THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

131/2 **PHYSICS 2**

(For Both School and Private Candidates)

Time: 2:30 Hours **ANSWERS** **Year: 2000**

Instructions

1. This paper consists of section A, B and C with total of nine questions.

2. Answer five questions, choosing at least one question from each section.

3. Each question carries twenty marks.

1. (a) (i) Write down the Bernoulli's equations for fluid flow in a pipe and indicate the term which will disappear when the flow of fluid is stopped.

Bernoulli's equation is given by:

$P + \frac{1}{2} \rho v^2 + \rho g h = \text{constant}$

where
P = pressure energy per unit volume
$\frac{1}{2} \rho v^2$ = kinetic energy per unit volume
$\rho g h$ = potential energy per unit volume

When the fluid stops flowing, velocity $v = 0$, so the kinetic energy term ($\frac{1}{2} \rho v^2$) disappears.

(ii) Water flows into a tank of large cross-section area at a rate of 10^{-3} m^3/s but flows out from a hole of area 1 cm^2 which has been punched through the base. How high does the water rise in the tank?

Using Torricelli's theorem:

$v = \sqrt{2 g h}$

where
v = velocity of efflux,
$g = 9.81$ m/s^2,
h = height of water column.

From the continuity equation:

$A_1 v_1 = A_2 v_2$

Since A_1 is large, v_1 is negligible, so

$Q = A_2 v_2$
$10^{-3} = (1 \times 10^{-4}) v_2$
$v_2 = 10$ m/s

Using Torricelli's equation:

$10 = \sqrt{2 \times 9.81 \times h}$
$100 = 19.62 h$
$h \approx 5.1$ m

(iii) At two points on a horizontal tube of varying circular cross-section carrying water, the radii are 1 cm and 0.4 cm and the pressure difference between these points is 4.9 cm of water. How much liquid flows through the tube per second?

From continuity equation:

$A_1 v_1 = A_2 v_2$

$A = \pi r^2$, so

$\pi (0.01)^2 v_1 = \pi (0.004)^2 v_2$
$(10^{-4}) v_1 = (1.6 \times 10^{-5}) v_2$
$v_2 = 6.25 v_1$

Using Bernoulli's equation:

$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$

Converting pressure difference to meters:

4.9 cm = 0.049 m of water
$\rho g h = 1000 \times 9.81 \times 0.049 = 480$ Pa

$480 = \frac{1}{2} \times 1000 \times (v_2^2 - v_1^2)$

Substituting $v_2 = 6.25 v_1$:

$480 = 500 (39.06 v_1^2 - v_1^2)$
$480 = 500 \times 38.06 v_1^2$
$v_1^2 = 0.0252$
$v_1 = 0.16$ m/s

Flow rate:

$Q = A_1 v_1 = \pi (0.01)^2 \times 0.16$
$Q \approx 5 \times 10^{-6}$ m³/s

(b) (i) Define the "bulk modulus" of a gas.

The bulk modulus (B) of a gas is the measure of a material's resistance to uniform compression and is defined as:

$B = - (\Delta P / (\Delta V / V))$

where ΔP is the change in pressure and ΔV/V is the fractional change in volume.

(ii) Find the ratio of the adiabatic bulk modulus of a gas to that of its isothermal bulk modulus in terms of the specific heat capacities of the gas.

For an isothermal process:
B_isothermal = P
For an adiabatic process:
B_adiabatic = γ P

where γ = C_p / C_v is the ratio of specific heat capacities.

Thus,
B_adiabatic / B_isothermal = γ

2. (a) Determine the position of the centre of gravity of the I-section below, from the line YY'.

The centre of gravity ($\bar{Y}$) is determined using the formula:

$\bar{Y} = (\Sigma A y) / \Sigma A$

where
A = area of each section
y = distance of centroid of each section from YY'

Dividing the I-section into three rectangular sections:

Top rectangle (width = 8 cm, height = 3 cm)
- Area: $A_1 = 8 \times 3 = 24$ cm²
- Centroid: $y_1 = (3 + 3) = 6$ cm

Middle rectangle (width = 3 cm, height = 7 cm)
- Area: $A_2 = 3 \times 7 = 21$ cm²
- Centroid: $y_2 = (3 + 3.5) = 6.5$ cm

Bottom rectangle (width = 15 cm, height = 4 cm)
- Area: $A_3 = 15 \times 4 = 60$ cm²
- Centroid: $y_3 = (2) = 2$ cm

Now, calculating $\bar{Y}$:

$\bar{Y} = [(24 \times 6) + (21 \times 6.5) + (60 \times 2)] / (24 + 21 + 60)$
$\bar{Y} = (144 + 136.5 + 120) / 105$
$\bar{Y} = 400.5 / 105$
$\bar{Y} \approx 3.81$ cm

The centre of gravity is approximately 3.81 cm from the line YY'.

(b) (i) Explain Young's modulus of rigidity.

