

**THE UNITED REPUBLIC OF TANZANIA**  
**NATIONAL EXAMINATIONS COUNCIL OF TANZANIA**  
**ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION**

**131/1**

**PHYSICS 1**

(For Both School and Private Candidates)

**Time: 2:30 Hours**

**ANSWERS**

**Year: 2000**

**Instructions**

1. This paper consists of section A, B and C with total of nine questions.
2. Answer five questions, choosing at least one question from each section.
3. Each question carries twenty marks.

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1. (a) (i) What is an error? Mention two causes of systematic and two causes of random errors.

An error is the difference between the measured value and the true or accepted value of a quantity. Errors occur due to limitations in measuring instruments, external influences, or human mistakes.

Causes of systematic errors:

- Instrumental errors – caused by faulty or improperly calibrated measuring instruments.
- Environmental errors – due to changes in external conditions such as temperature, pressure, or humidity affecting measurements.

Causes of random errors:

- Human errors – variations in readings due to observer limitations, such as reaction time.
- Fluctuations in experimental conditions – such as small unpredictable variations in voltage, temperature, or air currents.

(ii) The pressure  $P$  is calculated from the relation  $P = F / (\pi R^2)$  where  $F$  is the force and  $R$  the radius. If the percentage possible errors are  $\pm 2\%$  for  $F$  and  $\pm 1\%$  for  $R$ , calculate the possible percentage error for  $P$ .

The formula for percentage error propagation states:

For a function of the form  $P = F / (\pi R^2)$ , the percentage error is calculated as:

percentage error in  $P = \text{percentage error in } F + 2 \times \text{percentage error in } R$

Substituting given values:

percentage error in  $P = 2\% + 2(1\%)$

percentage error in  $P = 2\% + 2\%$

percentage error in  $P = 4\%$

The possible percentage error in  $P$  is  $4\%$ .

(b) The speed  $v$  of a wave is found to depend on the tension  $T$  in the string and the mass per unit length  $\mu$  (linear mass density). Using dimensional analysis, derive the relationship between  $v$ ,  $T$ , and  $\mu$ .

Let the wave speed  $v$  be related to the tension  $T$  and the linear mass density  $\mu$  as:

$$v = k T^a \mu^b$$

where  $k$  is a dimensionless constant, and  $a$  and  $b$  are exponents to be determined.

Writing the dimensions of each quantity:

v (velocity) has dimensions  $[L T^{-1}]$ .

T (tension, force) has dimensions  $[M L T^{-2}]$ .

$\mu$  (linear mass density) has dimensions  $[M L^{-1}]$ .

Equating dimensions:

$$[L T^{-1}] = [M L T^{-2}]^a [M L^{-1}]^b$$

Expanding:

$$[L T^{-1}] = [M^a L^a T^{-2a}] [M^b L^{-b}]$$

Grouping similar terms:

$$M^{(a+b)} L^{(a-b)} T^{(-2a)} = L^1 T^{-1}$$

Equating powers of M:

$$a + b = 0$$

Equating powers of L:

$$a - b = 1$$

Equating powers of T:

$$-2a = -1$$

$$a = 1/2$$

Substituting  $a = 1/2$  into  $a + b = 0$ :

$$1/2 + b = 0$$

$$b = -1/2$$

Thus,  $v = k T^{(1/2)} \mu^{(-1/2)}$ , or:

$$v = k \sqrt{(T / \mu)}$$

Since k is a dimensionless constant, we take  $k = 1$  for proportionality, giving:

$$v = \sqrt{(T / \mu)}$$

(c) The longitudinal wave speed in gases is given by  $v = \sqrt{\gamma P / \rho}$ , where  $\gamma = C_p / C_v$ ,  $P$  is the pressure, and  $\rho$  the density of gas. If  $v_1$  and  $v_2$  are the speeds of sound in air at temperatures  $T_1$  and  $T_2$  respectively, show that  $v_2 / v_1 = \sqrt{T_2 / T_1}$ .

The speed of sound in gases is given by:

$$v = \sqrt{\gamma P / \rho}$$

For an ideal gas, the equation of state is:

$$P = \rho R T$$

where  $R$  is the specific gas constant and  $T$  is the absolute temperature.

Substituting  $P$  in the speed equation:

$$v = \sqrt{\gamma (\rho R T) / \rho}$$

Canceling  $\rho$ :

$$v = \sqrt{\gamma R T}$$

Since  $\gamma$  and  $R$  are constants for the same gas, we can write:

$$v \propto \sqrt{T}$$

For two different temperatures  $T_1$  and  $T_2$ , the corresponding speeds  $v_1$  and  $v_2$  are:

$$v_1 \propto \sqrt{T_1}$$

$$v_2 \propto \sqrt{T_2}$$

Dividing the two equations:

$$v_2 / v_1 = \sqrt{T_2 / T_1}$$

This proves the required relationship.

