THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

141 BASIC APPLIED MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2024**

Instructions

1. This paper consists of TEN questions.
2. Answer all questions.

1. (a) Use a non-programmable calculator to:
Compute the value of $1000\ (1 + 0.12 \div 8760)^{8760}$ correct to 2 decimal places.
Answer: 1127.50
(b) Compute the value of $\lim_{x\to 0} (\sin x)/x$ correct to five significant figures.
Answer: 1.0000
(c) Evaluate $\int_0^2 e^{2x-3}\,dx$ correct to two decimal places.
Answer: 1.33

5. (a) Evaluate $\int(2x - 1)(4x^2 - 4x)^5\,dx$.

Let $u = (4x^2 - 4x)$
Then, $du/dx = 8x - 4$

Notice that $2x - 1$ is exactly (1/4) of $(8x - 4)$, so we rewrite:
$(2x - 1)\,dx = (1/4)\,du$

Therefore,
$\int(2x - 1)(4x^2 - 4x)^5\,dx$
$= \int(1/4)\,u^5\,du$
$= (1/4) \times (u^6 / 6) + C$
$= (1/24)(4x^2 - 4x)^6 + C$

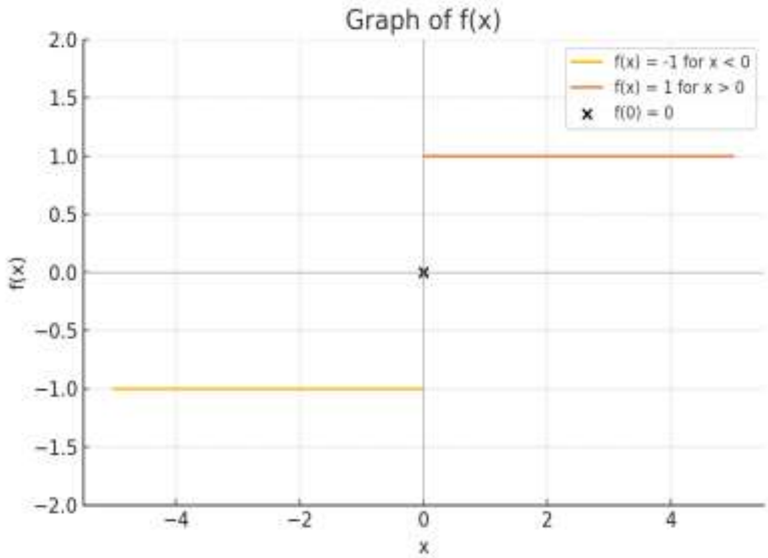
Answer: $(1/24)(4x^2 - 4x)^6 + C$

2. (a) Given the function $g(x) = 1 / (x - 3) + 2$, find the vertical and horizontal asymptotes.

Vertical asymptote occurs when the denominator is zero:
$x - 3 = 0$
$x = 3$
For horizontal asymptote, examine the behavior as $x \to \pm\infty$:
As x becomes large, $1 / (x - 3) \to 0$
So, $g(x) \to 2$
Vertical asymptote: $x = 3$
Horizontal asymptote: $y = 2$

2. (b) Given that
$f(x) =$
1 if $x > 0$
0 if $x = 0$
-1 if $x < 0$

(i) Sketch the graph of f(x)



(ii) State the domain and range of f(x)
Domain: All real numbers = $\mathbb{R}$
Range: {−1, 0, 1}

3. (a) Solve the following system of simultaneous equations:

$x + 2y = 4$ …(1)

$x^2 + 3xy = 10$ …(2)

From equation (1):
$x = 4 - 2y$

Substitute into equation (2):
$(4 - 2y)^2 + 3(4 - 2y)(y) = 10$
$16 - 16y + 4y^2 + 12y - 6y^2 = 10$
$-2y^2 - 4y + 16 = 10$
$-2y^2 - 4y + 6 = 0$
Divide by −2:
$y^2 + 2y - 3 = 0$
$(y + 3)(y - 1) = 0$
$y = -3$ or $y = 1$

If $y = -3$: $x = 4 - 2(-3) = 10$
If $y = 1$: $x = 4 - 2(1) = 2$

Solutions: $(x, y) = (10, -3)$ and $(2, 1)$

3. (b) Evaluate ∑ from r = 1 to 4 of $16(-\frac{1}{2})^r$

This is a geometric series:
First term $a = 16(-\frac{1}{2}) = -8$
Common ratio $r = -\frac{1}{2}$
$n = 4$

$S_4 = a(1 - r^4) / (1 - r)$
$= -8(1 - (-\frac{1}{2})^4) / (1 + \frac{1}{2})$
$= -8(1 - 1/16) / (3/2)$
$= -8(15/16) \times (2/3)$
$= (-240/48) = -5$

3. (c) The time (t) to complete a project varies inversely to the number of employees (e). If 3 people complete the project in 10 days, how many days will 5 people take?