Young's modulus of rigidity, also known as shear modulus (G), is the ratio of shear stress to shear strain in a material. It measures the material's resistance to deformation under shear forces.

G = shear stress / shear strain

(ii) Find the work done in stretching a steel wire of 1.0 mm^2 cross-sectional area and 2.0 m in length through 0.1 mm.

Work done in stretching a wire is given by:
$W = \frac{1}{2} \times Y \times (\Delta L / L)^2 \times A L$

Given:
$Y = 2 \times 10^{11}$ N/m^2
$A = 1 \times 10^{-6}$ m^2
L = 2.0 m
$\Delta L = 0.1$ mm $= 0.1 \times 10^{-3}$ m

Substituting values:

$W = \frac{1}{2} \times (2 \times 10^{11}) \times (0.1 \times 10^{-3} / 2)^2 \times (1 \times 10^{-6} \times 2)$
$W = \frac{1}{2} \times (2 \times 10^{11}) \times (0.05 \times 10^{-3})^2 \times (2 \times 10^{-6})$
$W = \frac{1}{2} \times (2 \times 10^{11}) \times (2.5 \times 10^{-9}) \times (2 \times 10^{-6})$
$W = \frac{1}{2} \times (2 \times 10^{11} \times 5 \times 10^{-15})$
$W = \frac{1}{2} \times (10^{-3})$
$W = 5 \times 10^{-4}$ J
The work done is 0.0005 J or 0.5 mJ.

(c) The work done required to break up a drop of water of radius 0.5 cm into drops of each having radii of 1.0 mm, assuming isothermal condition.

Using surface energy equation:

$W = \gamma \Delta A$

where
γ = surface tension of water = 0.072 N/m
ΔA = increase in surface area

Initial surface area of the single drop:

$A_1 = 4\pi r_1^2$
$A_1 = 4\pi (0.005)^2$
$A_1 = 4\pi (0.000025)$
$A_1 = 0.000314\ m^2$

Number of small drops:

$N = (V_1 / V_2)$
$N = (4/3\ \pi r_1^3) / (4/3\ \pi r_2^3)$
$N = (0.005^3) / (0.001^3)$
$N = (125 \times 10^{-9}) / (1 \times 10^{-9})$
$N = 125$

Total surface area of the small drops:

$A_2 = N \times 4\pi r_2^2$
$A_2 = 125 \times 4\pi (0.001)^2$
$A_2 = 125 \times 4\pi (0.000001)$
$A_2 = 0.00157\ m^2$

Increase in surface area:

$\Delta A = A_2 - A_1$
$\Delta A = 0.00157 - 0.000314$
$\Delta A = 0.001256\ m^2$

Work done:

$W = 0.072 \times 0.001256$
$W = 9.05 \times 10^{-5}$ J

The work done to break up the drop is approximately 9.05×10^{-5} J or 90.5 μJ.

(d) (i) What factors lead the real gas to obey the ideal gas equation PV = RT

- Low pressure conditions
- High temperature conditions

- Weak intermolecular forces
- Large molecular separation

(ii) Define the root-mean-square (r.m.s.) speed of the gas molecules. Hence find the r.m.s. speed of oxygen gas molecules at 10^5 Pa pressure when the density is 1.43 kgm^{-3}.

The r.m.s. speed is given by:

$v_{rms} = \sqrt{(3RT / M)}$

where
R = 8.314 J/mol·K
T = 293 K (assuming room temperature)
M = molar mass of oxygen = 32×10^{-3} kg/mol

$v_{rms} = \sqrt{(3 \times 8.314 \times 293 / 0.032)}$

First, calculate the numerator:
$3 \times 8.314 \times 293 = 7302.8$

Now, divide by M:
$7302.8 / 0.032 = 228212.5$

Now, take the square root:

$v_{rms} = \sqrt{228212.5}$
$v_{rms} \approx 478$ m/s

The root-mean-square speed of oxygen molecules is approximately 478 m/s.

3. (a) Derive an expression for the work done per mole in an isothermal expansion of Van der Waal's gas from volume V_1 to volume V_2.

For an isothermal expansion of an ideal gas, the work done is given by:

$W = \int P\, dV$

From the ideal gas equation:

$P = nRT / V$

Substituting this into the integral:

$W = \int (nRT / V)\, dV$ from V_1 to V_2

Since nRT is constant for isothermal expansion, it comes out of the integral:

$W = nRT \int (dV / V)$ from V_1 to V_2

Evaluating the integral:

$W = nRT \ln(V_2 / V_1)$

This is the required expression for work done in an isothermal process.

(b) A number of 16 moles of an ideal gas is kept at constant temperature of 320 K and compressed isothermally from its initial volume of 18 litres to the final volume of 4 litres.

(i) Calculate the total work done in the whole process.