2. (a) Show that the period of a body of mass  $m$  revolving in a horizontal circle with constant velocity  $v$  at the end of a string of length  $l$  is independent of the mass of the object.

The period  $T$  of circular motion is given by:

$$T = \text{circumference} / \text{velocity}$$

$$T = (2\pi r) / v$$

For an object moving in a circular path with radius  $l$  (assuming the string length is  $l$  and the motion is in a horizontal plane):

$$T = (2\pi l) / v$$

To express  $v$  in terms of  $l$  and other variables, we use centripetal force:

Tension in the string provides the necessary centripetal force:

$$T = m v^2 / l$$

Solving for  $v$ :

$$v = \sqrt{(T l / m)}$$

Substituting this in the period equation:

$$T = (2\pi l) / \sqrt{(T l / m)}$$

Since tension  $T$  is due to gravitational forces or other constraints and does not involve mass explicitly, we see that:

$T$  is independent of mass  $m$ .

(b) A ball of mass 100 g is attached to the end of a string and is swung in a circle of radius 100 cm at a constant velocity of 200 cm/s. While in motion, the string is shortened to 50 cm. Calculate:

(i) The new velocity of the motion.

(ii) The new period of the motion.

The motion follows the principle of conservation of angular momentum:

$$L = m v r$$

Since no external torque acts, angular momentum is conserved:

$$m v_1 r_1 = m v_2 r_2$$

Canceling mass:

$$v_1 r_1 = v_2 r_2$$

Substituting given values:

$$(200 \text{ cm/s}) (100 \text{ cm}) = v_2 (50 \text{ cm})$$

$$v_2 = (200 \times 100) / 50$$

$$v_2 = 400 \text{ cm/s}$$

The new velocity is 400 cm/s.

For the new period:

$$T = (2\pi r) / v$$

For the original motion:

$$T_1 = (2\pi \times 100) / 200$$

$$T_1 = (200\pi) / 200$$

$$T_1 = \pi \text{ seconds}$$

For the new motion:

$$T_2 = (2\pi \times 50) / 400$$

$$T_2 = (100\pi) / 400$$

$$T_2 = \pi / 4 \text{ seconds}$$

The new period is  $\pi / 4$  seconds.

(c) A car travels over a humpback bridge of radius of curvature 45 m. Calculate the maximum speed of the car if the wheels are to remain in contact with the bridge.

At the highest point of the bridge, the normal reaction  $N$  and the weight of the car  $mg$  act. For the car to just remain in contact with the bridge, the normal reaction becomes zero, meaning that the only force providing the necessary centripetal acceleration is the weight of the car.

$$mg = m v^2 / r$$

Canceling mass  $m$ :

$$g = v^2 / r$$

Solving for  $v$ :

$$v = \sqrt{(g r)}$$

Taking  $g = 9.81 \text{ m/s}^2$  and  $r = 45 \text{ m}$ :

$$v = \sqrt{(9.81 \times 45)}$$

$$v = \sqrt{(441.45)}$$

$$v \approx 21 \text{ m/s}$$

The maximum speed of the car to remain in contact with the bridge is 21 m/s.

3. (a) Mention two motions that add up to make projectile motion.

Projectile motion is a combination of:

- Horizontal motion – A uniform motion with constant velocity since there is no acceleration in the horizontal direction (neglecting air resistance).
- Vertical motion – A uniformly accelerated motion due to gravity, acting downward with an acceleration of  $9.81 \text{ m/s}^2$ .

(b) (i) In long jumps does it matter how high you jump? State the factors which determine the span of the jump.

Yes, in long jumps, the height of the jump matters because it affects the time of flight. The longer the jumper stays in the air, the farther they can travel horizontally.

Factors that determine the span (range) of the jump:

- Initial velocity – The greater the initial speed, the farther the jump.
- Angle of takeoff – The optimal angle for maximum range in projectile motion (neglecting air resistance) is  $45^\circ$ .
- Acceleration due to gravity – Affects the time of flight and the parabolic trajectory.
- Air resistance – Can reduce the range if significant.
- Initial height – A higher launch point increases the total time in the air.

(ii) Derive an expression that relates the span of the jump and the factors you have mentioned.

The range  $R$  of projectile motion is given by:

$$R = (v_0^2 \sin 2\theta) / g$$

where:

$v_0$  = initial velocity

$\theta$  = angle of projection

$g$  = acceleration due to gravity

Since the time of flight also affects the jump, we express it as:

$$t = (2 v_0 \sin \theta) / g$$

The horizontal distance traveled during this time is:

$$R = v_0 \cos \theta \times t$$

$$R = v_0 \cos \theta \times (2 v_0 \sin \theta) / g$$

$$R = (2 v_0^2 \sin \theta \cos \theta) / g$$

Using the trigonometric identity  $\sin 2\theta = 2 \sin \theta \cos \theta$ :

$$R = (v_0^2 \sin 2\theta) / g$$

This equation shows that the range depends on the initial velocity, angle of takeoff, and acceleration due to gravity.

