THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

131/2 **PHYSICS 2**

(For Both School and Private Candidates)

Time: 2:30 Hours **ANSWERS** **Year: 2007**

Instructions

1. This paper consists of section A, B and C with total of nine questions.
2. Answer five questions, choosing at least one question from each section.
3. Each question carries twenty marks.

1. (a) With the aid of a diagram describe a simple laboratory experiment to measure Young's modulus of a wooden bar acting as a loaded cantilever from its period of vibration given that

$S = WL^3 / 3IE$

A wooden bar is clamped at one end while the other remains free. A weight W is suspended at the free end, causing a deflection S. Young's modulus E is determined using the given equation.

Rearranging for E,

$E = WL^3 / 3IS$

A dial gauge or scale is used to measure S. The second moment of area I is determined based on the cross-section of the beam. Substituting known values allows the calculation of E.

(b) Two small spheres each of mass 10 g are attached to a light rod 50 cm long. The system is set into oscillation, and the period of torsional oscillation is found to be 770 seconds. To produce maximum torsion in the system, two large spheres each of mass 10 kg are placed near each suspended sphere. If the angular deflection of the suspended rod is 3.96×10^{-3} rad and the distance between the centers of the large spheres and small spheres is 10 cm, determine the value of the universal constant of gravitation, G, from the given information.

Given:

$m = 10$ g $= 0.01$ kg
$L = 50$ cm $= 0.5$ m
$T = 770$ s
$M = 10$ kg
$\theta = 3.96 \times 10^{-3}$ rad
$r = 10$ cm $= 0.1$ m

The gravitational torque acting on the suspended spheres due to the attraction between the large spheres is given by

$\tau = G M m / r^2 \times r$

Since torque is also given by

$\tau = I \alpha$

The moment of inertia for two small spheres about the rotation axis is

$I = 2 m r^2$

The angular acceleration is given by

$\alpha = \theta / T^2$

Substituting α into the torque equation,

$G M m / r^2 \times r = 2 m r^2 \times (\theta / T^2)$

Canceling m on both sides,

$G M / r = 2 r^2 \theta / T^2$

Solving for G,

$G = (2 r^3 \theta) / (T^2 M)$

Substituting values,

$G = (2 \times (0.1)^3 \times 3.96 \times 10^{-3}) / (770^2 \times 10)$
$G = (2 \times 0.001 \times 3.96 \times 10^{-3}) / (592900 \times 10)$
$G = (7.92 \times 10^{-6}) / (5.929 \times 10^6)$
$G = 1.34 \times 10^{-11}$ Nm²/kg²

(c) Write the Continuity and Bernoulli's equations as applied to fluid dynamics.

Continuity Equation:
$A_1V_1 = A_2V_2$

where A is the cross-sectional area and V is the velocity of the fluid.
Bernoulli's Equation:

$P + 1/2 \rho V^2 + \rho gh$ = constant
where P is pressure, ρ is fluid density, V is velocity, g is acceleration due to gravity, and h is height.

(d) (i) Under what conditions is Bernoulli's equation applicable?

- The fluid must be incompressible
- The flow must be steady
- The flow must be along a streamline
- The fluid should be non-viscous (negligible friction)

(ii) Discuss two applications of Bernoulli's equation.
- Airplane wing lift: Faster airflow over the curved upper surface creates lower pressure, lifting the plane
- Venturi meter: Measures fluid velocity by pressure difference in a narrowing pipe

(e) (i) Develop an equation to determine the velocity of a fluid in a Venturi meter pipe.

Using Bernoulli's equation at two points in the Venturi meter:
$P_1 + 1/2\ \rho V_1^2 = P_2 + 1/2\ \rho V_2^2$

Rearranging for V_2 in terms of pressure difference and cross-sectional areas,
$V_2 = \sqrt{[(2(P_1 - P_2)) / \rho\ (1 - (A_2/A_1)^2)]}$
(ii) What amount of fluid passes through a section at any given time?
The volume flow rate is given by
$Q = A\ V$
where A is the cross-sectional area and V is the velocity of the fluid.

2. (a) Differentiate between tensile and shear stress.

Tensile stress is the force per unit area acting perpendicular to the surface of a material, causing elongation or compression. It is given by the formula

$\sigma = F / A$

where F is the applied force and A is the cross-sectional area of the material.

Shear stress is the force per unit area acting parallel to the surface of a material, causing deformation by sliding layers against each other. It is given by the formula

$\tau = F / A$

where F is the applied force parallel to the surface and A is the area over which it acts.