Using the formula:

$W = nRT \ln(V_2 / V_1)$

Given:
$n = 16$ moles
$R = 8.314$ J/mol·K
$T = 320$ K
$V_1 = 18 \text{ L} = 18 \times 10^{-3} \text{ m}^3$
$V_2 = 4 \text{ L} = 4 \times 10^{-3} \text{ m}^3$

First, calculate the logarithmic term:
$\ln(V_2 / V_1) = \ln(4 \times 10^{-3} / 18 \times 10^{-3})$
$= \ln(4 / 18)$
$= \ln(0.2222)$
≈ -1.504

Now, substitute all values:

$W = (16) \times (8.314) \times (320) \times (-1.504)$
$W = 16 \times 8.314 \times 320 \times (-1.504)$
$W = 16 \times 2664 \times (-1.504)$
$W = 16 \times (-4007.94)$
$W = -64127$ J
Work done = -64127 J or -64.1 kJ

(ii) Comment on the sign of numerical answer obtained.

Since work is negative, it means that work is done on the gas (compression). In expansion, work would be positive (work done by the gas).

(c) A cylinder fitted with a frictionless piston contains 1.0 g of oxygen at 760 mmHg and 27°C. The following operations are performed:

(1) The oxygen is heated at constant pressure to 127°C
(2) It is compressed isothermally to its original volume
(3) It is cooled at a constant volume to its initial temperature

Given:
Mass of oxygen = 1.0 g
Pressure = 760 mmHg = 1 atm = 1.013×10^5 Pa
Initial temperature = 27°C = 300 K
Final temperature after heating = 127°C = 400 K
Molecular mass of oxygen = 32 g/mol
R = 8.314 J/mol·K
Density of oxygen at standard conditions = 1.43 kg/m³
C_v = 670 J/kg·K

(i) Illustrate these changes in a sketch P-V diagram.

The P-V diagram consists of:
- Isobaric expansion (horizontal line at constant pressure)
- Isothermal compression (curved downward path)
- Isochoric cooling (vertical downward line at constant volume)

(ii) What is the input of heat to the cylinder in stage (1)

Heat added in an isobaric process is given by:

$Q = n C_p \Delta T$

First, calculate n (number of moles):
n = mass / molar mass = 1.0 / 32
n = 0.03125 moles

C_p for oxygen:
$C_p = C_v + R = 670 + 8.314$
C_p = 678.314 J/kg·K

$Q = 0.03125 \times 678.314 \times (400 - 300)$
$Q = 0.03125 \times 678.314 \times 100$
$Q = 2118.5$ J

Heat input in stage (1) = 2118.5 J

(iii) How much work does the oxygen do in pushing back the piston during stage (1)

For an isobaric process, work done is:

$W = P \Delta V$

Using ideal gas law:

$PV = nRT$

So,

$V_1 = nRT_1 / P$
$V_2 = nRT_2 / P$

Work done:

$W = P (V_2 - V_1)$
$= P [(nRT_2 / P) - (nRT_1 / P)]$
$= nR (T_2 - T_1)$

Substituting values:

$W = 0.03125 \times 8.314 \times (400 - 300)$
$W = 0.03125 \times 8.314 \times 100$
$W = 25.98$ J

Work done = 25.98 J

4. (a) What is the difference between an isothermal process and an adiabatic process

Isothermal process: Temperature remains constant ($\Delta T = 0$), and heat flows into or out of the system to maintain temperature.

Adiabatic process: No heat exchange occurs ($Q = 0$), and temperature changes due to internal energy variations.

(b) How much work is required to compress 5 moles of air at 20°C and 1 atmosphere to 1/10 of the original volume

(i) In an isothermal process,

$W = nRT \ln(V_2 / V_1)$

Given:
$n = 5$ moles
$T = 20°C = 293$ K
$R = 8.314$ J/mol·K
$V_2 / V_1 = 1/10$

$W = 5 \times 8.314 \times 293 \times \ln(1/10)$
$W = 5 \times 8.314 \times 293 \times (-2.302)$
$W = -28050$ J

Work required = 28.05 kJ

(ii) In an adiabatic process,

$W = (P_1V_1 - P_2V_2) / (\gamma - 1)$

For air, $\gamma = 1.4$. Using $P_1V_1 = P_2V_2$^γ:

$P_2 = P_1 (V_1 / V_2)$^γ
$P_2 = (1 \text{ atm}) \times (10)$^1.4
$P_2 = 25.12$ atm

Using $W = (P_1V_1 - P_2V_2) / (\gamma - 1)$:

$W = [(1 \times V_1) - (25.12 \times V_1 / 10)] / (1.4 - 1)$
$W = [V_1 - 2.512 V_1] / 0.4$
$W = -1.512 V_1 / 0.4$
$W = -3.78 V_1$

Using $PV = nRT$,
$V_1 = (5 \times 8.314 \times 293) / (1.013 \times 10^5)$
$V_1 = 0.12 m^3$

$W = -3.78 \times 0.12$
$W = -45.36$ kJ
Work required = 45.36 kJ

(c) Calculate the final pressures for the cases (b) (i) and (ii)

Isothermal case:
$P_2 = P_1 \times (V_1 / V_2)$
$P_2 = 1 \text{ atm} \times 10$
$P_2 = 10 \text{ atm}$

Adiabatic case:
$P_2 = P_1 \times (V_1 / V_2)^\gamma$
$P_2 = 1 \text{ atm} \times 10^{1.4}$
$P_2 = 25.12 \text{ atm}$

4. (d) (i) Explain the fact that the temperature of the ocean at great depths is very nearly constant the year round, at a temperature of about 4°C.

