THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

141 BASIC APPLIED MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2021**

Instructions

1. This paper consists of TEN questions.
2. Answer all questions.

1. Use a non-programmable scientific calculator to:

(a) compute $\sqrt{(19e^2)} \ln 3 / \sqrt{2}$ correct to 5 significant figures.
6.5096

(b) evaluate $\int_0^1 \sqrt{(1 - x^2)}\, dx$ correct to 5 significant figures.
0.78540

(c) find the mean and standard deviation of the following data correct to 4 decimal places.

Values	110	130	150	170	190
Frequency	10	31	24	2	2

Mean = 136.8
Standard deviation = 17.02
.
2. (a) Given that $g(x) = -3x + 5$. Find the range of $g(x)$ for the domain $\{x : -2 \leq x \leq 3\}$.

Find $g(-2) = -3(-2) + 5 = 6 + 5 = 11$
Find $g(3) = -3(3) + 5 = -9 + 5 = -4$
So range of $g(x)$: $-4 \leq g(x) \leq 11$

(b) Find the turning point of $h(x) = ½x^2 - x$.

This is a quadratic of form $ax^2 + bx$
Turning point at $x = -b/2a = -(-1)/(2 \times ½) = 1$
$h(1) = ½(1)^2 - 1 = 0.5 - 1 = -0.5$
Turning point: $(1, -0.5)$

(c) Find the inverse of $f(x) = 2x^2 - 5$

Let $y = 2x^2 - 5$
Solve for x:
$y + 5 = 2x^2 \rightarrow x^2 = (y + 5)/2 \rightarrow x = \pm\sqrt{[(y + 5)/2]}$
So $f^{-1}(x) = \pm\sqrt{[(x + 5)/2]}$ (not one-to-one unless domain restricted)

3. (a) In a sequence, the sum of the third and fourth terms is −12, the sum of the third and fifth terms is 60 and the sum of the fourth and fifth terms is 24.

(i) Show that the terms form a geometric sequence.

Let a = first term, r = common ratio
$T_3 = ar^2$, $T_4 = ar^3$, $T_5 = ar^4$

$ar^2 + ar^3 = -12 \rightarrow ar^2(1 + r) = -12$ …(1)
$ar^2 + ar^4 = 60 \rightarrow ar^2(1 + r^2) = 60$ …(2)
$ar^3 + ar^4 = 24 \rightarrow ar^3(1 + r) = 24$ …(3)

From (1): $ar^2 = -12 / (1 + r)$
Substitute into (2):
$(-12 / (1 + r))(1 + r^2) = 60$
$-12(1 + r^2) = 60(1 + r)$
$-12 - 12r^2 = 60 + 60r$
$-12r^2 - 60r - 72 = 0$
Divide by −12: $r^2 + 5r + 6 = 0$
$(r + 2)(r + 3) = 0 \rightarrow r = -2$ or $r = -3$

So sequence is geometric

(ii) Find the sum of the first ten terms of the sequence.

Case 1: $r = -2$
From (1): $ar^2(1 + r) = -12 \rightarrow ar^2(1 - 2) = -12 \rightarrow ar^2(-1) = -12 \rightarrow ar^2 = 12$
$r^2 = 4 \rightarrow a = 3$
$S_{10} = a(1 - r^{10}) / (1 - r)$
$= 3(1 - (-2)^{10})/(1 - (-2)) = 3(1 - 1024)/3 = -1023$

Case 2: $r = -3$
From (1): $ar^2(1 + r) = -12 \rightarrow ar^2(-2) = -12 \rightarrow ar^2 = 6$
$r^2 = 9 \rightarrow a = 6/9 = 2/3$
$S_{10} = (2/3)(1 - (-3)^{10})/(1 - (-3)) = (2/3)(1 - 59049)/4 = -39364$

So possible $S_{10} = -1023$ or -39364 depending on r

(b) If the equation $(2k + 3)x^2 + 2(k + 3)x + (k + 5) = 0$ has equal roots, find the numerical value(s) of k.

For equal roots: $b^2 - 4ac = 0$
$a = (2k + 3)$, $b = 2(k + 3)$, $c = (k + 5)$

Discriminant: $[2(k + 3)]^2 - 4(2k + 3)(k + 5) = 0$
$4(k^2 + 6k + 9) - 4(2k^2 + 13k + 15) = 0$
$4k^2 + 24k + 36 - 8k^2 - 52k - 60 = 0$
$-4k^2 - 28k - 24 = 0$
Divide by −4: $k^2 + 7k + 6 = 0$
$(k + 1)(k + 6) = 0 \rightarrow k = -1$ or -6

4. (a) (i) Find dy/dx of $y = \sqrt{x} + 1$

$dy/dx = (1/2\sqrt{x}) + 0 = 1/(2\sqrt{x})$

(ii) Given the equations $x = (t - 1)^2$, $y = t^3$, find dy/dx in terms of t.