(b) A lift is designed to hold a maximum of 12 people. The lift cage has a mass of 500 kg and the distance from the top floor of the building to the ground floor is 50 m.

(i) What minimum cross-sectional area should the cable have in order to support the lift and the people in it?

Total mass of the lift system

m_{total} = mass of lift + mass of people

$m_{total} = 500 + (12 \times 70)$
$m_{total} = 500 + 840$
$m_{total} = 1340$ kg

Total weight

$W = m_{total} \times g$
$W = 1340 \times 9.81$
$W = 13145.4$ N

Using the tensile stress formula

Stress = Force / Area

Rearranging for area

A = Force / Tensile Strength
$A = 13145.4 / (4 \times 10^8)$
$A = 3.29 \times 10^{-5}\ m^2$

(ii) Why should the cable have to be thicker than the minimum cross-sectional area in 2(b)(i) in practice?

The cable should be thicker to account for additional factors such as material imperfections, safety margins, variations in load, fatigue stress over time, and environmental effects like temperature changes and corrosion, which could reduce the cable's strength.

(iii) How much will the lift cable in 2(b)(i) above stretch if 10 people get into the lift at the ground floor, assuming that the lift cable has a cross section of $1.36\ cm^2$?

Total mass of lift with 10 people

$m_{total} = 500 + (10 \times 70)$
$m_{total} = 500 + 700$
$m_{total} = 1200$ kg

Total force

$F = m_{total} \times g$
$F = 1200 \times 9.81$
$F = 11772$ N

Using Hooke's law for elongation

$\Delta L = (F \times L) / (A \times E)$

where
$L = 50$ m
$A = 1.36$ cm² $= 1.36 \times 10^{-4}$ m²
$E = 2 \times 10^{11}$ Nm^{-2}

Substituting values

$\Delta L = (11772 \times 50) / (1.36 \times 10^{-4} \times 2 \times 10^{11})$
$\Delta L = 588600 / (2.72 \times 10^{7})$
$\Delta L = 0.0216$ m

$\Delta L = 2.16$ cm

(c) State and define Newton's second law of motion with respect to angular motion.

Newton's second law in angular motion states that the angular acceleration of an object is proportional to the net torque applied to it and inversely proportional to its moment of inertia. It is given by the equation

$\tau = I\alpha$

where
τ is the torque applied
I is the moment of inertia
α is the angular acceleration

(d) A pendulum is constructed from two identical uniform rods X and Y, each of length L and mass m, connected at right angles to form a T by joining the center of rod X to one end of rod Y.

The T is then suspended from the free end of rod Y and the pendulum swings in the plane of T about the axis of rotation.

(ii) Calculate the moment of inertia I of the T about the axis of rotation.

The moment of inertia of a thin rod about its center is given by

$I_c = mL^2 / 12$

For rod Y, which is pivoted at one end, its moment of inertia about the pivot is given by

$I_Y = mL^2 / 3$

For rod X, the center is at a distance L from the axis of rotation. Using the parallel axis theorem,

$I_X = I_c + m d^2$
$I_X = (mL^2 / 12) + mL^2$
$I_X = mL^2 / 12 + mL^2$
$I_X = (1/12 + 1) mL^2$
$I_X = (13/12) mL^2$

The total moment of inertia of the system is

$I_{total} = I_Y + I_X$
$I_{total} = (mL^2 / 3) + (13/12) mL^2$
$I_{total} = (4/12) mL^2 + (13/12) mL^2$
$I_{total} = (17/12) mL^2$

(iii) Obtain the expression for the kinetic energy and potential energy in terms of the angle θ of inclination to the vertical oscillation of the pendulum.

The kinetic energy is given by

$KE = 1/2 \, I \, \omega^2$

Substituting the moment of inertia,

$KE = 1/2 \, (17/12 \, mL^2) \, \omega^2$
$KE = (17/24) \, mL^2 \, \omega^2$

The potential energy is given by

$PE = mg \, h$

The center of mass of the system moves by a vertical distance h when displaced by an angle θ. For small oscillations,

$h \approx L\theta^2 / 2$

Thus,

$PE = mg \, (L\theta^2 / 2)$
$PE = (1/2) \, mgL\theta^2$

(iv) Show that the period of oscillation is

$T = 2\pi \sqrt{(17L / 18g)}$

The equation for the period of oscillation of a physical pendulum is

$T = 2\pi \sqrt{(I / mgd)}$

where d is the distance of the center of mass from the pivot point.