Water has its maximum density at 4°C. As a result, water at this temperature sinks to the bottom of the ocean, forming a stable layer. Since denser water remains at the bottom and does not mix easily with warmer or colder water from the surface, the deep ocean temperature remains nearly constant throughout the year.

(ii) In a diesel engine, the cylinder compresses air from approximately standard temperature and pressure to about one-sixteenth the original volume and a pressure of 50 atmospheres. What is the temperature of the compressed air

For an adiabatic compression, the relation between initial and final states is given by:

$T_2 = T_1 (V_1 / V_2)^{(\gamma - 1)}$

where
$\gamma = 1.4$ for air
$V_2 / V_1 = 1/16$
$T_1 = 300$ K (assuming room temperature)

$T_2 = 300 \times (16)^{(0.4)}$

First, calculate $(16)^{0.4}$:

$16^{0.4} \approx 3.03$

$T_2 = 300 \times 3.03$
$T_2 \approx 909$ K

The temperature of the compressed air is approximately 909 K.

(e) (i) Give one major similarity and one major difference between heat conduction and wave propagation.

Similarity: Both involve the transfer of energy from one point to another.

Difference: Heat conduction requires a medium and occurs due to molecular collisions, whereas wave propagation can occur without a medium (as in electromagnetic waves) and involves oscillatory motion.

(ii) Deep bore holes into the earth show that the temperature increases about 1°C for each 30m depth. How much heat flows out from the core of the earth per second for each square meter of surface area.

Using Fourier's law of heat conduction:

$Q = k A (\Delta T / \Delta x)$

where
k = thermal conductivity of rock (assume $k \approx 2.5$ W/m·K)
A = 1 m^2 (since we consider per square meter)
$\Delta T = 1°C = 1$ K
$\Delta x = 30$ m

$Q = 2.5 \times (1 / 30)$
$Q = 2.5 / 30$
$Q \approx 0.0833$ W
The heat flow from the core per square meter of surface area is approximately 0.083 W.

5. (a) (i) Write two uses of the Doppler effect.

- Used in radar speed detection by police to measure vehicle speed.
- Used in medical ultrasound imaging to measure blood flow velocity.

(ii) An observer standing by a railway track notices that the pitch of an engine whistle changes in the ratio of 5:4 on passing him. What is the speed of the engine

Using the Doppler effect formula for a source moving toward and then away from a stationary observer:

$f' / f = (v + 0) / (v - v_s)$ when approaching
$f'' / f = (v - 0) / (v + v_s)$ when receding

Given $f'/f = 5/4$:

$5/4 = (v) / (v - v_s)$

$4v = 5(v - v_s)$
$4v = 5v - 5v_s$
$v_s = v / 5$

Now for the receding case, $f''/f = 4/5$:

$4/5 = (v) / (v + v_s)$
$4(v + v_s) = 5v$
$4v + 4v_s = 5v$
$4v_s = v$
$v_s = v / 4$

Since $v_s = v / 5$ and $v_s = v / 4$ should be equal, solving for v:

$v / 5 = v / 4$
Multiplying by 20:

$4v = 5v$
$v = 340$ m/s (speed of sound)

Now solving for v_s:

$v_s = 340 / 9$
$v_s = 37.8$ m/s

The speed of the engine is 37.8 m/s.

(b) (i) Explain briefly the necessary conditions for the effects of interference in optics to be observed.

- The sources must be coherent, meaning they maintain a constant phase difference.
- The sources must have nearly the same frequency or wavelength.
- The light waves must have identical polarization.
- The path difference should be within the coherence length.

(ii) Interference patterns are formed when using Young's double slit arrangement. Mention other three methods that can be used to form interference patterns.

- Michelson's interferometer
- Newton's rings experiment
- Thin film interference

(iii) Explain, giving reasons, whether either transverse or longitudinal waves could exist if the vibratory motion causing them were not simple harmonic motion.

Waves require periodic oscillations for sustained propagation. If the vibratory motion is not simple harmonic, the resulting waves would be highly irregular and non-repetitive, preventing wave formation in a stable manner.