$t \propto 1/e$
$t = k/e$
$10 = k / 3 \rightarrow k = 30$
Now, $t = 30 / 5 = 6$ days

4. (a) Use the first principles to find the first derivative of the function $f(x) = 3x^2 - 2$.

$f'(x) = \lim_{h\to 0} [f(x + h) - f(x)] / h$
$= \lim_{h\to 0} [3(x + h)^2 - 2 - (3x^2 - 2)] / h$
$= \lim_{h\to 0} [3(x^2 + 2xh + h^2) - 3x^2] / h$
$= \lim_{h\to 0} [3x^2 + 6xh + 3h^2 - 3x^2] / h$
$= \lim_{h\to 0} [6xh + 3h^2] / h$
$= \lim_{h\to 0} [6x + 3h]$
$= 6x$

4. (b) Given that $y^3 + x^3 - 3xy = 4$, find dy/dx.

Differentiate both sides:
$d/dx(y^3) + d/dx(x^3) - d/dx(3xy) = 0$
$3y^2(dy/dx) + 3x^2 - 3(x(dy/dx) + y) = 0$
$3y^2\, dy/dx + 3x^2 - 3x\, dy/dx - 3y = 0$
Group dy/dx terms:
$(3y^2 - 3x)\, dy/dx = 3y - 3x^2$
$dy/dx = (3y - 3x^2) / (3y^2 - 3x)$

4. (c) Find the slope of the curve $y^3 = 64x$ at $x = -1$.

Differentiate both sides:

$3y^2\, dy/dx = 64$
$dy/dx = 64 / (3y^2)$

When $x = -1$:
$y^3 = 64(-1) = -64 \rightarrow y = -4$
$dy/dx = 64 / (3 \times 16) = 64 / 48 = 4/3$

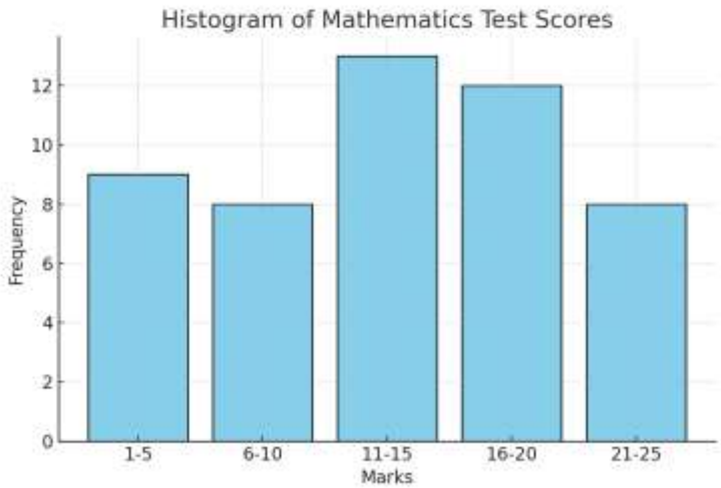
5. (b) Find the area enclosed between the curves of the functions $y = 4 - x^2$ and $y = x^2 - 2x$.

Set the two functions equal to find points of intersection:
$4 - x^2 = x^2 - 2x$
$4 = 2x^2 - 2x$
$0 = 2x^2 - 2x - 4$
Divide by 2:
$x^2 - x - 2 = 0$
Factor: $(x - 2)(x + 1) = 0$
$x = -1$ and $x = 2$

Now compute the area between the curves from $x = -1$ to $x = 2$.
Area = ∫ from −1 to 2 of $[(4 - x^2) - (x^2 - 2x)]\, dx$
= ∫ from −1 to 2 of $[4 - x^2 - x^2 + 2x]\, dx$
= ∫ from −1 to 2 of $[4 - 2x^2 + 2x]\, dx$