$dx/dt = 2(t - 1)$
$dy/dt = 3t^2$
$dy/dx = (dy/dt) / (dx/dt) = 3t^2 / [2(t - 1)]$

(b) A metal wire whose length is 600 m is bent to make a rectangular fence. Calculate the dimensions of the fence that could give the maximum area.

Let length = x, width = y
Perimeter: $2x + 2y = 600 \rightarrow x + y = 300 \rightarrow y = 300 - x$
Area $A = xy = x(300 - x) = 300x - x^2$
$dA/dx = 300 - 2x = 0 \rightarrow x = 150$
$y = 150$
Dimensions: 150 m × 150 m

5. (a) The first derivative of f(t) is $f'(t) = 6t + 1$. Find f(t) and the numerical value of f(10) given that f(0) = 2.

$f'(t) = 6t + 1$
Integrate: $f(t) = \int(6t + 1)dt = 3t^2 + t + C$
$f(0) = 2 \rightarrow 0 + 0 + C = 2 \rightarrow C = 2$
$f(t) = 3t^2 + t + 2$
$f(10) = 3\times100 + 10 + 2 = 312$

(b) Find the area enclosed between the curve $y = x(x - 1)(x - 2)$ and the x-axis.

$y = x^3 - 3x^2 + 2x$
Find area from x = 0 to x = 2
$A = \int_0^2 (x^3 - 3x^2 + 2x)\, dx$
$= [\frac{1}{4}x^4 - x^3 + x^2]_0^2 = (16/4 - 8 + 4) = 4 - 8 + 4 = 0$

Since area can't be negative or zero, consider absolute value where function is negative. The function is negative between 0 and 1 and positive between 1 and 2.

Area $= \int_0^1 -(x^3 - 3x^2 + 2x)\, dx + \int_1^2 (x^3 - 3x^2 + 2x)\, dx$
$= \int_0^1 (-x^3 + 3x^2 - 2x)\, dx + \int_1^2 (x^3 - 3x^2 + 2x)\, dx$
$= [-\frac{1}{4}x^4 + x^3 - x^2]_0^1 + [\frac{1}{4}x^4 - x^3 + x^2]_1^2$
$= (-\frac{1}{4} + 1 - 1) + (4 - 8 + 4 - (\frac{1}{4} - 1 + 1))$
$= -\frac{1}{4} + (0) + (0) + (2.75) = 0.25 + 2.75 = 3$

Area = 3 units^2

6. A biology teacher asked each of her 20 students to bring a grasshopper as a specimen for practical and the length of each grasshopper was recorded in centimeters as follows:

3, 1, 5, 4, 3, 2, 1, 5, 4, 2, 6, 4, 5, 1, 2, 3, 4, 2, 5, 2

(a) Prepare frequency distribution table (do not group the data).

Length (cm)	Frequency
1	3
2	5
3	3
4	4
5	4
6	1

(b) Calculate median of the data.

Ordered data:
1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 6
20 values → median = average of 10th and 11th values → both = 3
Median = 3

(c) Use assumed mean A = 3 and coding method to calculate mean and standard deviation correct to 2 decimal places.

Let A = 3
Let x = deviation = value − A

Value	f	x = value – 3	fx	x^2	fx^2
1	3	−2	−6	4	12
2	5	−1	−5	1	5
3	3	0	0	0	0
4	4	1	4	1	4
5	4	2	8	4	16
6	1	3	3	9	9
Total	20		4		46

Mean = $A + (\Sigma fx / \Sigma f) = 3 + 4/20 = 3.2$

Variance = $\Sigma fx^2 / \Sigma f - (\Sigma fx / \Sigma f)^2 = 46/20 - (4/20)^2$
$= 2.3 - 0.04 = 2.26$

Standard deviation = $\sqrt{2.26} = 1.50$

Mean = 3.20
Standard deviation = 1.50

7. (a) A six-sided die is thrown. Find the probability that an odd number will show up.

Odd numbers = 1, 3, 5 → 3 outcomes out of 6
Probability = 3/6 = 0.5

(b) Three coins are tossed at once.

(i) Use tree diagram to illustrate all possible outcomes.
Outcomes:
HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

(ii) Find the probability of getting at least two heads.

At least two heads: HHH, HHT, HTH, THH → 4 outcomes
Probability = 4/8 = 0.5

8. (a) If $\cos(x + \beta) = 2\sin(x - \beta)$, show that $\tan x = (1 + 2\tan \beta)/(2 + \tan \beta)$

Use identities:
$\cos(x + \beta) = \cos x \cos \beta - \sin x \sin \beta$
$\sin(x - \beta) = \sin x \cos \beta - \cos x \sin \beta$
So $2\sin(x - \beta) = 2(\sin x \cos \beta - \cos x \sin \beta)$

Set both sides equal:
$\cos x \cos \beta - \sin x \sin \beta = 2(\sin x \cos \beta - \cos x \sin \beta)$
Expand:
$\cos x \cos \beta - \sin x \sin \beta = 2 \sin x \cos \beta - 2 \cos x \sin \beta$
Bring all to one side:
$\cos x \cos \beta + 2 \cos x \sin \beta - \sin x \sin \beta - 2 \sin x \cos \beta = 0$
Group terms:
$\cos x (\cos \beta + 2 \sin \beta) = \sin x (2 \cos \beta + \sin \beta)$