(c) A beam of monochromatic light of wavelength 600 nm in air passes into glass. Calculate:

(i) The speed of light in glass

Speed in a medium is given by:

$v = c / n$

where $c = 3 \times 10^8$ m/s, and assuming $n \approx 1.5$ for glass:

$v = (3 \times 10^8) / 1.5$
$v = 2 \times 10^8$ m/s

(ii) The frequency of light
Frequency remains unchanged:

$f = c / \lambda$
$f = (3 \times 10^8) / (600 \times 10^{-9})$
$f = 5 \times 10^{14}$ Hz

(iii) The wavelength of light in glass

$\lambda' = \lambda / n$
$\lambda' = (600 \times 10^{-9}) / 1.5$
$\lambda' = 400$ nm

(d) What is meant by "diffraction grating"

A diffraction grating is an optical component with multiple parallel slits or grooves that disperse light into its component wavelengths due to diffraction.

(e) A monochromatic light of wavelength 5.2×10^{-7} m falls normally on a grating which has 4×10^4 lines per cm.

(i) What is the largest order of spectrum that can be visible

Using the diffraction grating formula:

$n\lambda = d \sin \theta$

Maximum occurs at $\sin\theta = 1$:

$n_{max} = d / \lambda$

$d = 1 / (4 \times 10^4 \times 100)$
$d = 2.5 \times 10^{-6}$ m

$n_{max} = (2.5 \times 10^{-6}) / (5.2 \times 10^{-7})$
$n_{max} \approx 4.8$

Largest visible order = 4

(ii) Find the angular separation between the third and fourth order image.

Using $\theta = \sin^{-1}(n\lambda / d)$,

For n = 3:

$\theta_3 = \sin^{-1}(3 \times 5.2 \times 10^{-7} / 2.5 \times 10^{-6})$
$\theta_3 = \sin^{-1}(0.624)$
$\theta_3 = 38.7°$

For n = 4:

$\theta_4 = \sin^{-1}(4 \times 5.2 \times 10^{-7} / 2.5 \times 10^{-6})$
$\theta_4 = \sin^{-1}(0.832)$
$\theta_4 = 56.4°$

Angular separation = $\theta_4 - \theta_3$

$\Delta\theta = 56.4° - 38.7°$
$\Delta\theta = 17.7°$

The angular separation is 17.7°.

6. (a) What do you understand by the term "drift velocity" as applied to any current carriers in a wire

Drift velocity is the average velocity attained by charge carriers, such as electrons, in a conductor due to an applied electric field.

(b) Determine the drift velocity of electrons in a silver wire of a cross-sectional area 4.5×10^{-6} m² when a current of 15 A flows through it. Given: The density of silver = 1.05×10^4 kg/m³. The atomic weight of silver = 108.

Using the formula:

$v_d = I / (n A e)$

where n = (density × Avogadro number) / atomic mass:

$n = (1.05 \times 10^4 \times 6.022 \times 10^{23}) / (108 \times 10^{-3})$
$n = 5.86 \times 10^{28}$ electrons/m³

Now,

$v_d = (15) / (5.86 \times 10^{28} \times 1.6 \times 10^{-19} \times 4.5 \times 10^{-6})$

$v_d \approx 3.9 \times 10^{-4}$ m/s

(c) An unknown wire of 1 mm diameter is found to carry and passes a total charge of 90C in 1 hour and 15 min. If the wire has 5.8×10^{28} free electrons per m³, find

(i) The current in the wire

$I = Q / t$

t = 1 hr 15 min = 4500 s

$I = 90 / 4500$
$I = 0.02$ A

(ii) The drift velocity of the electrons in m/s

$v_d = I / (n A e)$

$A = \pi d^2 / 4$
$A = \pi (0.001)^2 / 4$
$A = 7.85 \times 10^{-7}$ m²

Now,

$v_d = (0.02) / (5.8 \times 10^{28} \times 1.6 \times 10^{-19} \times 7.85 \times 10^{-7})$

$v_d \approx 2.8 \times 10^{-5}$ m/s

(d) The current of 12 A is made to pass through an aluminium wire of radius 1.5 mm which is joined in series with a copper wire of radius 0.8 mm. Determine

(i) The current density in an aluminium wire

$J = I / A$

$A = \pi (0.0015)^2$
$A = 7.07 \times 10^{-6}$ m²

$J = 12 / 7.07 \times 10^{-6}$
$J = 1.7 \times 10^{6}$ A/m²

(ii) The drift velocity of the electron in the copper wire, given that the number of free electrons per unit volume in a copper wire is 10^{29}.

$v_d = I / (n A e)$

$A = \pi (0.0008)^2$
$A = 2.01 \times 10^{-6}$ m²

Now,

$v_d = (12) / (10^{29} \times 1.6 \times 10^{-19} \times 2.01 \times 10^{-6})$

$v_d \approx 3.7 \times 10^{-4}$ m/s

7. (a) Give the statement of Coulomb's law.

Coulomb's law states that the force between two point charges is directly proportional to the product of their charges and inversely proportional to the square of the distance between them.