Integrate:
$\int[4 - 2x^2 + 2x]\, dx$
$= 4x - (2/3)x^3 + x^2$ from −1 to 2
At $x = 2$:
$4(2) - (2/3)(8) + 4 = 8 - 16/3 + 4 = 12 - 16/3 = (36 - 16)/3 = 20/3$
At $x = -1$:
$4(-1) - (2/3)(-1)^3 + (-1)^2 = -4 + 2/3 + 1 = -3 + 2/3 = -7/3$
Now compute total area:
$20/3 - (-7/3) = 27/3 = 9$
Answer: 9 square units

6. (a) Represent the data using histogram.
Marks intervals:
1–5: frequency 9
6–10: frequency 8
11–15: frequency 13
16–20: frequency 12
21–25: frequency 8



6. (b) Calculate the variance correct to 4 significant figures.
Midpoints:
3, 8, 13, 18, 23
f × x:
9×3 = 27
8×8 = 64
13×13 = 169
12×18 = 216
8×23 = 184
Σf = 50, Σfx = 660
Mean = 660 / 50 = 13.2
f × (x − mean)²:
9(−10.2)² = 936.36
8(−5.2)² = 216.32
13(−0.2)² = 0.52
12(4.8)² = 276.48
8(9.8)² = 768.32
Σf(x − mean)² = 2197.99
Variance = 2197.99 / 50 = 43.96

7. (a) A random experiment can result in one of the outcomes a, b, c and d whose probabilities are 0.1, 0.3, 0.5 and 0.1, respectively. If A denotes the event {a, b, d} and B denotes the event {b, c, d}, determine the probability of the event A ∩ B.
A ∩ B = {b, d}
P(A ∩ B) = P(b) + P(d) = 0.3 + 0.1 = 0.4

7. (b) A four-digit number is to be formed from the digits 1, 2, 3 and 5. If the repetition of a digit is not allowed, find the probability that the number formed is divisible by 5.

To be divisible by 5, the number must end with 5
Fix 5 at unit place
Remaining: 1, 2, 3 → arrangements = 3! = 6
Total 4-digit numbers = 4! = 24
Probability = 6 / 24 = 0.25

7. (c) The probability that a certain type of machine will break down in the first month of operation is 0.1. If a firm has two machines of such type installed at the same time, find the probability that, at the end of the first month, just one machine will be broken down.
Required: P(1st breaks, 2nd does not) + P(1st does not, 2nd breaks)
= (0.1)(0.9) + (0.9)(0.1) = 0.09 + 0.09 = 0.18

8. (a) Given that cos A = 1/2, evaluate cos(A/2). Express your answer in surd form.
cos A = 1/2 → A = 60°
cos(A/2) = cos(30°) = $\sqrt{3}$ / 2

8. (b) Given that sinθ − 2sinθcosθ = 0 and 0° ≤ θ ≤ 180°, determine the values of θ.
sinθ(1 − 2cosθ) = 0
Either sinθ = 0 → θ = 0°, 180°
or cosθ = 1/2 → θ = 60°, 300°
Restricting to 0° ≤ θ ≤ 180°: θ = 0°, 60°, 180°

8. (c) Calculate the length of the side EF of the following diagram and write the answer correct to four significant figures.
Triangle EFG with angle at F = 40°, angle at G = 75°, side EG = 14 cm
Angle E = 180° − (40° + 75°) = 65°
Use sine rule:
EF / sin(75°) = 14 / sin(65°)
EF = [14 × sin(75°)] / sin(65°)
= [14 × 0.9659] / 0.9063
= 13.5226 / 0.9063 = 14.92 cm (4 significant figures)

9. (a) Evaluate $\int_0^1$ 4 / (7x + 2) dx.
Let u = 7x + 2 → du/dx = 7 → dx = du / 7
When x = 0, u = 2
When x = 1, u = 9
$\int_0^1$ 4 / (7x + 2) dx = $\int_2^9$ 4 / u × 1/7 du
= (4/7) $\int_2^9$ 1/u du = (4/7)(ln 9 − ln 2) = (4/7) ln(9/2)

9. (b) The following graph describes the population of bacteria (N(t)) after a particular period of time (t) in hours. Formulate the equation that relates N(t) and t.
From the graph:
At t = 0, N = 10

At t = 1, N = 20
At t = 2, N = 40
At t = 3, N = 80
So, population doubles every hour → exponential growth
$N(t) = 10 \times 2^t$

10. (a) Given the matrices

$$A = \begin{pmatrix} 3 & 1 & 2 \\ 1 & 5 & 2 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 4 & -1 & 2 \\ 3 & 1 & 3 \end{pmatrix}$$

