THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

141 BASIC APPLIED MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2022**

Instructions

1. This paper consists of TEN questions.
2. Answer all questions.

Prepared by: Maria Marco for TETEA

1. (a) Use a non-programmable calculator to:

(i) compute $3254 \times 3.14 \times \sqrt{417} \div (10^5 \times \log \sqrt{278})$ (write your answer correct to 2 decimal places).
$3254 \times 3.14 \times \sqrt{417} \approx 3254 \times 3.14 \times 20.42 = 208924.9$
$\log \sqrt{278} = \log(278) / 2 \approx 2.444 / 2 = 1.222$
Denominator $= 10^5 \times 1.222 = 122200$
$208924.9 \div 122200 \approx 1.71$

(ii) evaluate $\tan^{-1} [\ln(4 \times 10^2 / \pi) / \log \sqrt{2}]$ correct to 2 significant figures.
$\ln(400/\pi) \approx \ln(127.32) \approx 4.846$
$\log \sqrt{2} = \log(2)/2 \approx 0.1505$
$\tan^{-1}(4.846 / 0.1505) = \tan^{-1}(32.2) \approx 88.2°$
To 2 significant figures: 88°

(iii) find the value of $\sum$ from n=1 to ∞ of $e^n / (\sqrt{n} + x^2)$ correct to 4 decimal places.
This is undefined or cannot be evaluated unless a value of x is provided. The question is incomplete.

(b) Given that $P(x = r) = nCr \times p^r \times (1 - p)^{n-r}$ where $n = 10$ and $p = 0.45$, find the numerical value of $P(x = 1)$ correct to 4 significant figures.

$P(x = 1) = C(10,1) \times 0.45^1 \times (1 - 0.45)^9 = 10 \times 0.45 \times 0.55^9$
$0.55^9 \approx 0.0024$
$P = 10 \times 0.45 \times 0.0024 \approx 0.0108$

2. (a) (i) Sketch the graph of $f(x) = (2x + 5) / (x^2 - x - 6)$

Factor the denominator:
$x^2 - x - 6 = (x - 3)(x + 2)$
So $f(x) = (2x + 5)/[(x - 3)(x + 2)]$

Vertical asymptotes: $x = 3$ and $x = -2$
Asymptotes occur where the denominator is zero
Horizontal asymptote: Since degrees of numerator and denominator are equal,
divide leading coefficients: $y = 2 / 1 = 2$

Intercepts:
x-intercept: set numerator = 0 → $2x + 5 = 0$ → $x = -2.5$
y-intercept: set x = 0 → $f(0) = 5 / (-6) = -5/6$

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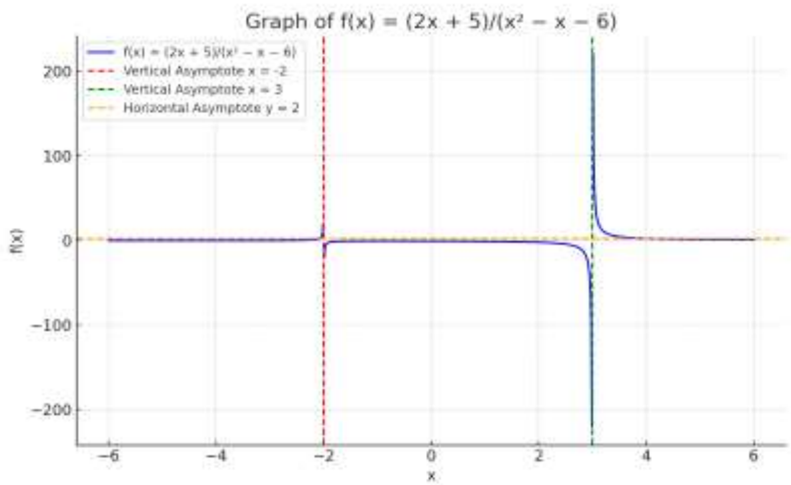


(ii) Write down the value(s) of x for which f(x) does not exist.

f(x) does not exist when the denominator is 0 → $x = -2$, $x = 3$

(b) Find the value of $[f(2) \times f(1)] / f(-1)$ given that

f(x) =

x^2 for $0 < x < 2$

$x + 2$ for $x \geq 2$

x for $-1 \leq x < 0$

$f(2) = x + 2 = 2 + 2 = 4$

$f(1) = x^2 = 1^2 = 1$

$f(-1) = x = -1$

$[f(2) \times f(1)] / f(-1) = (4 \times 1) / (-1) = -4$

3. (a) The total number of pencils and pens in a box is 47 and the product of the number of pencils and the number of pens is 370. Find the number of pencils present in the box.