(d) A bullet is fired from a gun on the top of a cliff 140 m high with a velocity of 150 m/s at an elevation of  $30^\circ$  to the horizontal. Find the horizontal distance from the foot of a cliff to the point where the bullet lands on the ground.

Given:

Initial velocity,  $v_0 = 150$  m/s

Angle of projection,  $\theta = 30^\circ$

Height of the cliff,  $h = 140$  m

Acceleration due to gravity,  $g = 9.81$  m/s<sup>2</sup>

Resolving velocity components:

$$v_{0x} = v_0 \cos \theta = 150 \cos 30^\circ$$

$$v_{0x} \approx 150 \times 0.866$$

$$v_{0x} \approx 129.9 \text{ m/s}$$

$$v_{0y} = v_0 \sin \theta = 150 \sin 30^\circ$$

$$v_{0y} \approx 150 \times 0.5$$

$$v_{0y} \approx 75 \text{ m/s}$$

Time of flight is determined from vertical motion. Using the equation:

$$y = v_{0y} t + \frac{1}{2} g t^2$$

Taking downward as positive and setting  $y = -140$  m:



$$\begin{aligned}-140 &= 75t - \frac{1}{2}(9.81)t^2 \\ -140 &= 75t - 4.905t^2\end{aligned}$$

Rearrange into quadratic form:

$$4.905t^2 - 75t - 140 = 0$$

Solving for t using the quadratic formula:

$$\begin{aligned}t &= \frac{-(-75) \pm \sqrt{(-75)^2 - 4(4.905)(-140)}}{2(4.905)} \\ t &= \frac{75 \pm \sqrt{5625 + 2746.8}}{9.81} \\ t &= \frac{75 \pm \sqrt{8371.8}}{9.81} \\ t &= \frac{75 \pm 91.5}{9.81}\end{aligned}$$

Taking the positive root:

$$\begin{aligned}t &= (75 + 91.5) / 9.81 \\ t &\approx 16.97 \text{ s}\end{aligned}$$

Calculating horizontal range:

$$\begin{aligned}R &= v_{0x} \times t \\ R &= 129.9 \times 16.97 \\ R &\approx 2205 \text{ m}\end{aligned}$$

The bullet lands approximately 2205 m from the base of the cliff.

4. (a) Define simple harmonic motion.

Simple harmonic motion (SHM) is a type of periodic motion in which a body moves back and forth about a mean position under a restoring force that is directly proportional to the displacement and directed towards the equilibrium position. Mathematically, it is defined by the equation:

$$F = -kx$$

where:

F = restoring force

k = force constant

x = displacement from the equilibrium position

The motion follows the equation:

$$x = A \cos(\omega t + \phi)$$

where:

A = amplitude

$\omega$  = angular frequency

t = time

$\phi$  = phase constant

(b) (i) After what further time will the two pendulums be in step again?

Given:

Lengths of the pendulums:

$l_1 = 0.4 \text{ m}$

$l_2 = 0.6 \text{ m}$

The period of a simple pendulum is given by:

$$T = 2\pi \sqrt{l / g}$$

For the first pendulum:

$$T_1 = 2\pi \sqrt{(0.4 / 9.81)}$$

$$T_1 \approx 2\pi \sqrt{(0.0408)}$$

$$T_1 \approx 2\pi (0.202)$$

$$T_1 \approx 1.27 \text{ s}$$

For the second pendulum:

$$T_2 = 2\pi \sqrt{(0.6 / 9.81)}$$

$$T_2 \approx 2\pi \sqrt{(0.0612)}$$

$$T_2 \approx 2\pi (0.247)$$

$$T_2 \approx 1.55 \text{ s}$$

The least common multiple (LCM) of  $T_1$  and  $T_2$  determines the time after which the pendulums will be in step again:

$$\text{LCM}(T_1, T_2) \approx \text{LCM}(1.27, 1.55) \approx 5.0 \text{ s}$$

The pendulums will be in step again after approximately 5.0 s.

(ii) Find the number of oscillations made by each pendulum during the time in (i) above.

The number of oscillations for each pendulum is given by:

$$n = \text{total time} / \text{period}$$

For the first pendulum:

$$n_1 = 5.0 / 1.27$$

$$n_1 \approx 3.94 \approx 4 \text{ oscillations}$$

For the second pendulum:

$$n_2 = 5.0 / 1.55$$

$$n_2 \approx 3.23 \approx 3 \text{ oscillations}$$

Thus, the first pendulum completes 4 oscillations, and the second pendulum completes 3 oscillations in 5.0 s.

(c) Cite two examples of SHM which are of importance to everyday life experience.

1. Oscillations of a pendulum clock – The periodic motion of a pendulum in clocks helps in timekeeping by maintaining a regular interval.
2. Vibrations of a stretched guitar string – The harmonic motion of strings in musical instruments produces sound with specific frequencies, enabling the creation of music.

5. (a) What does one require in order to establish a scale of temperature?

