THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

141 BASIC APPLIED MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2023**

Instructions

1. This paper consists of TEN questions.
2. Answer all questions.

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1. Use a non-programmable scientific calculator to:
(a) compute the value of $(67.9^3 / 68.53) \div (\sqrt[3]{e} \times \ln 2)$ correct to 5 significant figures.
4729.5
(b) evaluate $\int_0^1 e^{x^2}\,dx$ correct to 4 decimal places.
1.4626
(c) approximate the value(s) of x (correct to 3 decimal places) which satisfy the equation $x^3 + 5x^2 + 3x - 7 = 0$.
0.706

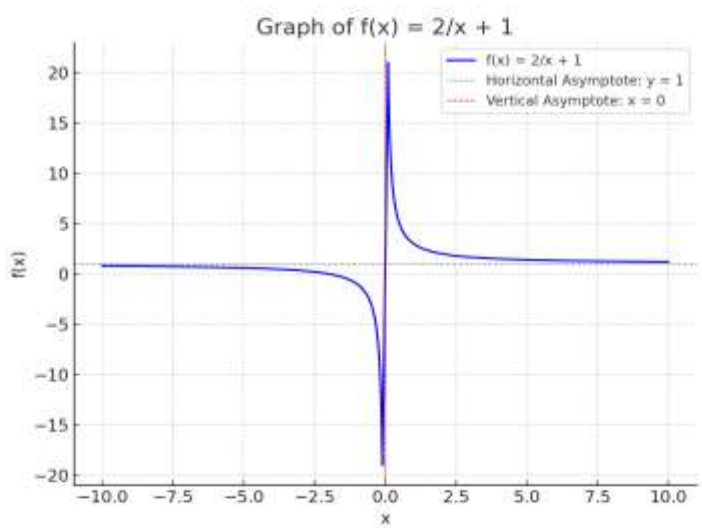
2. The function f is defined as $f(x) = a/x + b$ such that $f(2) = 2$ and $f(-1) = -1$.

(a) Find the values of a and b.

$f(2) = a/2 + b = 2 \rightarrow a + 2b = 4$ …(1)
$f(-1) = a/(-1) + b = -1 \rightarrow -a + b = -1$ …(2)
Add (1) and (2):
$a + 2b - a + b = 4 - 1$
$3b = 3 \rightarrow b = 1$
Substitute into (1):
$a + 2(1) = 4 \rightarrow a = 2$

$a = 2, b = 1$

(b) Sketch the graph of f.



The graph is a rectangular hyperbola shifted up by 1 unit.
It has a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 1$.

(c) State the domain and range of f.

Domain: $x \neq 0$
Range: $y \neq 1$

3. (a) The sum of the first three terms of an arithmetic progression is 3 and the sum of the first five terms is 20. Find the first term and the common difference.

Let the first term be a, common difference d.
Sum of 3 terms: $S_3 = 3a + 3d = 3 \rightarrow a + d = 1$ …(1)
Sum of 5 terms: $S_5 = 5a + 10d = 20 \rightarrow a + 2d = 4$ …(2)

Subtract (1) from (2):
$a + 2d - a - d = 4 - 1$
$d = 3$

Substitute into (1): $a + 3 = 1 \rightarrow a = -2$

$a = -2, d = 3$

(b) The volume of a cone varies jointly as its height and the square of its radius. The cone with r = 6 cm and h = 10 cm has volume 120π cm³. Find the volume of the cone having r = 15 cm and h = 7 cm.

$V = k \times r^2 \times h$
$120\pi = k \times 36 \times 10 = 360k \rightarrow k = \pi/3$

Now for new values:
$V = (\pi/3) \times 15^2 \times 7 = (\pi/3) \times 225 \times 7 = 525\pi$

$V = 525\pi \text{ cm}^3$

4. (a) Find the first derivative for each of the following functions:

(i) $f(x) = \cos(2x + 1)$
$f'(x) = -\sin(2x + 1) \times 2 = -2\sin(2x + 1)$

(ii) $g(x) = x / (1 + x^2)$
Use quotient rule:
$g'(x) = [(1 + x^2)(1) - x(2x)] / (1 + x^2)^2$
$= (1 + x^2 - 2x^2) / (1 + x^2)^2 = (1 - x^2) / (1 + x^2)^2$

(iii) $h(x) = 3^x$
$h'(x) = 3^x \times \ln 3$

(b) The temperature (T) in °C of meat in a freezer after t hours is given by $T = 70 - 12t + 4 / (t + 1)$

(i) What is the temperature of the meat after 3 hours?

$T = 70 - 12(3) + 4 / (3 + 1) = 70 - 36 + 4/4 = 34 + 1 = 35°C$

(ii) How fast is the temperature of the meat falling after 3 hours?

$T = 70 - 12t + 4 / (t + 1)$
dT/dt = derivative of $70 - 12t + 4 / (t + 1)$
$= -12 - 4 / (t + 1)^2$
At t = 3:
$= -12 - 4 / 16 = -12.25$

Rate of fall is 12.25°C/hour

5. (a) Given that $\int_3^5 h(x)\,dx = 4$,

(i) evaluate $\int_3^5 (h(x) + 3)\,dx$.