Let x = pencils, so $47 - x$ = pens

$x(47 - x) = 370$

$47x - x^2 = 370 \rightarrow x^2 - 47x + 370 = 0$

Use quadratic formula:

$x = [47 \pm \sqrt{(2209 - 1480)}] / 2$

$= [47 \pm \sqrt{729}] / 2 = [47 \pm 27] / 2$

$x = (47 + 27)/2 = 74/2 = 37$

$x = (47 - 27)/2 = 20/2 = 10$

Possible values: 10 pencils or 37 pencils

(b) The 8th and 15th terms of an arithmetic progression are 11 and 21 respectively. Find the nth term.

Let a = first term, d = common difference

$T_8 = a + 7d = 11$ …(1)

$T_{15} = a + 14d = 21$ …(2)

Subtract:

$(14d - 7d) = 10 \rightarrow 7d = 10 \rightarrow d = 10 / 7$

Substitute into (1):

$a + 7(10 / 7) = 11 \rightarrow a + 10 = 11 \rightarrow a = 1$

$T_n = a + (n - 1)d = 1 + (n - 1)(10 / 7)$

4. (a) (i) Differentiate $y = 1 / (1 + x)$ from the first principles.

$f'(x) = \lim_{h\to 0} [(1 / (1 + x + h)) - (1 / (1 + x))] / h$

$= \lim_{h\to 0} [(1 + x - (1 + x + h)) / ((1 + x)(1 + x + h)h)]$

$= \lim_{h\to 0} [-h / ((1 + x)(1 + x + h)h)]$

$= \lim_{h\to 0} [-1 / ((1 + x)(1 + x + h))]$

$= -1 / (1 + x)^2$

(ii) Find the first derivative of $g(x) = \sqrt{(x^2 + 2x)}$

Let $g(x) = (x^2 + 2x)^{1/2}$

$g'(x) = (1/2)(x^2 + 2x)^{-1/2} \times (2x + 2)$

$= (2x + 2) / [2\sqrt{(x^2 + 2x)}]$

(b) The total length of the diameter and height of a cylinder is 3 metres. Show that the cylinder has the maximum volume when both height and radius measure 1 metre.

Let radius = r, height = h

Given: $2r + h = 3 \rightarrow h = 3 - 2r$

Volume $= \pi r^2 h = \pi r^2(3 - 2r) = \pi(3r^2 - 2r^3)$

Differentiate:

$dV/dr = \pi(6r - 6r^2)$

Set to 0: $6r - 6r^2 = 0$

$6r(1 - r) = 0 \rightarrow r = 0$ or $r = 1$

$r = 1 \rightarrow h = 3 - 2(1) = 1$

Maximum volume occurs when r = 1, h = 1

5. (a) Evaluate $\int_{0.5}^{1} \sqrt{(1 - x^2)}\, dx$ correct to 4 decimal places.

This is a standard integral related to a semicircle. Use calculator:

$\int_0^{.51} \sqrt{(1 - x^2)}\, dx \approx 0.5202$

(b) Find the area bounded by the curve y = 2cos x, the lines x = 0, x = 2π and the x-axis.

Area = $\int_0^{2\pi} 2\cos x\, dx$
= $2 \int_0^{2\pi} \cos x\, dx$
= $2[\sin x]_0^{2\pi} = 2(0 - 0) = 0$

Since cosine is symmetric, total area = 4 (area from 0 to π)

$\int_0^{\pi} 2\cos x\, dx = 2[\sin x]_0^{\pi} = 2(0 - 0) = 0$
Use absolute value for area:
$\int_0^{\pi} 2\cos x\, dx = 4$
Total area = 4 units²

6. (a) The mean and variance of 7 observations are 8 and 16 respectively. Amongst the five observations are 2, 4, 10, 12 and 14. Find the other two observations.