To establish a scale of temperature, the following requirements are necessary:

- Fixed Reference Points – Two standard reference temperatures must be chosen, such as the freezing and boiling points of water ( $0^\circ\text{C}$  and  $100^\circ\text{C}$  in the Celsius scale).
- Thermometric Property – A measurable physical property that changes with temperature must be selected, such as the expansion of a liquid (mercury or alcohol), change in electrical resistance, or pressure of a gas.
- Calibration Method – A consistent method to divide the temperature range into equal intervals, such as the Celsius scale dividing the range between freezing and boiling points of water into 100 equal parts.
- Standardization – The temperature scale should be universally accepted and based on reproducible phenomena to ensure consistency in measurements.

(b) A copper-constantan thermocouple with its cold junction at  $0^\circ\text{C}$  had an emf of 4.28 mV when its other hot junction was at  $100^\circ\text{C}$ . The emf became 9.29 mV when the temperature of the hot junction was  $200^\circ\text{C}$ . If the emf  $E$  is related to the temperature difference  $\theta$  between hot and cold junctions by the equation  $E = A\theta + B\theta^2$ , calculate:

(i) The values of  $A$  and  $B$ .

Given data:

$$E_1 = 4.28 \text{ mV at } \theta_1 = 100^\circ\text{C}$$

$$E_2 = 9.29 \text{ mV at } \theta_2 = 200^\circ\text{C}$$

The equation for emf is:

$$E = A\theta + B\theta^2$$

Substituting values:

For  $\theta_1 = 100^\circ\text{C}$ :

$$4.28 = A(100) + B(100)^2$$

$$4.28 = 100A + 10000B \quad \dots (1)$$

For  $\theta_2 = 200^\circ\text{C}$ :

$$9.29 = A(200) + B(200)^2$$

$$9.29 = 200A + 40000B \quad \dots (2)$$

Solving equations (1) and (2) simultaneously:

Multiply equation (1) by 2:

$$8.56 = 200A + 20000B$$

Subtract from equation (2):

$$(9.29 - 8.56) = (200A + 40000B) - (200A + 20000B)$$

$$0.73 = 20000B$$

$$B = 0.73 / 20000$$

$$B = 3.65 \times 10^{-5}$$

Substituting B into equation (1):

$$4.28 = 100A + (10000 \times 3.65 \times 10^{-5})$$

$$4.28 = 100A + 0.365$$

$$100A = 4.28 - 0.365$$

$$100A = 3.915$$

$$A = 3.915 / 100$$

$$A = 0.03915$$

The values are:

$$A = 0.03915$$

$$B = 3.65 \times 10^{-5}$$

(ii) The range of temperature for which E may be assumed proportional to  $\theta$  without incurring an error of more than 1%.

The emf equation is:

$$E = A\theta + B\theta^2$$

For E to be proportional to  $\theta$ , the quadratic term  $B\theta^2$  must contribute at most 1% of the total emf:

$$B\theta^2 \leq 0.01(A\theta)$$

Dividing both sides by  $\theta$ :

$$B\theta \leq 0.01A$$

Substituting  $A = 0.03915$  and  $B = 3.65 \times 10^{-5}$ :

$$(3.65 \times 10^{-5})\theta \leq 0.01(0.03915)$$

$$\theta \leq (0.0003915) / (3.65 \times 10^{-5})$$

$$\theta \leq 10.72^\circ\text{C}$$

The temperature range for which the emf is approximately proportional to  $\theta$  without more than 1% error is  $0^\circ\text{C}$  to about  $10.72^\circ\text{C}$ .

(c) The resistance  $R_t$  of a platinum wire varies with temperature  $t$  according to the equation

$$R_t = R_0(1 + 8000t - bt^2)$$

where  $b$  is a constant. Calculate the temperature on the platinum scale corresponding to  $400^\circ\text{C}$  on the gas scale.

The temperature on the platinum scale,  $t$ , corresponds to  $T = 400^\circ\text{C}$  on the gas scale. Since at low temperatures, the platinum scale and the gas scale are assumed to be the same, we assume the equation holds for  $T = 400^\circ\text{C}$ .

At  $T = 0^\circ\text{C}$ ,  $R_0$  is the resistance. The relationship between the platinum scale and the gas scale is found by solving for  $t$ .

Using the given equation:

$$R_{400} = R_0(1 + 8000t - bt^2)$$

For an ideal thermometer,  $R_{400}$  should be proportional to  $T$ , meaning:

$$R_{400} = R_0(1 + 8000 \times 400 - b \times 400^2)$$

Rearranging:

$$1 + 8000t - bt^2 = 1 + 8000(400) - b(400)^2$$

Solving for  $t$ ,

$$8000t - bt^2 = 8000(400) - b(400)^2$$

Since the equation describes a second-degree polynomial, we solve for  $t$  assuming the relation holds linearly for small variations. For exact calculations,  $b$  must be known.

This requires experimental values for  $b$ , so the final temperature on the platinum scale depends on  $b$ 's value.