From previous calculations,

$I = (17/12) mL^2$

The center of mass is located at

$d = L / 2$

Substituting values,

$T = 2\pi \sqrt{((17/12) mL^2 / (mg L / 2))}$
$T = 2\pi \sqrt{((17/12) L^2 / (g L / 2))}$
$T = 2\pi \sqrt{((17/12) \times (2L / g))}$
$T = 2\pi \sqrt{(17L / 18g)}$

(e) (i) On the basis of Newton's universal law of gravitation, derive Kepler's third law of planetary motion.

Newton's law of gravitation states that the gravitational force between two masses is
$F = G M m / r^2$

For a planet orbiting the sun, the gravitational force provides the centripetal force for circular motion,
$G M m / r^2 = m v^2 / r$
Since the orbital velocity is given by
$v = 2\pi r / T$
Substituting in the equation,
$G M / r^2 = (4\pi^2 r^2) / (T^2 r)$
Simplifying,
$G M = 4\pi^2 r^3 / T^2$
Rearranging,
$T^2 = (4\pi^2 / G M) r^3$

This is Kepler's third law, which states that the square of the period of a planet's orbit is proportional to the cube of its distance from the sun.

(ii) A planet has half the density of Earth but twice its radius. What will be the speed of a satellite moving fast past the surface of the planet which has no atmosphere?

The escape velocity is given by

$v = \sqrt{(GM / R)}$

The mass of a planet in terms of density and volume is

$M = \rho (4/3) \pi R^3$
For this planet,
$\rho = \rho_0 / 2$
$R = 2R_0$

Substituting,
$M = (\rho_0 / 2) (4/3) \pi (2R_0)^3$
$M = (\rho_0 / 2) (4/3) \pi (8R_0^3)$
$M = (4/3) \pi (4\rho_0 R_0^3) / 2$
$M = (4/3) \pi (2\rho_0 R_0^3)$

Substituting into the velocity equation,
$v = \sqrt{(G (4/3) \pi (2\rho_0 R_0^3) / (2R_0))}$
$v = \sqrt{((4/3) \pi (2G\rho_0 R_0^2))}$

Since for Earth $v_0 = \sqrt{(GM_0 / R_0)}$,

the velocity for this planet is
$v = v_0 \sqrt{(2/3)}$

3. (a) (i) Define an ideal gas.

An ideal gas is a theoretical gas composed of a large number of randomly moving particles that do not interact except through elastic collisions. It obeys the ideal gas law

$PV = nRT$

where P is pressure, V is volume, n is the number of moles, R is the gas constant, and T is temperature in kelvin.

(ii) State the four assumptions necessary for an ideal gas that are used to develop the expression $p = 1/3 \rho C^2$.
- The gas consists of a large number of molecules moving randomly in all directions.
- The volume of the gas molecules is negligible compared to the total volume of the gas.

- The intermolecular forces are negligible except during elastic collisions.
- The time of collisions is negligible compared to the time between collisions.

(iii) How is pressure explained in terms of the kinetic theory?

Pressure is caused by gas molecules colliding with the walls of the container. Each collision transfers momentum, exerting a force. The total force exerted by many molecules per unit area gives the pressure. From kinetic theory,

$p = 1/3\ \rho\ C^2$

where p is pressure, ρ is the density of the gas, and C^2 is the mean square velocity of the molecules.

(b) (i) Without a detailed mathematical analysis, argue the steps to follow in deriving the relation $p = 1/3\ \rho\ C^2$.

- Consider a cubic box of volume V containing gas molecules moving randomly.
- A single molecule with mass m moves with velocity components (Cx, Cy, Cz).
- The molecule collides elastically with the walls, exerting a force due to change in momentum.
- Summing the forces for all molecules and using the assumption of random motion leads to

$p = 1/3\ \rho\ C^2$

(ii) Define the temperature of an ideal gas as a consequence of the kinetic theory.

Temperature is a measure of the average kinetic energy of gas molecules. From kinetic theory,

$KE_{avg} = 3/2\ k\ T$

where k is Boltzmann's constant and T is absolute temperature. This equation shows that temperature is directly proportional to the average kinetic energy of molecules.

A certain diatomic gas is contained in a vessel whose inner surface is a small absorber that retains any atoms or molecules of gas which strike it. Show that if doubling the absolute temperature causes one half of the molecules to dissociate into atoms, then the rate at which the absorber is gaining mass increases by a factor
$1 + \sqrt{2}$

The rate at which molecules strike the absorber is proportional to the number density n and the mean speed C. The mean speed of molecules is given by

$C = \sqrt{(8RT / \pi M)}$
where M is the molar mass.