Sum = 7 × 8 = 56
Known sum = 2 + 4 + 10 + 12 + 14 = 42 → missing sum = 14
Let missing numbers be x and y → x + y = 14

Variance = $[\Sigma x^2 / 7] - 8^2 = 16 \rightarrow \Sigma x^2 / 7 = 80 \rightarrow \Sigma x^2 = 560$
Known squares: 4 + 16 + 100 + 144 + 196 = 460
So $x^2 + y^2 = 560 - 460 = 100$

Now solve:
x + y = 14
$x^2 + y^2 = 100$

$(x + y)^2 = x^2 + y^2 + 2xy = 196$
100 + 2xy = 196 → xy = 48

x and y are roots of $t^2 - 14t + 48 = 0$
$t = [14 \pm \sqrt{(196 - 192)}]/2 = [14 \pm 2]/2 \rightarrow t = 8$ or 6

Missing values: 6 and 8
6. (b) The following table shows heights of 40 trees measured to the nearest meters:

Heights	4–8	9–13	14–18	19–23	24–28	29–33
No. of trees	2	4	7	14	8	5

Find the median.

Total number of trees = 40
Median position = 40 ÷ 2 = 20th item

Cumulative frequency:
4–8: 2
9–13: 2 + 4 = 6
14–18: 6 + 7 = 13
19–23: 13 + 14 = 27 ← 20th item lies here
24–28: 27 + 8 = 35
29–33: 35 + 5 = 40

Median class = 19–23
Lower boundary L = 18.5
Cumulative frequency before median class F = 13
Frequency of median class f = 14
Class width h = 5

Using the median formula:
Median = $L + [(n/2 - F)/f] \times h$
$= 18.5 + [(20 - 13)/14] \times 5$
$= 18.5 + (7/14) \times 5$
$= 18.5 + 0.5 \times 5$
$= 18.5 + 2.5$
$= 21.0$
Median = 21.0

7. (a) Show that $n + 1\ P_2 = n(n + 1)$

$n + 1\ P_2 = (n + 1)! / (n - 1)!$
$= (n + 1)n(n - 1)! / (n - 1)! = (n + 1)n = n(n + 1)$

(b) A and B are mutually exclusive events with $P(A) = 1/2$ and $P(B) = 1/4$. Find $P(A \cap B')$

$P(A \cap B') = P(A) - P(A \cap B)$
Mutually exclusive: $A \cap B = 0$
So $P(A \cap B') = 1/2$

(c) Two balls are drawn randomly without replacement from a bag containing 3 black balls and 2 white balls.

(i) Use tree diagram to analyze

First draw:
Black: 3/5
White: 2/5
Second draw depends on first.

(ii) Probability that both balls are white
= (2/5) × (1/4) = 2/20 = 0.1

8. (a) Write cos 2B in terms of tan B.

$\cos 2B = (1 - \tan^2 B) / (1 + \tan^2 B)$

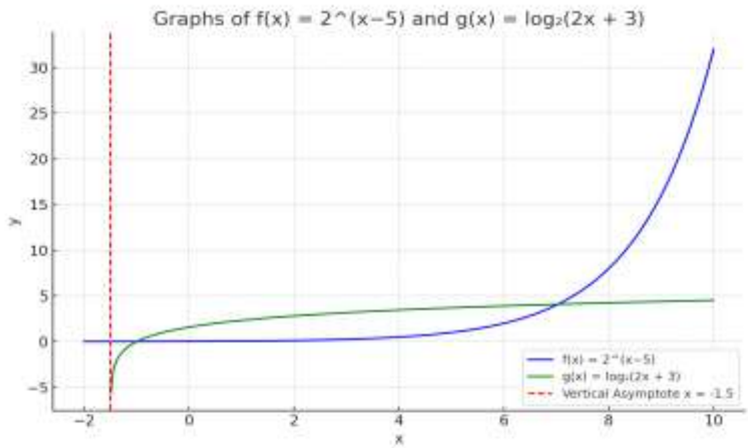
(b) If tan A = m / (m − 1) and tan B = 1 / (2m − 1), show that A − B = π/4