6. (a) Define the thermal conductivity of a material.

Thermal conductivity is the property of a material that determines its ability to conduct heat. It is defined as the amount of heat transferred per unit time through a unit area of a material, per unit temperature gradient.

Mathematically, it is given by:

$$Q = -k A (dT/dx)$$

where:

$Q$  = rate of heat transfer

$k$  = thermal conductivity of the material

$A$  = cross-sectional area

$dT/dx$  = temperature gradient along the material

The negative sign indicates that heat flows from higher to lower temperature regions.

(b) Write down a formula for the rate of cooling under natural convection and define all the symbols used.

The rate of cooling under natural convection is given by Newton's Law of Cooling:

$$dT/dt = -h A (T - T_a) / m c$$

where:

$dT/dt$  = rate of temperature change

$h$  = heat transfer coefficient

$A$  = surface area of the object

$T$  = temperature of the body

$T_a$  = ambient temperature

$m$  = mass of the object

$c$  = specific heat capacity of the material

(c) Heat is supplied at a rate of 80 W to one end of a well-lagged copper bar of uniform cross-section area  $10 \text{ cm}^2$  having a total length of 20 cm. The heat is removed by water cooling at the other end of the bar. Temperature recorded by two thermometers  $T_1$  and  $T_2$  at distances 5 cm and 15 cm from the hot end are  $48^\circ\text{C}$  and  $28^\circ\text{C}$  respectively.

(i) Calculate the thermal conductivity of copper.

The heat conduction equation is given by Fourier's law:

$$Q = -k A (dT/dx)$$

Rearranging for  $k$ :

$$k = Q / (A (dT/dx))$$

Given data:

$$Q = 80 \text{ W}$$

$$A = 10 \text{ cm}^2 = 10 \times 10^{-4} \text{ m}^2$$

$$T_1 = 48^\circ\text{C}$$

$$T_2 = 28^\circ\text{C}$$

$$x_1 = 5 \text{ cm} = 0.05 \text{ m}$$

$$x_2 = 15 \text{ cm} = 0.15 \text{ m}$$

Temperature gradient:

$$dT/dx = (T_2 - T_1) / (x_2 - x_1)$$

$$= (28 - 48) / (0.15 - 0.05)$$

$$= (-20) / (0.10)$$

$$= -200^\circ\text{C/m}$$

Substituting values:

$$k = 80 / (10 \times 10^{-4} \times 200)$$

$$k = 80 / (0.2)$$

$$k = 400 \text{ W/mK}$$

The thermal conductivity of copper is 400 W/mK.

(ii) Estimate the rate of flow (in g/min) of cooling water sufficient for the water temperature to rise 5K.

The heat lost by the copper bar is absorbed by the water, given by:

$$Q = m c \Delta T$$

Rearranging for m:

$$m = Q / (c \Delta T)$$

Given:

$c = 4200 \text{ J/kgK}$  (specific heat capacity of water)

$\Delta T = 5 \text{ K}$

$Q = 80 \text{ J/s}$

$$m = 80 / (4200 \times 5)$$

$$m = 80 / 21000$$

$$m = 0.00381 \text{ kg/s}$$

Converting to g/min:

$$m = 0.00381 \times 1000 \times 60$$

$$m \approx 229 \text{ g/min}$$

The required flow rate of cooling water is approximately 229 g/min.

(iii) What is the temperature at the cold end of the bar?

The temperature distribution along a conducting bar follows the linear relation:

$$T = T_1 + (x / L) (T_2 - T_1)$$

At the cold end ( $x = 20 \text{ cm} = 0.20 \text{ m}$ ):

$$T_0 = 48 + (0.20 / 0.20) \times (28 - 48)$$

$$T_0 = 48 + (1 \times -20)$$

$$T_0 = 28^\circ\text{C}$$

The temperature at the cold end of the bar is  $28^\circ\text{C}$ .

I will extract the questions from the image and continue solving them sequentially in the same format as before. This will take some time. I will update you once I complete each section.



7. (a) What vibrates in the following types of wave motion?

- (i) Light waves – Electric and magnetic fields oscillate perpendicular to each other and to the direction of wave propagation.
- (ii) Sound waves – Air molecules or particles in a medium vibrate back and forth in the direction of wave propagation.
- (iii) X-rays – Electric and magnetic fields oscillate, similar to light waves, as X-rays are a form of electromagnetic radiation.
- (iv) Water waves – Water molecules move in circular or elliptical paths due to the combination of transverse and longitudinal motion.

(b) A plane progressive wave on a water surface is given by the equation  $y = 2 \sin 2\pi(100t - x / 30)$ ; where  $x$  is the distance covered in a time  $t$ .  $x$ ,  $y$ , and  $t$  are in cm and seconds respectively.

Find:

- (i) The wavelength, and frequency of the wave motion.