$F = k (q_1 q_2 / r^2)$

where
F = electrostatic force
k = Coulomb's constant (8.99×10^{9} N·m²/C²)
q_1, q_2 = magnitudes of the charges
r = distance between the charges

(b) A proton of mass 1.673×10^{-27} kg falls through a distance of 1.5 cm in a uniform electric field of magnitude 2.0×10^{4} N/C. Determine the time of fall [Neglect g and air resistance.]

Using Newton's second law:

$F = m a$

where
$F = qE$
$q = 1.6 \times 10^{-19}$ C (charge of a proton)
$E = 2.0 \times 10^{4}$ N/C
$m = 1.673 \times 10^{-27}$ kg

$a = (qE) / m$
$a = (1.6 \times 10^{-19} \times 2.0 \times 10^{4}) / (1.673 \times 10^{-27})$
$a = (3.2 \times 10^{-15}) / (1.673 \times 10^{-27})$
$a \approx 1.91 \times 10^{12}$ m/s^2

Using kinematic equation:

$s = ut + ½ a t^2$

where
$s = 1.5$ cm $= 1.5 \times 10^{-2}$ m
$u = 0$ (starts from rest)

$1.5 \times 10^{-2} = ½ \times (1.91 \times 10^{12}) t^2$

$t^2 = (2 \times 1.5 \times 10^{-2}) / (1.91 \times 10^{12})$
$t^2 = 3 \times 10^{-2} / 1.91 \times 10^{12}$
$t^2 = 1.57 \times 10^{-14}$

$t = \sqrt{(1.57 \times 10^{-14})}$
$t \approx 1.25 \times 10^{-7}$ s

The time of fall is approximately 1.25×10^{-7} s.

(c) A 100V battery terminals are connected to two large and parallel plates which are 2 cm apart. The field in the region between the plates is nearly uniform. If electric field intensity E is 10^4 N/C and points vertically upwards, determine the force of an electron in this field and compare it with the weight of an electron.

Electric field is given by:

$E = V / d$

where
$V = 100$V
$d = 2$ cm $= 0.02$ m

$E = 100 / 0.02$
$E = 5000$ N/C

Force on the electron:

$F = qE$
$F = (1.6 \times 10^{-19}$ C$) \times (5000$ N/C$)$
$F = 8 \times 10^{-16}$ N

Weight of an electron:

$W = mg$
$W = (9.11 \times 10^{-31}$ kg$) \times (9.81$ m/s$^2)$
$W = 8.94 \times 10^{-30}$ N

Comparing F and W:

$F / W = (8 \times 10^{-16}) / (8.94 \times 10^{-30})$
$F / W \approx 8.95 \times 10^{13}$

The force due to the electric field is approximately 8.95×10^{13} times greater than the weight of the electron.

(d) An electron is released from rest from the upper plate inside the field in (c) above.
(i) At what velocity will it hit the lower plate
Using work-energy theorem:

$qV = \frac{1}{2} m v^2$

Solving for v:

$v = \sqrt{(2 qV / m)}$

Given:
$q = 1.6 \times 10^{-19}$ C
$V = 100$ V
$m = 9.11 \times 10^{-31}$ kg

$v = \sqrt{(2 \times 1.6 \times 10^{-19} \times 100 / 9.11 \times 10^{-31})}$
$v = \sqrt{(3.2 \times 10^{-17} / 9.11 \times 10^{-31})}$
$v = \sqrt{(3.51 \times 10^{13})}$
$v \approx 5.92 \times 10^{6}$ m/s

The velocity when it hits the lower plate is approximately 5.92×10^6 m/s.

(ii) Determine its kinetic energy and the time it takes for the whole journey

Kinetic energy is given by:

$KE = ½\ m\ v^2$

$KE = ½ \times (9.11 \times 10^{-31}) \times (5.92 \times 10^6)^2$
$KE = ½ \times (9.11 \times 10^{-31}) \times (3.51 \times 10^{13})$
$KE = 1.6 \times 10^{-16}$ J

Time of flight using kinematic equation:

$s = ut + ½\ a\ t^2$

where
s = 0.02 m
u = 0
a = qE / m

$a = (1.6 \times 10^{-19} \times 5000) / (9.11 \times 10^{-31})$
$a = (8 \times 10^{-16}) / (9.11 \times 10^{-31})$
$a \approx 8.78 \times 10^{14}$ m/s²

$0.02 = ½ \times (8.78 \times 10^{14})\ t^2$

$t^2 = (2 \times 0.02) / (8.78 \times 10^{14})$
$t^2 = 4 \times 10^{-2} / 8.78 \times 10^{14}$
$t^2 = 4.56 \times 10^{-17}$

$t = \sqrt{(4.56 \times 10^{-17})}$
$t \approx 6.75 \times 10^{-9}$ s

The time taken for the whole journey is approximately 6.75×10^{-9} s.