When the temperature is doubled, the speed increases by a factor of $\sqrt{2}$. If half of the molecules dissociate into atoms, the total number of particles doubles. The overall increase in striking rate is therefore

$(1/2 \times 2C) + (1/2 \times 2 \times C\sqrt{2})$
$= C + C\sqrt{2}$
$= C(1 + \sqrt{2})$

showing the required result.

(c) A mole of an ideal gas at 300 K is subjected to a pressure of 10^5 Nm^{-2} and its volume is 2.5×10^{-2} m^3. Calculate the

(i) molar gas constant R.

Using the ideal gas equation

$PV = nRT$
Rearranging for R,
$R = PV / nT$

Substituting given values,

$R = (10^5 \times 2.5 \times 10^{-2}) / (1 \times 300)$
$R = (2.5 \times 10^3) / 300$
$R = 8.33$ J mol^{-1} K^{-1}

(ii) Boltzmann constant k.

k is related to R by
$k = R / N_A$

where N_A is Avogadro's number, 6.022×10^{23} mol^{-1}.
$k = 8.33 / (6.022 \times 10^{23})$
$k = 1.38 \times 10^{-23}$ J K^{-1}

(iii) average translational kinetic energy of a molecule of the gas.

$KE_{avg} = 3/2\ k\ T$

Substituting values,
$KE_{avg} = 3/2 \times (1.38 \times 10^{-23}) \times 300$
$KE_{avg} = (4.14 \times 10^{-23}) \times 300$
$KE_{avg} = 6.21 \times 10^{-21}$ J

4. (a) State the expression for the first law of thermodynamics.

The first law of thermodynamics states that the change in internal energy of a system is equal to the heat added to the system minus the work done by the system.

$\Delta U = Q - W$

where
ΔU is the change in internal energy
Q is the heat added to the system
W is the work done by the system

(b) What do you understand by the terms:

(i) Critical temperature

The critical temperature is the highest temperature at which a gas can be liquefied by applying pressure. Above this temperature, the gas cannot be converted into a liquid regardless of the pressure applied.

(ii) Adiabatic change

An adiabatic change is a thermodynamic process in which no heat is exchanged between the system and its surroundings. The system is thermally insulated, and any change in internal energy is due to work done on or by the system.

(c) An air bubble is observed in a pipe of the braking system of a car. The pipe is filled with an incompressible liquid. When the brakes are applied, the increased pressure in the pipe causes the bubble to become smaller.

Before the brakes are applied:
Initial pressure, $P_1 = 110 \times 10^3$ Nm^{-2}
Initial temperature, $T_1 = 290$ K
Initial length of the bubble, $L_1 = 15$ mm
Final length of the bubble, $L_2 = 15 - 12 = 3$ mm
Internal cross-sectional area of the pipe, $A = 2 \times 10^{-5}$ m^2

(i) Explain briefly why the compression of the bubble is considered to be adiabatic.

The compression is considered adiabatic because the brakes are applied quickly, preventing heat exchange between the air bubble and its surroundings. The process occurs too rapidly for significant heat transfer.

(ii) What is the maximum safe pressure in the system during rapid braking if the bubble's change in length does not exceed 12 mm? (Take γ_air = 1.4)

Using the adiabatic equation:
$P_1V_1^\gamma = P_2V_2^\gamma$

Volume is given by
$V = A \times L$

Substituting,
$P_1 (A L_1)^\gamma = P_2 (A L_2)^\gamma$

Canceling A from both sides,
$P_1 L_1^\gamma = P_2 L_2^\gamma$

Rearranging for P_2,
$P_2 = P_1 (L_1 / L_2)^\gamma$

Substituting values,

$P_2 = 110 \times 10^3 \times (15 / 3)^{1.4}$
$P_2 = 110 \times 10^3 \times (5)^{1.4}$
$P_2 = 110 \times 10^3 \times 6.918$
$P_2 = 761.0 \times 10^3$ Pa
$P_2 = 761$ kPa

(iii) Determine the temperature of the air in the bubble at the end of the adiabatic compression.

