) = 0.4 and P(A ∩ B) = 0.1, find P(A ∪ B)

P(A ∪ B) = P(A) + P(B) − P(A ∩ B)
= 0.3 + 0.4 − 0.1 = 0.6

7. (c) A bag contains 3 red and 5 blue marbles. Two are drawn with replacement. Find the probability that:

(i) Both are red
= 3/8 × 3/8 = 9/64

(ii) One is red and one is blue
= (3/8 × 5/8) + (5/8 × 3/8) = 30/64

(iii) Both are blue
= 5/8 × 5/8 = 25/64

8. (a) Given triangle XYZ, XY = 3.5 cm, YZ = 4.5 cm, ZX = 6.5 cm. Calculate the angle Y.

Use cosine rule:
$\cos Y = (XY^2 + YZ^2 - ZX^2)/(2\cdot XY\cdot YZ)$
$= (3.5^2 + 4.5^2 - 6.5^2) / (2\cdot3.5\cdot4.5)$
$= (12.25 + 20.25 - 42.25)/(31.5) = -9.75 / 31.5 = -0.3095$
$Y = \cos^{-1}(-0.3095) \approx 108.1°$

8. (b) Solve the equation $1 + \cos\theta = 2\sin^2\theta$ for $\theta \in [0, 2\pi]$

Use identity $\sin^2\theta = 1 - \cos^2\theta$
$1 + \cos\theta = 2(1 - \cos^2\theta)$

$1 + \cos\theta = 2 - 2\cos^2\theta$
$2\cos^2\theta + \cos\theta - 1 = 0$
Solve: $\cos\theta = \frac{1}{2}$ or $-1$
$\cos\theta = \frac{1}{2} \rightarrow \theta = \pi/3, 5\pi/3$
$\cos\theta = -1 \rightarrow \theta = \pi$
Solutions: $\pi/3, 5\pi/3, \pi$

9. Given matrix A =
| 1  1  1 |
| 1  2  1 |
| 2  1 −1 |

(i) Find |A|

Use cofactor expansion:

$|A| = 1(2\cdot(-1) - 1\cdot1) - 1(1\cdot(-1) - 1\cdot2) + 1(1\cdot1 - 2\cdot2)$
$= 1(-2 - 1) - 1(-1 - 2) + 1(1 - 4)$
$= 1(-3) - 1(-3) + 1(-3) = -3 + 3 - 3 = -3$

$|A| = -3$

(ii) Find $A^{-1}$
Given matrix A:

A =
| 1   1   1 |
| 1   2   1 |
| 2   1  −1 |

We are to compute the inverse $A^{-1}$ using the formula:
$A^{-1} = (1/|A|) \times \text{adj}(A)$

First, compute the determinant |A|:

$|A| = 1\cdot(2\cdot(-1) - 1\cdot1) - 1\cdot(1\cdot(-1) - 1\cdot2) + 1\cdot(1\cdot1 - 2\cdot2)$
$= 1\cdot(-2 - 1) - 1\cdot(-1 - 2) + 1\cdot(1 - 4)$
$= 1(-3) - 1(-3) + 1(-3)$
$= -3 + 3 - 3 = -3$

So $|A| = -3$

Now compute the matrix of minors:

Minor of $a_{11}$ = |2 1|
|1 −1| = (2)(−1) − (1)(1) = −2 − 1 = −3
Minor of $a_{12}$ = |1 1|
|2 −1| = (1)(−1) − (2)(1) = −1 − 2 = −3
Minor of $a_{13}$ = |1 2|
|2 1| = (1)(1) − (2)(2) = 1 − 4 = −3

Minor of $a_{21}$ = |1 1|
|1 −1| = (1)(−1) − (1)(1) = −1 − 1 = −2
Minor of $a_{22}$ = |1 1|
|2 −1| = (1)(−1) − (2)(1) = −1 − 2 = −3
Minor of $a_{23}$ = |1 1|
|2 1| = (1)(1) − (2)(1) = 1 − 2 = −1

Minor of $a_{31}$ = |1 1|
|2 1| = (1)(1) − (2)(1) = 1 − 2 = −1
Minor of $a_{32}$ = |1 1|
|1 1| = (1)(1) − (1)(1) = 1 − 1 = 0
Minor of $a_{33}$ = |1 1|
|1 2| = (1)(2) − (1)(1) = 2 − 1 = 1

Now apply the checkerboard signs to get the matrix of cofactors:

Cofactor matrix:
| + − + |
| − + − |
| + − + |

→
| −3 3 −3 |
| 2 −3 1 |
| −1 0 1 |

Now take the transpose of the cofactor matrix to get the adjugate matrix:

Adj(A) =
| −3 2 −1 |
| 3 −3 0 |
| −3 1 1 |

Now compute the inverse:
$A^{-1} = (1/-3) \times \text{adj}(A)$

So:

$$A^{-1} = \begin{vmatrix} 1 & -2/3 & 1/3 \\ -1 & 1 & 0 \\ 1 & -1/3 & -1/3 \end{vmatrix}$$

Final result:

$$A^{-1} = \begin{vmatrix} -1 & 2/3 & -1/3 \\ 1 & -1 & 0 \\ -1 & 1/3 & 1/3 \end{vmatrix}$$

10. (a) A furniture dealer makes chairs and tables. A chair requires one hour on machine I and two hours on machine II. A table requires two hours on machine I and one hour on machine II. Each day, machine I is available for 10 hours and machine II is available for 15 hours. The profit on a chair is Tsh 30 and on a table is Tsh 45. Find how many chairs and how many tables should be made to maximize profit.

Let
$x$ = number of chairs
$y$ = number of tables

From the question:

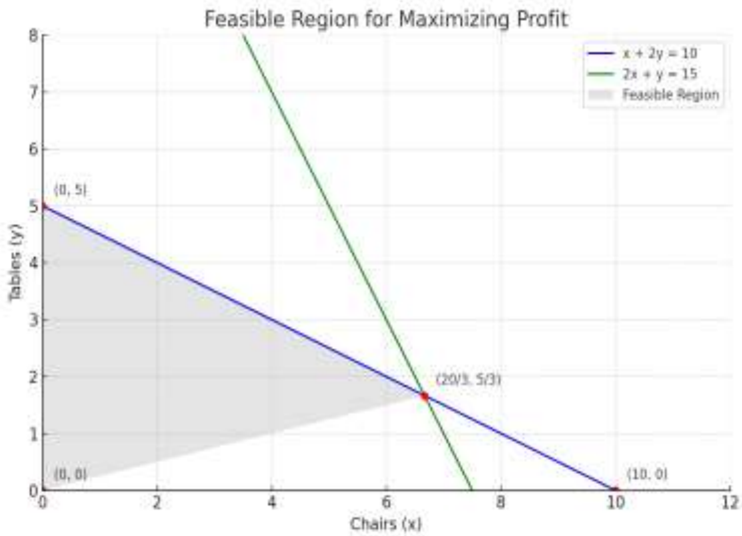
Machine I constraint:
$1x + 2y \leq 10 \rightarrow x + 2y \leq 10$

Machine II constraint:
$2x + 1y \leq 15 \rightarrow 2x + y \leq 15$

Non-negativity:
$x \geq 0, y \geq 0$

Objective function:
Maximize profit $P = 30x + 45y$

10. (b) Draw the feasible region and determine the values of x and y which maximize the profit.




Now solve the system of inequalities graphically.

Step 1: Convert inequalities to equalities for graphing:
1) x + 2y = 10
2) 2x + y = 15

Find intersections:

From (1):
x + 2y = 10 → y = (10 − x)/2
When x = 0 → y = 5
When y = 0 → x = 10
So line (1) passes through (0, 5) and (10, 0)

From (2):
2x + y = 15 → y = 15 − 2x
When x = 0 → y = 15
When y = 0 → x = 7.5
So line (2) passes through (0, 15) and (7.5, 0)

Now find intersection of the two lines:
x + 2y = 10
2x + y = 15

Multiply second equation by 2:
4x + 2y = 30
Now subtract:
(4x + 2y) − (x + 2y) = 30 − 10

$3x = 20 \rightarrow x = 20/3$
Substitute into $x + 2y = 10$:
$20/3 + 2y = 10 \rightarrow 2y = 10 - 20/3 = (30 - 20)/3 = 10/3$
$y = 5/3$

So point of intersection: $(20/3, 5/3)$

Now evaluate profit at corner points:
1) (0, 0): $P = 0$
2) (10, 0): $P = 30 \times 10 = 300$
3) (0, 5): $P = 45 \times 5 = 225$
4) (20/3, 5/3):
$P = 30(20/3) + 45(5/3) = 200 + 75 = 275$

Maximum profit is Tsh 300 at point (10, 0)

Hence, to maximize profit, the dealer should make 10 chairs and 0 tables.