A general wave equation is given as:

$$y = A \sin 2\pi(ft - x / \lambda)$$

Comparing with the given equation:

$$y = 2 \sin 2\pi(100t - x / 30)$$

The frequency  $f = 100$  Hz and the wavelength  $\lambda = 30$  cm.

- (ii) The phase difference between two points on the water surface that are 60 cm apart.

The phase difference  $\Delta\phi$  is given by:

$$\Delta\phi = (2\pi / \lambda) \Delta x$$

Substituting values:

$$\Delta\phi = (2\pi / 30) \times 60$$

$$\Delta\phi = 4\pi \text{ radians}$$

- (c) (i) Show how wavelength and frequency of a wave are related.

The relationship between wavelength  $\lambda$ , frequency  $f$ , and wave speed  $v$  is given by:

$$v = f \lambda$$

This equation shows that the speed of a wave is the product of its frequency and wavelength.

(ii) Two open organ pipes of length 50 cm and 51 cm respectively give beat frequency of 6.0 Hz when sounding their fundamental notes together, neglecting end corrections. What value does this give for the velocity of sound in air?

For an open pipe, the fundamental frequency is given by:

$$f = v / 2L$$

For the two pipes:

$$f_1 = v / (2 \times 0.50)$$

$$f_2 = v / (2 \times 0.51)$$

The beat frequency is:

$$|f_1 - f_2| = 6 \text{ Hz}$$

Substituting the frequency expressions:

$$|(v / 1.00) - (v / 1.02)| = 6$$

Taking v as the common factor:

$$v [(1 / 1.00) - (1 / 1.02)] = 6$$

$$v [0.02 / (1.00 \times 1.02)] = 6$$

$$v \times (0.0196) = 6$$

$$v = 6 / 0.0196$$

$$v \approx 306 \text{ m/s}$$

The velocity of sound in air is approximately 306 m/s.

8. (a) (i) What is electric potential at a point in an electrostatic field?

Electric potential at a point is the amount of work done in bringing a unit positive charge from infinity to that point in the field without acceleration. It is measured in volts (V).

(ii) Derive an expression for an electric potential at a point a distance r from a positive point charge Q.

The electric potential V at a distance r from a point charge Q is given by:

$$V = W / q$$

The work done in bringing a small charge  $q$  from infinity to distance  $r$  is obtained by integrating the electric field:

$$dW = F dr = (q E) dr$$

Since  $E = kQ / r^2$ , we integrate from  $r$  to  $\infty$ :

$$V = \int (kQ / r^2) dr$$

$$V = kQ \int (1 / r^2) dr$$

$$V = kQ (-1 / r) \big| \text{from } r \text{ to } \infty$$

$$V = kQ (0 - (-1 / r))$$

$$V = kQ / r$$

Thus, the electric potential at a distance  $r$  from a charge  $Q$  is:

$$V = (1 / 4\pi\epsilon_0) \times (Q / r)$$

(b) Positive charge is distributed over a solid spherical volume of radius  $R$  and the charge per unit volume is  $\sigma$ .

(i) Show that the electric field inside the volume at a distance  $r < R$  from the centre is given by  $E = \sigma r / 3\epsilon_0$ .

Using Gauss's Law:

$$\oint E dA = Q_{\text{enclosed}} / \epsilon_0$$

The charge enclosed within a sphere of radius  $r$  is:

$$Q_{\text{enclosed}} = \sigma \times (4/3) \pi r^3$$

The Gaussian surface for a sphere has area  $A = 4\pi r^2$ , so:

$$E \times 4\pi r^2 = (\sigma \times (4/3) \pi r^3) / \epsilon_0$$

$$E = (\sigma r / 3\epsilon_0)$$

(ii) What is the electric field at a point  $r > R$  (i.e. outside the spherical volume).

For  $r > R$ , the charge behaves as if it were concentrated at the center, so:

$$E = (1 / 4\pi\epsilon_0) \times (Q / r^2)$$

$$\text{Since } Q = \sigma \times (4/3) \pi R^3,$$

$$E = (\sigma \times (4/3) \pi R^3) / (4\pi\epsilon_0 r^2)$$

$$E = (\sigma R^3) / (3\epsilon_0 r^2)$$

9. (a) What is meant by the terms electrical resistivity and ohmic conductor?

Electrical resistivity is a material property that quantifies how strongly a material opposes the flow of electric current. It is given by  $\rho = R A / L$ , where  $R$  is resistance,  $A$  is cross-sectional area, and  $L$  is length.

An ohmic conductor is a material that obeys Ohm's law, meaning that its voltage and current are directly proportional, and its resistance remains constant regardless of the applied voltage.

(b) A 4m long resistance wire has a cross-sectional area of  $0.8\text{mm}^2$  and a resistance of  $2.80\Omega$ .

(i) Determine the resistivity of the wire.

Using  $\rho = R A / L$ :

$$\rho = (2.80 \times 0.8 \times 10^{-6}) / 4$$

$$\rho = (2.24 \times 10^{-6}) / 4$$

$$\rho = 5.6 \times 10^{-7} \Omega\text{m}$$

(ii) Determine the length of a similar wire which when joined in parallel will give a total resistance of  $2.0\Omega$ .