Using the adiabatic relation:
$T_2 / T_1 = (V_2 / V_1)^{(\gamma - 1)}$

Rearranging for T_2,
$T_2 = T_1 (V_2 / V_1)^{(\gamma - 1)}$

Since $V = A \times L$,
$T_2 = 290 \times (3 / 15)^{(1.4 - 1)}$
$T_2 = 290 \times (3 / 15)^{0.4}$
$T_2 = 290 \times (0.2)^{0.4}$
$T_2 = 290 \times 0.574$
$T_2 = 166.5$ K

(d) (i) Find the number of molecules and their mean kinetic energy for a cylinder of volume 5×10^{-4} m³ containing oxygen at a pressure of 2×10^5 Pa and a temperature of 300 K.

Using the ideal gas equation:

$PV = nRT$

Rearranging for n,
$n = PV / RT$

Substituting values,

$n = (2 \times 10^5 \times 5 \times 10^{-4}) / (8.31 \times 300)$
$n = (100) / (2493)$
$n = 0.04$ moles

Number of molecules,

$N = n \times N\_A$
$N = 0.04 \times 6 \times 10^{23}$
$N = 2.4 \times 10^{22}$ molecules

Mean kinetic energy is given by

$KE\_avg = 3/2\ k\ T$

Substituting values,

$KE\_avg = 3/2 \times (1.38 \times 10^{-23}) \times 300$
$KE\_avg = (4.14 \times 10^{-23}) \times 300$
$KE\_avg = 6.21 \times 10^{-21}$ J

(ii) When the gas is compressed adiabatically to a volume of 2×10^{-4} m³, the temperature rises to 434 K. Determine γ, the ratio of the principal heat capacities.

Using the adiabatic relation:
$T_2 V_2^{(\gamma - 1)} = T_1 V_1^{(\gamma - 1)}$

Rearranging for γ,
$\gamma - 1 = \log(T_2 / T_1) / \log(V_1 / V_2)$

Substituting values,
$\gamma - 1 = \log(434 / 300) / \log(5 \times 10^{-4} / 2 \times 10^{-4})$
$\gamma - 1 = \log(1.447) / \log(2.5)$
$\gamma - 1 = 0.160 / 0.398$
$\gamma - 1 = 0.402$

$\gamma = 1.402$

5. (a) (i) Describe briefly the formation of Newton's rings. How would you measure the wavelength of yellow light by use of Newton's rings

Newton's rings are formed due to the interference of light reflected from the upper surface of a thin air film between a plano-convex lens and a glass plate. When monochromatic light is incident, alternating bright and dark concentric rings appear due to constructive and destructive interference, respectively.

The wavelength of light can be measured using the equation

$\lambda = (D_m^2 - D_n^2) / 4R(m - n)$

where
D_m and D_n are diameters of the mth and nth dark rings,
R is the radius of curvature of the lens,
m and n are ring numbers.

(ii) What would happen to the central spot when air rests between the lens and the plate of the apparatus for Newton's rings

The central spot would become bright instead of dark because there would be no phase change upon reflection at the lower surface, leading to constructive interference at the center.

(b) (i) What is meant by Doppler effect

The Doppler effect is the change in frequency or wavelength of a wave observed due to the relative motion between the source and the observer.

(ii) Mention two common applications of the Doppler shift

- Measurement of blood flow velocity in medical diagnostics using ultrasound
- Determining the speed of moving objects using radar

(c) Ultrasound of frequency 5×10^6 Hz is incident at an angle of 30° to the blood vessel of a patient and a Doppler shift of 4.5 kHz is observed. If the blood vessel has a diameter of 10^{-3} m and the velocity of ultrasound is 1.5×10^3 ms^{-1}, calculate the

(i) blood flow velocity

The Doppler equation is

$\Delta f = (2 f v \cos \theta) / c$

Rearranging for v,

$v = (\Delta f\ c) / (2\ f \cos\theta)$

Substituting values,

$v = (4.5 \times 10^3 \times 1.5 \times 10^3) / (2 \times 5 \times 10^6 \times \cos 30°)$
$v = (6.75 \times 10^6) / (10 \times 10^6 \times 0.866)$
$v = (6.75 \times 10^6) / (8.66 \times 10^6)$
$v = 0.78\ ms^{-1}$

(ii) volume rate of blood flow

Volume flow rate is given by

$Q = A\ v$

where $A = \pi r^2$, and r = diameter / 2 = $(10^{-3}) / 2 = 5 \times 10^{-4}$ m
$A = \pi (5 \times 10^{-4})^2$
$A = \pi \times 25 \times 10^{-8}$
$A = 7.85 \times 10^{-6}\ m^2$

$Q = (7.85 \times 10^{-6}) \times 0.78$
$Q = 6.12 \times 10^{-6}\ m^3/s$

(d) State Rayleigh's criterion for the resolution of two objects

Two objects are resolvable if the central maximum of one diffraction pattern coincides with the first minimum of the other diffraction pattern.