$\int_3^5 h(x)\,dx + \int_3^5 3\,dx = 4 + 3(5 - 3) = 4 + 6 = 10$

(ii) find the value of k if $\int_3^5 (h(x) + kx)\,dx = 28$.

$\int_3^5 h(x)\,dx + \int_3^5 kx\,dx = 28$
$4 + k\int_3^5 x\,dx = 28$
$\int_3^5 x\,dx = (1/2)(5^2 - 3^2) = (1/2)(25 - 9) = 8$
$4 + 8k = 28 \rightarrow k = 3$

(b) Find $\int_1^2 (1 + 5t)^7\,dt$

Let $u = 1 + 5t \rightarrow du = 5\,dt \rightarrow dt = du/5$
When t = 1, u = 6; when t = 2, u = 11

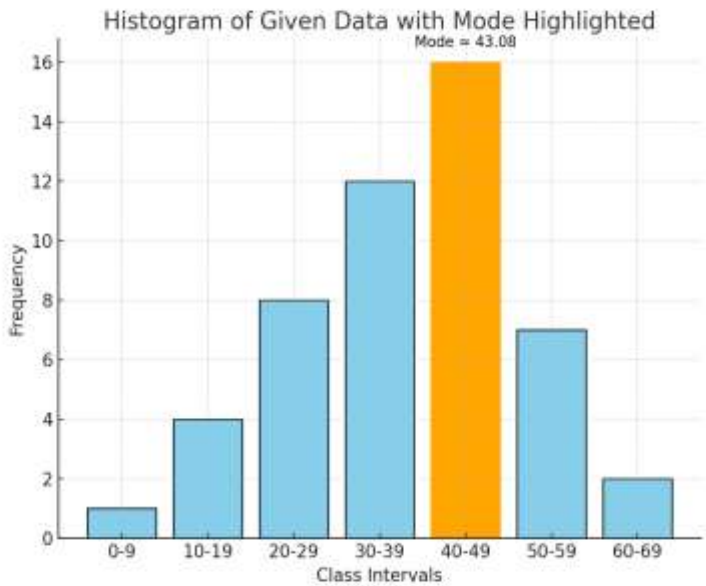
$\int_1^2 (1 + 5t)^7\,dt = \int_6^{11} u^7 \times (1/5)\,du = (1/5) \times \int_6^{11} u^7\,du$
$= (1/5)[u^8 / 8]$ from 6 to 11
$= (1/5)(11^8 - 6^8) / 8$
$= (1/40)(214358881 - 1679616) = (1/40)(212679265)$
$= 5316981.625$

6. Consider the following data:
7, 12, 14, 15, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 36, 37, 38, 39, 40, 41, 41, 42, 43, 43, 44, 44, 45, 46, 46, 47, 48, 48, 49, 50, 51, 52, 54, 56, 57, 58, 59, 60, 61, 62

(a) Construct a frequency distribution table using the intervals 0–9, 10–19, ..., etc.

Class Intervals	frequency
0–9	1
10–19	4
20–29	8
30–39	12
40–49	16
50–59	7
60–69	2

(b) Draw a histogram and use it to estimate the mode correct to 2 decimal places.



Modal class = 40–49 (highest frequency = 16)
Class before = 30–39 (frequency = 12)
Class after = 50–59 (frequency = 7)
Using formula:
$\text{Mode} = L + [(f_1 - f_0) / (2f_1 - f_0 - f_2)] \times h$
$L = 40$, $f_1 = 16$, $f_0 = 12$, $f_2 = 7$, $h = 10$

$\text{Mode} = 40 + [(16 - 12) / (2\times16 - 12 - 7)] \times 10$
$= 40 + [4 / (32 - 19)] \times 10$
$= 40 + (4 / 13) \times 10$
$= 40 + 3.08$
$= 43.08$

(c) Calculate:

(i) the median (correct to 3 decimal places)

Number of values = 50 → median = average of 25^{th} and 26^{th} values
From ordered data: 25^{th} = 40, 26^{th} = 41
Median = (40 + 41) / 2 = 40.5

(ii) the 70^{th} percentile (correct to 3 decimal places)

70% of 50 = 35^{th} value
35^{th} value = 47
70^{th} percentile = 47.000

7. (a) A certain family consists of mother, father and their ten children. The family is invited to send a group of four representatives to a wedding. In how many ways can the group be formed if it must include both parents?

From 12 people, if both parents are included, choose 2 children out of 10:
C(10, 2) = 45

(b) A fair coin is tossed three times. Using tree diagram, find the probability of obtaining exactly two heads.

Possible outcomes: HHT, HTH, THH → 3 outcomes
Total outcomes = 8
Probability = 3 / 8

8. (a) If $x = \sin(A + B)$ and $y = \sin(A - B)$, prove that $xy = \sin^2 A - \sin^2 B$.