For parallel resistance:

$$1 / R_{\text{total}} = 1 / R_1 + 1 / R_2$$

$$1 / 2 = 1 / 2.8 + 1 / R_2$$

$$1 / R_2 = 1 / 2 - 1 / 2.8$$

$$1 / R_2 \approx 0.1786$$

$$R_2 \approx 5.6 \Omega$$

Using  $R = \rho L / A$ :

$$L = (R A) / \rho$$

$$L = (5.6 \times 0.8 \times 10^{-6}) / (5.6 \times 10^{-7})$$

$$L = 8 \text{ m}$$

10. (a) An electron with charge  $e$  and mass  $m_e$  is initially projected with a speed  $v$  at right angles to a uniform magnetic field of flux density  $B$ .

(i) Explain why the path of the electron is circular.

The magnetic force on the electron is given by  $F = e v B$ . This force acts perpendicular to the velocity, causing the electron to move in a circular path due to centripetal force:

$$e v B = m_e v^2 / r$$

(ii) Show that the time to describe one complete circle is independent of the speed of the electron.

The time for one cycle is:

$$T = (2\pi r) / v$$

From  $e v B = m_e v^2 / r$ , solving for  $r$ :

$$r = m_e v / (e B)$$

Substituting in  $T$ :

$$T = (2\pi m_e v / e B) / v$$

$$T = 2\pi m_e / (e B)$$

This shows that  $T$  is independent of  $v$ .

(b) Calculate the radius of the path traversed by an electron of energy 450 eV moving at right angles to a uniform magnetic field of flux density  $1.5 \times 10^{-3} \text{ T}$ .

The radius of the circular path of a charged particle in a magnetic field is given by:

$$r = (m v) / (e B)$$

First, determine the velocity of the electron using the energy equation:

$$\text{Kinetic energy, } K = \frac{1}{2} m_e v^2$$

Solving for  $v$ :

$$v = \sqrt{(2K / m_e)}$$

Given:

$$K = 450 \text{ eV} = 450 \times 1.6 \times 10^{-19} \text{ J}$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$B = 1.5 \times 10^{-3} \text{ T}$$

$$v = \sqrt{(2 \times (450 \times 1.6 \times 10^{-19}) / (9.11 \times 10^{-31}))}$$

$$v = \sqrt{(1.44 \times 10^{-16} / 9.11 \times 10^{-31})}$$

$$v = \sqrt{(1.58 \times 10^{14})}$$

$$v \approx 1.26 \times 10^7 \text{ m/s}$$

Now, calculating the radius:

$$r = (9.11 \times 10^{-31} \times 1.26 \times 10^7) / (1.6 \times 10^{-19} \times 1.5 \times 10^{-3})$$

$$r = (1.15 \times 10^{-23}) / (2.4 \times 10^{-22})$$

$$r \approx 4.8 \times 10^{-2} \text{ m}$$

The radius of the path traversed by the electron is approximately **4.8 cm**.

11. (a) Distinguish between metals and semiconductors in terms of energy bands.

Metals have a partially filled conduction band or overlapping valence and conduction bands, allowing free movement of electrons and high electrical conductivity.

Semiconductors have a small energy gap ( $\approx 1 \text{ eV}$ ) between the valence and conduction bands. At low temperatures, electrons remain in the valence band, but at higher temperatures or with doping, some electrons gain enough energy to jump to the conduction band, allowing controlled conductivity.

(b) Briefly discuss the formation of the potential difference barrier (depletion layer) of a p-n junction diode.

When a p-type semiconductor (with holes as majority carriers) and an n-type semiconductor (with electrons as majority carriers) are joined, free electrons from the n-region diffuse into the p-region and recombine with holes. This creates a depletion region near the junction, devoid of mobile charge carriers, leaving behind immobile positive ions in the n-region and negative ions in the p-region. The electric field formed opposes further diffusion, leading to the formation of a potential difference barrier, preventing further carrier movement unless an external voltage is applied.

(c) (i) What is a rectifier?

A rectifier is an electrical device that converts alternating current (AC) into direct current (DC). It allows current to flow in only one direction using semiconductor diodes.

(ii) Using p-n junction diodes, draw the arrangement of a full-wave rectifier and briefly explain how it works.

A full-wave rectifier consists of two diodes connected in a center-tap transformer or four diodes in a bridge configuration. In each half-cycle of AC input, one set of diodes conducts, allowing only positive polarity to appear across the load, thus converting AC to pulsating DC.

12. (a) Define the electron-volt.

An electron-volt (eV) is the amount of energy gained or lost by an electron when it moves through a potential difference of one volt. It is equal to  $1.6 \times 10^{-19}$  joules.

(b) Electrons in a certain television tube are accelerated through a potential difference of 2.0 kV.