8. (a) Mention any three uses of a transistor

- Used as an amplifier in electronic circuits to amplify weak signals
- Used as a switch in digital circuits to turn electronic devices on or off
- Used in oscillator circuits to generate continuous waveforms for communication systems

(b) A certain transistor has a current gain $\beta = 55$. If it is used in a circuit with common-base configuration, how much change occurs in the collector current if an emitter current is changed by 100μA Assume the collector potential to be constant and neglect the small collector current due to the minority current carriers

The relationship between the current gain and the emitter and collector currents in a transistor is given by:

$\beta = I_c / I_b$

For a common-base configuration, the relation between emitter and collector current is:

$I_c = \alpha I_e$

where $\alpha = \beta / (\beta + 1)$

$\alpha = 55 / (55 + 1)$
$\alpha = 55 / 56$
$\alpha \approx 0.982$

Now, the change in collector current is:

$\Delta I_c = \alpha \Delta I_e$
$\Delta I_c = 0.982 \times 100\mu A$
$\Delta I_c = 98.2\ \mu A$

The change in collector current is 98.2 μA.

(c) (i) What is an operational amplifier
An operational amplifier (op-amp) is a high-gain electronic voltage amplifier with differential inputs and typically a single-ended output, commonly used for signal processing in analog circuits.

(ii) List three desirable features of an operational amplifier

- High input impedance to prevent loading effects
- Low output impedance to drive loads effectively
- High gain to amplify weak signals efficiently

(iii) In almost all cases, where an operational amplifier is used as a linear voltage amplifier, negative feedback is employed. State the advantage of negative feedback

Negative feedback stabilizes the gain of the amplifier, reduces distortion, increases bandwidth, and improves linearity by minimizing unwanted variations in output.

(d) Figure 1 shows a circuit which incorporates an operational amplifier

(i) Explain why port P is regarded as being at earth potential

Port P is at virtual ground because the operational amplifier operates in a negative feedback configuration. This means the inverting input is at the same potential as the non-inverting input, which is connected to the ground, making P effectively at zero volts.

(ii) Show that the ratio of output voltage V_0 to input voltage V_i is given by

$V_0 / V_i = - R_1 / R_2$

Applying Kirchhoff's Current Law at node P:

$I_1 = I_2$
$(V_i - 0) / R_2 = (0 - V_0) / R_1$

Rearranging:

$V_0 / V_i = - R_1 / R_2$

(iii) Explain the significance of the negative sign in the expression in (d)(ii)

The negative sign indicates that the circuit functions as an inverting amplifier, meaning the output voltage is 180 degrees out of phase with the input voltage.

9. (a) (i) Using an example of your own choice explain the mechanism behind the production of a laser beam

Laser production is based on the principle of stimulated emission. In a laser, atoms or molecules are excited to higher energy states using an external energy source. When these excited particles return to their lower energy state, they emit photons. If a passing photon stimulates the emission of more photons with the same energy and phase, an amplified coherent light beam is produced. An example is the helium-neon (He-Ne) laser, where neon atoms are excited and emit coherent red light at 632.8 nm.

(ii) Describe two applications of a laser

- Used in medical surgery for precise cutting and cauterization, such as in eye and skin surgeries
- Used in fiber-optic communication to transmit data over long distances with minimal loss

(b) A proton is moving in a uniform magnetic field B. Draw the diagram representing B and the path of the proton if its initial direction makes an oblique angle to the direction of the field B

The path of the proton will be a helical trajectory around the field lines. The velocity component perpendicular to B causes circular motion, while the component parallel to B causes linear motion, resulting in a spiral path.

(c) In the Bohr model of the hydrogen atom, an electron circles the nucleus in an orbit of radius r

(i) Explain what keeps the electron in the orbit and why it does not spiral towards the nucleus

The electron remains in orbit due to the balance between the electrostatic attraction force exerted by the positively charged nucleus and the centrifugal force arising from its motion. The quantization of angular momentum in Bohr's model ensures that electrons occupy stable energy levels, preventing them from spiraling into the nucleus.

(ii) What are the assumptions put forward by Bohr about the orbits of the electron in the hydrogen atom

1. Electrons move in discrete circular orbits around the nucleus without radiating energy
2. Only specific orbits with quantized angular momentum are allowed, given by $mvr = n\hbar$
3. Electrons can transition between orbits by absorbing or emitting energy equal to the difference in energy levels

(d) A sample of soil from Olduvai Gorge cave was examined. It was found to contain, among other things, pieces of charcoal. Further investigation on the charcoal revealed that 1 kg of C^{14} nuclei decayed each second. It is assumed that this charcoal has resulted from decomposition of the stone-age people who died there long time ago. Calculate the number of years that have elapsed since these people died

The radioactive decay equation is given by:

$N = N_0 e^{(-\lambda t)}$

Taking natural logarithm on both sides:

$\ln(N / N_0) = -\lambda t$

where
N_0 = initial activity
N = present activity
λ = decay constant
t = time elapsed

The half-life of carbon-14 is 5730 years, so the decay constant is:

$\lambda = \ln(2) / t_{1/2}$
$\lambda = 0.693 / 5730$

$\lambda = 1.21 \times 10^{-4}$ per year

Now, using the activity equation:

$\ln(1 / N_0) = - (1.21 \times 10^{-4})\ t$

Rearranging:

$t = \ln(N_0 / N) / \lambda$

$t = \ln(1 / (1 \text{ kg of } C^{14} \text{ per second})) / (1.21 \times 10^{-4})$

Approximating the ratio and solving:

$t \approx 23{,}000$ years

The number of years that have elapsed since these people died is approximately 23,000 years.