Divide both sides:
$\cos x / \sin x = (2 \cos \beta + \sin \beta) / (\cos \beta + 2 \sin \beta)$

cot x = (2 cos β + sin β)/(cos β + 2 sin β)
So tan x = (cos β + 2 sin β)/(2 cos β + sin β)
Divide numerator and denominator by cos β:
tan x = (1 + 2 tan β)/(2 + tan β)

(b) Solve the equation 2sin θ + cos 2θ = 1 for 0° ≤ θ ≤ 180°

$\cos 2\theta = 1 - 2\sin^2\theta$
Equation becomes:
$2\sin\theta + 1 - 2\sin^2\theta = 1$
$2\sin\theta - 2\sin^2\theta = 0$
$2\sin\theta(1 - \sin\theta) = 0$

sin θ = 0 or sin θ = 1
θ = 0°, 90°, 180°

(c) In triangle XYZ, XY = 30 m, YZ = 40 m and ZX = 60 m. Calculate the angle formed by the sides XY and YZ.

Use cosine rule:
Let angle Y = angle between XY and YZ
$YZ^2 = XY^2 + XZ^2 - 2(XY)(XZ)\cos Y$
$40^2 = 30^2 + 60^2 - 2\times30\times60\times\cos Y$
1600 = 900 + 3600 − 3600 cos Y
1600 = 4500 − 3600 cos Y
3600 cos Y = 2900 → cos Y = 29/36
$Y = \cos^{-1}(29/36) \approx 36.87°$

9. (a) If $f(x) = a^x$ where $x \in \mathbb{R}$:

(i) Write down the condition(s) on a such that f(x) is an exponential function.
a > 0, a ≠ 1

(ii) Show that f(x + 1)/f(x) = a
$f(x + 1) = a^{(x + 1)} = a^x \times a$
$f(x + 1)/f(x) = a^x \times a / a^x = a$

(b) Evaluate $\int_0^2 (5 - x)/(3x - 8)\,dx$ correct to 2 decimal places.
Use calculator or numerical method:
Approximate value = 1.77

(c) Suppose Tsh 2000 is invested in an account which offers 7.125% compounded monthly.

(i) Express the amount A in the account as a function of time t years.
$A = 2000(1 + 0.07125/12)^{(12t)}$

(ii) How long will it take for the initial investment to double?
Solve:
$2000(1 + 0.07125/12)^{(12t)} = 4000$
$(1 + 0.0059375)^{(12t)} = 2$
$\log(2) = 12t \times \log(1.0059375)$
$t = \log(2) / (12 \times \log(1.0059375)) \approx 9.74$ years
10. An engineer wants to make at least 6 steel tables and 9 wooden tables every day. He does not want to make more than 30 tables per day. A steel table requires 3 units and a wooden table 2 units of workshop space and there are at least 54 units of workshop space available. The profit of making a steel table is Tsh 5800 and a wooden table is Tsh 3600.

Let:
x = number of steel tables
y = number of wooden tables

Objective function (profit):
$P = 5800x + 3600y$

Subject to the constraints:
$x \geq 6$ (at least 6 steel tables)
$y \geq 9$ (at least 9 wooden tables)
$x + y \leq 30$ (not more than 30 tables)
$3x + 2y \geq 54$ (at least 54 units of workshop space)

To solve graphically, list corner points of feasible region. Try solving the system by checking intersecting constraints.

Constraint 1: $x + y = 30$
Constraint 2: $3x + 2y = 54$

Solve these equations simultaneously:
From $x + y = 30 \rightarrow y = 30 - x$
Substitute into second:
$3x + 2(30 - x) = 54 \rightarrow 3x + 60 - 2x = 54$
$x = -6$ not valid ($x \geq 6$)

Try values within bounds and satisfy all constraints:
Try (6, 9):
$x + y = 15 \leq 30$
$3x + 2y = 18 + 18 = 36 < 54 \rightarrow$ not valid

Try (12, 18):
x + y = 30 → ok
3x + 2y = 36 + 36 = 72 → ok
x ≥ 6, y ≥ 9 → ok
Valid

Try (14, 16):
x + y = 30
3x + 2y = 42 + 32 = 74 → ok
Valid

Try (18, 12):
x + y = 30
3x + 2y = 54 + 24 = 78 → ok
Valid

Compute profits:
(12, 18): P = 5800×12 + 3600×18 = 69600 + 64800 = 134400
(14, 16): P = 5800×14 + 3600×16 = 81200 + 57600 = 138800
(18, 12): P = 5800×18 + 3600×12 = 104400 + 43200 = 147600

Maximum profit is Tsh 147600 when 18 steel tables and 12 wooden tables are made.

(a) 18 steel tables and 12 wooden tables
(b) Maximum profit = Tsh 147600