$xy = \sin(A + B) \times \sin(A - B)$
Use identity:
$\sin(A + B) \times \sin(A - B) = \sin^2 A - \sin^2 B$
Therefore, $xy = \sin^2 A - \sin^2 B$

(b) Solve the following equations for $0° \leq \theta \leq 360°$:

(i) $2\sin^2\theta - 3\cos\theta = 3$
Use identity: $\sin^2\theta = 1 - \cos^2\theta$
$2(1 - \cos^2\theta) - 3\cos\theta = 3$
$2 - 2\cos^2\theta - 3\cos\theta - 3 = 0$
$-2\cos^2\theta - 3\cos\theta - 1 = 0$
$2\cos^2\theta + 3\cos\theta + 1 = 0$

Solve using quadratic formula:
$\cos\theta = [-3 \pm \sqrt{(9 - 8)}] / 4$
$\cos\theta = (-3 \pm 1)/4$
$\cos\theta = -0.5$ or -1

For $\cos\theta = -0.5 \rightarrow \theta = 120°, 240°$
For $\cos\theta = -1 \rightarrow \theta = 180°$

$\theta = 120°, 180°, 240°$

(ii) $\sqrt{2}\cos\theta - \sin 2\theta = 0$
Use identity: $\sin 2\theta = 2\sin\theta\cos\theta$
$\sqrt{2}\cos\theta - 2\sin\theta\cos\theta = 0$
$\cos\theta(\sqrt{2} - 2\sin\theta) = 0$

Either $\cos\theta = 0 \rightarrow \theta = 90°, 270°$
Or $\sin\theta = \sqrt{2} / 2 \rightarrow \theta = 45°, 135°$

$\theta = 45°, 90°, 135°, 270°$

9. The following figure shows part of the curve of the function $y = g(x)$, where $g(x) = |4e^{2x} - 25|$, $x \in \mathbb{R}$.

(a) The y-coordinate of point A.

At $x = 0$: $g(x) = |4e^0 - 25| = |4 - 25| = 21$
$y = 21$

(b) The x-coordinate of point B.

$g(x) = 0 \rightarrow 4e^{2x} - 25 = 0$
$e^{2x} = 25/4$
$2x = \ln(25/4)$
$x = (1/2)\ln(6.25) = (1/2) \times 1.8326 = 0.916$

$x = 0.916$

(c) The value of k.

As $x \rightarrow -\infty$, $e^{2x} \rightarrow 0$
$g(x) = |4e^{2x} - 25| \rightarrow |-25| = 25$
$k = 25$

10. (a) (i) Write down all possible orders for a matrix with 6 elements.

Possible orders:
1×6, 2×3, 3×2, 6×1

(ii) Suppose $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by $a_{ij} = (j - i)/2$. Determine the elements of matrix A.

$i = 1, 2$; $j = 1, 2$

$a_{11} = (1-1)/2 = 0$
$a_{12} = (2-1)/2 = 0.5$
$a_{21} = (1-2)/2 = -0.5$
$a_{22} = (2-2)/2 = 0$

Matrix A =
| 0 0.5 |
| −0.5 0 |

10. (b) The following graph represents business optimization possibilities for a company which sells two types of stoves, S_1 and S_2. The variable x represents the number of S_1 type while y represents the number of S_2 type. The time available for the company to make both S_1 type and S_2 type is 80 hours and the space available can hold not more than 50 stoves.

Constraints:
$x + y \leq 50$
$2x + y \leq 80$
$x \geq 0, y \geq 0$

(i) How many hours are used to make one stove of each type?

From the constraint equations:
- $2x + y = 80$ represents the time in hours
- $x + y = 50$ represents total number of stoves

Let:
x = number of S_1 stoves
y = number of S_2 stoves

To find hours per stove, solve:
$2x + y = 80$
$x + y = 50$

Subtract equations:

$(2x + y) - (x + y) = 80 - 50$
$x = 30$

Substitute into second equation:
$30 + y = 50 \rightarrow y = 20$

So for 30 S_1 and 20 S_2 stoves:
Total time = 80 hours → 80 / (30 + 20) = 80 / 50 = 1.6 hours per stove (on average)
S_1: 2 hours per stove
S_2: 1 hour per stove

(ii) If one stove of S_1 type is sold at Tshs. 300 and one stove of S_2 type is sold at Tshs. 200, how many stoves of each type could be sold in order to maximize revenue?

Revenue function:
$R = 300x + 200y$
Constraints:
$x + y \leq 50$
$2x + y \leq 80$
$x \geq 0, y \geq 0$

Feasible region corners from graph:
- (0, 0)
- (0, 50)
- (30, 20)
- (40, 10)
- (50, 0)

Evaluate revenue at each corner:

At (0, 0): $R = 0$
At (0, 50): $R = 0 + 200\times50 = 10000$
At (30, 20): $R = 300\times30 + 200\times20 = 9000 + 4000 = 13000$
At (40, 10): $R = 300\times40 + 200\times10 = 12000 + 2000 = 14000$
At (50, 0): $R = 300\times50 = 15000$

Maximum revenue is Tshs. 15000 at (50, 0)

So, 50 stoves of S_1 type and 0 stoves of S_2 type should be sold to maximize revenue.