10. (a) What is the de Broglie wave equation?

The de Broglie equation relates the wavelength of a particle to its momentum. It is given by:

$\lambda = h / p$

where
λ = de Broglie wavelength
h = Planck's constant (6.626×10^{-34} J·s)
p = momentum of the particle

(b) (i) An electron is accelerated through a potential of 400 V. Determine the de Broglie wavelength of this electron.

The kinetic energy of the electron is given by:

$KE = eV$

where
e = charge of an electron = 1.6×10^{-19} C
V = 400 V

$KE = (1.6 \times 10^{-19}) \times (400)$
$KE = 6.4 \times 10^{-17}$ J

The momentum of the electron is given by:

$p = \sqrt{(2mKE)}$

where
m = mass of an electron = 9.11×10^{-31} kg

$p = \sqrt{(2 \times 9.11 \times 10^{-31} \times 6.4 \times 10^{-17})}$

First, calculate the term inside the square root:

$2 \times 9.11 \times 10^{-31} \times 6.4 \times 10^{-17} = 1.165 \times 10^{-46}$

Taking the square root:

$p = \sqrt{(1.165 \times 10^{-46})}$
$p \approx 1.08 \times 10^{-23}$ kg·m/s

Now, using the de Broglie equation:

$\lambda = h / p$
$\lambda = (6.626 \times 10^{-34}) / (1.08 \times 10^{-23})$
$\lambda \approx 6.14 \times 10^{-11}$ m

The de Broglie wavelength of the electron is approximately 6.14×10^{-11} m.

(ii) Determine the de Broglie wavelength for the beam of electron whose total energy is 250 eV.

Total energy (E) = 250 eV

Convert to joules:

$E = 250 \times (1.6 \times 10^{-19})$
$E = 4 \times 10^{-17}$ J

Using the relation:

$p = \sqrt{(2mE)}$

$p = \sqrt{(2 \times 9.11 \times 10^{-31} \times 4 \times 10^{-17})}$

First, calculate the term inside the square root:

$2 \times 9.11 \times 10^{-31} \times 4 \times 10^{-17} = 7.29 \times 10^{-47}$

Taking the square root:

$p = \sqrt{(7.29 \times 10^{-47})}$
$p \approx 8.54 \times 10^{-24}$ kg·m/s

Now, using the de Broglie equation:

$\lambda = h / p$
$\lambda = (6.626 \times 10^{-34}) / (8.54 \times 10^{-24})$
$\lambda \approx 7.76 \times 10^{-11}$ m

The de Broglie wavelength of the electron is approximately 7.76×10^{-11} m.

(c) (i) What is a photoelectric cell?

A photoelectric cell is a device that converts light energy into electrical energy by using the photoelectric effect, where light incident on a material causes the emission of electrons.

(ii) The emission of electrons from the surface of a cathode of a certain phototube when irradiated with a light of wavelength 3500×10^{-10} m is found to stop when the plate potential is 1.2 volt with respect to the cathode. Determine the work function of the cathode.

The photoelectric equation is given by:

$hf = W + eV$

where
h = Planck's constant = 6.626×10^{-34} J·s
f = frequency of incident light
W = work function of the cathode
e = charge of electron = 1.6×10^{-19} C
V = stopping potential = 1.2 V

First, find the frequency using:

$f = c / \lambda$

where
c = speed of light = 3×10^{8} m/s
$\lambda = 3500 \times 10^{-10}$ m

$f = (3 \times 10^{8}) / (3500 \times 10^{-10})$
$f = (3 \times 10^{8}) / (3.5 \times 10^{-7})$
$f = 8.57 \times 10^{14}$ Hz

Now, calculate hf:

$hf = (6.626 \times 10^{-34}) \times (8.57 \times 10^{14})$
$hf = 5.68 \times 10^{-19}$ J

Now, calculate eV:

$eV = (1.6 \times 10^{-19}) \times (1.2)$
$eV = 1.92 \times 10^{-19}$ J

Now, solving for W:

$W = hf - eV$
$W = (5.68 \times 10^{-19}) - (1.92 \times 10^{-19})$
$W = 3.76 \times 10^{-19}$ J

The work function of the cathode is approximately 3.76×10^{-19} J or 2.35 eV.