(i) Calculate the velocity acquired by the electrons.

Using the energy relation:

$$KE = eV = \frac{1}{2} m v^2$$

Solving for v:

$$v = \sqrt{(2eV / m)}$$

Given:

$$V = 2.0 \times 10^3 \text{ V}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$m = 9.11 \times 10^{-31} \text{ kg}$$

$$v = \sqrt{(2 \times 1.6 \times 10^{-19} \times 2000 / 9.11 \times 10^{-31})}$$

$$v = \sqrt{(6.4 \times 10^{-16} / 9.11 \times 10^{-31})}$$

$$v = \sqrt{(7.02 \times 10^{14})}$$

$$v \approx 8.38 \times 10^7 \text{ m/s}$$

(ii) If these electrons lose all their energy on impact and given that  $10^{12}$  electrons pass per second in the TV tube, calculate the power dissipated.

Power dissipated is given by:

$$P = \text{energy per electron} \times \text{number of electrons per second}$$

$$P = (eV) \times N$$

$$P = (1.6 \times 10^{-19} \times 2000) \times 10^{12}$$

$$P = (3.2 \times 10^{-16}) \times 10^{12}$$

$$P = 3.2 \times 10^{-4} \text{ W}$$

(c) (i) Explain why audio amplification is necessary for a practical radio set.

Audio amplification is necessary because the signal received by a radio antenna is weak and cannot directly drive a speaker. An amplifier increases the amplitude of the audio signal, making it audible with sufficient power to drive loudspeakers efficiently.

(ii) A coil and a capacitor in parallel are used to make a tuning circuit for a radio receiver. Sketch the resonance curve for the circuit. State two ways of changing the circuit to increase the resonant frequency.

The resonance curve shows a peak at the resonant frequency, where the circuit achieves maximum current due to equal inductive and capacitive reactance.

Ways to increase resonant frequency:

- Decreasing the capacitance of the capacitor
- Decreasing the inductance of the coil

13. (a) Mention any three uses of a CRO.

- Measuring voltage and current waveforms in electrical circuits
- Displaying and analyzing frequency and phase relationships in signals
- Monitoring and testing communication signals and radio frequencies

(b) A proton is placed in a uniform electric field  $E$ . What must be the magnitude and direction of the field if the electrostatic force acting on the proton is just to balance its weight?

For equilibrium:

$$F_e = F_g$$

$$eE = mg$$

$$E = mg / e$$

Given:

$$m = 1.67 \times 10^{-27} \text{ kg}$$

$$g = 9.81 \text{ m/s}^2$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$E = (1.67 \times 10^{-27} \times 9.81) / (1.6 \times 10^{-19})$$

$$E = (1.64 \times 10^{-26}) / (1.6 \times 10^{-19})$$

$$E \approx 1.03 \times 10^{-7} \text{ V/m}$$

The direction of the field must be upward to oppose the downward gravitational force.

(c) A small charged oil drop is allowed to fall under gravity in the Millikan experiment, it is then made to remain stationary under the application of an electric field. Show that the charge  $Q$  of the oil drop is given by

$$Q = (6\pi\eta / E) \times \sqrt{((9\eta v) / (2(\rho_o - \rho_a)g) \times (v - v'))}$$

Using Stoke's law for terminal velocity:

$$F_v = 6\pi\eta r v$$

For equilibrium in the electric field,

$$Q E = 6\pi\eta r v$$



Solving for Q:

$$Q = (6\pi\eta r v) / E$$

Using the density relation and solving for r in terms of v and g,

$$r = \sqrt[3]{(9\eta v) / (2(\rho_0 - \rho_a)g)}$$

Substituting r in the equation for Q,

$$Q = (6\pi\eta / E) \times \sqrt[3]{(9\eta v) / (2(\rho_0 - \rho_a)g)} \times (v - v')$$

14. (a) With reference to an earthquake on a certain point of the earth explain the terms ‘Focus’ and ‘Epicentre’.

Focus: The point inside the Earth where an earthquake originates due to the sudden release of stress along a fault line.

Epicentre: The point on the Earth's surface directly above the focus where the earthquake's effects are strongest.

(b) What is the importance of the following layers of the atmosphere?

(i) The lowest layer

This is the troposphere, where weather events such as rain, wind, and storms occur. It contains most of the atmospheric water vapor and is crucial for sustaining life.

(ii) The Ionosphere

The ionosphere contains charged particles that reflect and transmit radio waves, making it essential for long-distance communication and GPS signal transmission.

(c) (i) Describe two ways by which seismic waves may be produced.

- Naturally occurring earthquakes due to tectonic plate movement along faults.
- Artificial explosions such as nuclear tests or controlled blasting in mines.

(ii) Describe briefly the meaning and application of “seismic prospecting.”

Seismic prospecting is a geophysical method used to explore underground structures by analyzing the propagation of seismic waves through different rock layers. It is widely used in oil and gas exploration, mineral prospecting, and studying geological formations.