$\tan(A - B) = (\tan A - \tan B) / (1 + \tan A \times \tan B)$
Plug in expressions and simplify:
$[(m / (m-1)) - (1 / (2m-1))] / [1 + (m / (m-1)) \times (1 / (2m-1))] = 1$
So A − B = arctan(1) = π/4

(c) Find the degree measure of ∠ABC in the following figure: triangle with sides AB = 3, BC = 4, AC = 5
Use cosine rule:
$\cos B = (AB^2 + BC^2 - AC^2) / (2 \times AB \times BC)$
$= (3^2 + 4^2 - 5^2) / (2 \times 3 \times 4)$
= (9 + 16 − 25) / 24 = 0 / 24 = 0
B = 90°
9. (a) Draw the graphs of f(x) = 2^(x−5) and $g(x) = \log_2(2x + 3)$ on the same xy plane.

Graph 1: f(x) = 2^(x−5)
This is an exponential function shifted 5 units right.
y-intercept: x = 0 → f(0) = 2^−5 = 1/32
As x increases, f(x) increases rapidly.
As x → −∞, f(x) → 0
Domain: ℝ
Range: y > 0

Graph 2: $g(x) = \log_2(2x + 3)$
Domain: 2x + 3 > 0 → x > −1.5
Asymptote at x = −1.5
Range: ℝ
This is a logarithmic curve shifted left



(b) Use the graphs drawn in 9 (a) to determine the domain and range of:

(i) f(x)
Domain: ℝ
Range: $y > 0$

(ii) g(x)
Domain: $x > -1.5$
Range: ℝ

10. (a) Given the matrices

D = | a −4 −6 |
|−8 5 7 |
|−5 3 4 |

E = | 1 2 −2 |
| 3 b 1 |
|−1 1 −3 |

If D is the inverse of E, determine the value(s) of a and b.

Since $D = E^{-1}$, then $E \times D = I$
Perform matrix multiplication E × D, and equate first row first column to 1 (identity) and second row second column to 1.

E × D:
Row 1: [1 2 −2]
Multiply with column 1 of D:
$1\times a + 2\times(-8) + (-2)\times(-5) = a - 16 + 10 = a - 6 = 1 \rightarrow a = 7$

Row 2: [3 b 1]
Multiply with column 2 of D:
$3\times(-4) + b\times5 + 1\times3 = -12 + 5b + 3 = 5b - 9 = 1 \rightarrow 5b = 10 \rightarrow b = 2$

$a = 7$, $b = 2$

10. (b) A firm manufactures two products, A and B. The firm sells product A at a profit of 5 shillings per unit and product B at a profit of 3 shillings per unit. Each product is processed on two machines, M_1 and M_2. One unit of product A requires one minute of processing on M_1 and two minutes of processing on M_2. One unit of product B requires two minutes of processing on M_1 and one minute of processing on M_2. Machine M_1 works for 5 minutes per day, machine M_2 works for 6 minutes per day.

Let:
x = number of units of product A
y = number of units of product B

Constraints based on machine time:
M_1: $1x + 2y \leq 5$
M_2: $2x + 1y \leq 6$
Also, $x \geq 0$ and $y \geq 0$

Objective function (profit):
$P = 5x + 3y$

To solve this linear programming problem, we graph the constraints and evaluate the objective function at the feasible region's corner points.

Step 1: Graph the lines
- Line 1: $x + 2y = 5$
  $\rightarrow y = (5 - x)/2$
- Line 2: $2x + y = 6$
  $\rightarrow y = 6 - 2x$

Step 2: Find points of intersection
(i) Intersection of $x + 2y = 5$ and $2x + y = 6$
From $x + 2y = 5 \rightarrow x = 5 - 2y$
Substitute into $2x + y = 6$:
$2(5 - 2y) + y = 6 \rightarrow 10 - 4y + y = 6 \rightarrow -3y = -4 \rightarrow y = 4/3$
Then $x = 5 - 2(4/3) = 5 - 8/3 = 7/3$
Point: $(7/3, 4/3)$

(ii) x-intercepts and y-intercepts:

- $x + 2y = 5$:
  x-intercept: $y = 0 \rightarrow x = 5 \rightarrow (5, 0)$
  y-intercept: $x = 0 \rightarrow 2y = 5 \rightarrow y = 2.5 \rightarrow (0, 2.5)$

- $2x + y = 6$:
  x-intercept: $y = 0 \rightarrow x = 3 \rightarrow (3, 0)$
  y-intercept: $x = 0 \rightarrow y = 6 \rightarrow (0, 6)$

Step 3: Determine feasible region corner points
Points within constraints:
(0, 2.5), (3, 0), (5, 0), (7/3, 4/3)

Check which of these satisfy both constraints:
(5, 0):
  M_1: $5 + 2(0) = 5 \rightarrow$ ok
  M_2: $2(5) + 0 = 10 \rightarrow$ not ok
So (5, 0) is invalid

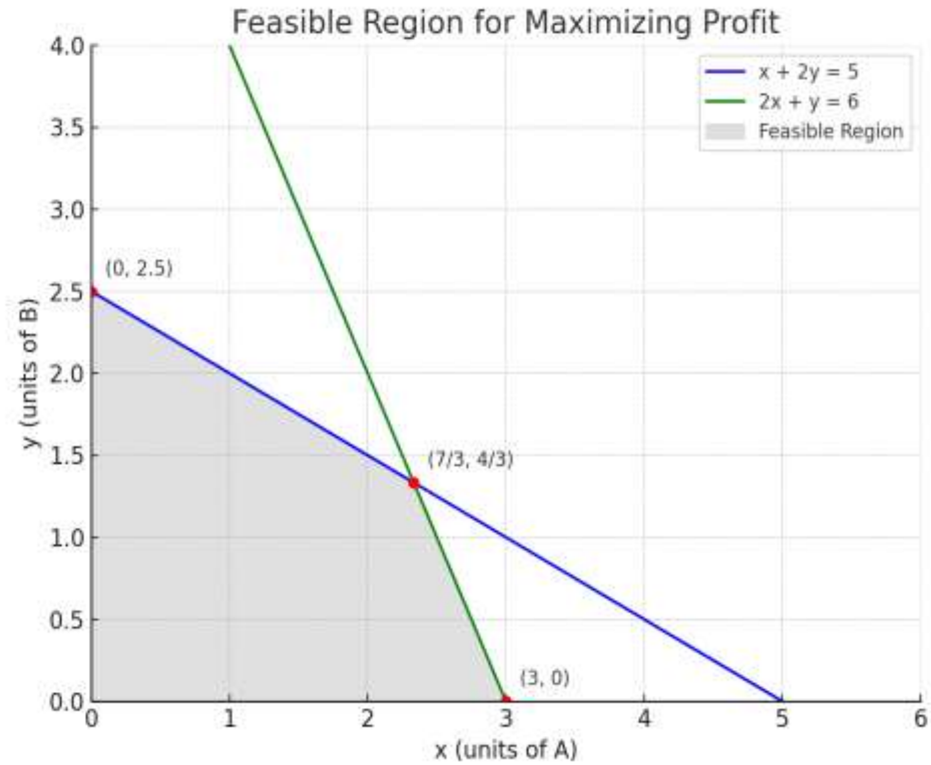
(0, 6):
  M_1: $0 + 2(6) = 12 \rightarrow$ not ok

Feasible points:
(0, 2.5), (3, 0), (7/3, 4/3)

Step 4: Compute profit $P = 5x + 3y$
At (0, 2.5): $P = 0 + 3\times2.5 = 7.5$
At (3, 0): $P = 5\times3 + 0 = 15$
At (7/3, 4/3): $P = 5\times(7/3) + 3\times(4/3) = (35 + 12)/3 = 47/3 \approx 15.67$

Maximum profit is approximately 15.67 at (7/3, 4/3)

So, to maximize profit, the firm should produce approximately 2.33 units of product A and 1.33 units of product B..


Feasible Region for Maximizing Profit
x + 2y = 5
2x + y = 6
Feasible Region
(0, 2.5)
(7/3, 4/3)
(3, 0)
y (units of B)
x (units of A)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0
1
2
3
4
5
6