Evaluate 3A − 2B

First compute 3A:

$$3A = \begin{pmatrix} 9 & 3 & 6 \\ 3 & 15 & 6 \end{pmatrix}$$

Next compute 2B:

$$2B = \begin{pmatrix} 8 & -2 & 4 \\ 6 & 2 & 6 \end{pmatrix}$$

Now subtract:

$$3A - 2B = \begin{pmatrix} 9-8 & 3-(-2) & 6-4 \\ 3-6 & 15-2 & 6-6 \end{pmatrix}$$

$$3A - 2B = \begin{pmatrix} 1 & 5 & 2 \\ -3 & 13 & 0 \end{pmatrix}$$

10. (b) A manufacturer produces nuts and bolts for machines. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts, while it takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. The manufacturer works for 12 hours a day and earns a profit of Tsh. 250,000 per package of nuts and Tsh. 100,000 per package of bolts. How many packages of nuts and bolts should be produced to realize a maximum profit?

Let x = number of packages of nuts
Let y = number of packages of bolts

From the problem:
Machine A time: $1x + 3y \leq 12$
Machine B time: $3x + 1y \leq 12$
$x \geq 0, y \geq 0$

Objective function (profit):
$P = 250000x + 100000y$

Now apply linear programming:

Rewrite constraints:
$x + 3y \leq 12$ …(1)
$3x + y \leq 12$ …(2)

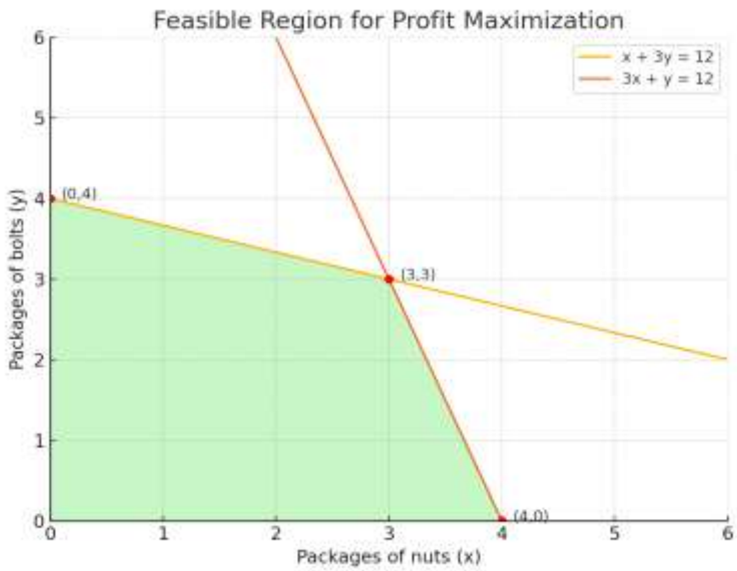
Now find feasible region by solving these inequalities

From (1):
$x + 3y = 12$
$\rightarrow y = (12 - x)/3$

From (2):
$3x + y = 12$
$\rightarrow y = 12 - 3x$

Find points of intersection:
- Intersection of (1) and (2):
$x + 3y = 12$
$3x + y = 12$
Multiply second equation by 3:
$9x + 3y = 36$
Now subtract:
$(9x + 3y) - (x + 3y) = 36 - 12$
$8x = 24 \rightarrow x = 3$
Substitute into (1): $3 + 3y = 12 \rightarrow y = 3$
Point: (3, 3)

Other boundary points:
- When $x = 0$ in (1): $3y = 12 \rightarrow y = 4 \rightarrow (0, 4)$
- When $y = 0$ in (1): $x = 12 \rightarrow (12, 0)$ — but must check with (2):
$3\times12 + 0 = 36 > 12 \rightarrow$ not valid
Try $x = 0$ in (2): $y = 12 \rightarrow$ check with (1): $0 + 3\times12 = 36 > 12 \rightarrow$ invalid
Try $y = 0$ in (2): $3x = 12 \rightarrow x = 4 \rightarrow (4, 0)$



Feasible region corners:
(0, 4), (3, 3), (4, 0)

Evaluate P at each:
At (0, 4): P = 0 + 100000×4 = 400000
At (3, 3): P = 250000×3 + 100000×3 = 750000 + 300000 = 1050000
At (4, 0): P = 250000×4 = 1000000

Maximum profit occurs at (3, 3)

Therefore, 3 packages of nuts and 3 packages of bolts should be produced.