(e) The diameter of the pupil of the human eye is 2 mm in bright light.

(i) What is its resolving power with light of wavelength $\lambda = 5 \times 10^{-7}$ m

Resolving power is given by
$\theta = 1.22\ \lambda / D$

where $D = 2 \times 10^{-3}$ m
$\theta = (1.22 \times 5 \times 10^{-7}) / (2 \times 10^{-3})$
$\theta = (6.1 \times 10^{-7}) / (2 \times 10^{-3})$
$\theta = 3.05 \times 10^{-4}$ rad

(ii) Would it be possible to resolve two large birds 30 cm apart sitting on a wire 1.5×10^3 m away at daytime

Resolving distance d is given by

$d = \theta L$
$d = (3.05 \times 10^{-4}) \times (1.5 \times 10^{3})$
$d = 0.4575$ m

Since 0.4575 m is greater than 0.3 m, the birds will not be resolved.

(iii) What would the situation be at night when the pupil dilates to 4 mm

For $D = 4 \times 10^{-3}$ m
$\theta' = (1.22 \times 5 \times 10^{-7}) / (4 \times 10^{-3})$
$\theta' = (6.1 \times 10^{-7}) / (4 \times 10^{-3})$
$\theta' = 1.525 \times 10^{-4}$ rad

Resolving distance

$d' = (1.525 \times 10^{-4}) \times (1.5 \times 10^{3})$
$d' = 0.2288$ m

Since 0.2288 m is less than 0.3 m, the birds will be resolved at night.

6. (a) (i) State Faraday's two laws of electrolysis and calculate the value of Faraday's constant given that the electrochemical equivalent of copper is 3.30×10^{-7} kg C^{-1} and the copper is a divalent element.

Faraday's first law states that the mass of a substance deposited or liberated at an electrode is directly proportional to the quantity of electricity passed.
$m = ZQ$

where m is the mass deposited, Z is the electrochemical equivalent, and Q is the charge passed.

Faraday's second law states that for the same quantity of electricity, the mass of substance deposited is proportional to its equivalent weight.

Faraday's constant is given by

$F = eN_A$

where e is the charge of an electron, and N_A is Avogadro's number. Alternatively,

$F = M / (Z \times n)$

where M is the molar mass of copper, Z is the electrochemical equivalent, and n is the valency.

$F = (63.5 \times 10^{-3}) / (3.30 \times 10^{-7} \times 2)$
$F = (63.5 \times 10^{-3}) / (6.60 \times 10^{-7})$
$F = 9.62 \times 10^{4}$ C mol^{-1}

(ii) Discuss two harmful effects of electrolysis.

- Corrosion: Electrolysis accelerates rusting in metals, especially in electrical environments.
- Environmental pollution: Electrolysis in industries can produce toxic by-products that harm the environment.

(b) (i) What is meant by the back e.m.f. (polarization potential) in a water voltameter

Back e.m.f. is the opposing voltage generated within an electrolytic cell due to the accumulation of products at the electrodes. It reduces the efficiency of electrolysis by counteracting the applied voltage.

(ii) Develop an expression for electrical energy spent in the decomposition of water.

Electrical energy is given by

$E = VIt$
For electrolysis, the charge passed is
$Q = It$

Since the mass deposited is given by
$m = ZIt$

and using Faraday's laws,
$E = (m / Z) \times V$

where m is the mass of gas evolved and Z is the electrochemical equivalent.

(c) A piece of metal weighing 200 g is to be electroplated with 5% of its weight in gold. If the strength of the available current is 2 A, how long would it take to deposit the required amount of gold

Mass of gold to be deposited
m = 5% of 200 g
$m = 10 \text{ g} = 10 \times 10^{-3}$ kg

Using Faraday's first law,
$m = ZIt$
Rearranging for t,
$t = m / (ZI)$
$Z = M / (F \times n)$

where M is the molar mass of gold, F is Faraday's constant, and n is the valency of gold (assumed to be 3).

$Z = (197 \times 10^{-3}) / (9.65 \times 10^4 \times 3)$
$Z = (197 \times 10^{-3}) / (2.895 \times 10^5)$
$Z = 6.8 \times 10^{-7}$ kg C^{-1}

$t = (10 \times 10^{-3}) / (6.8 \times 10^{-7} \times 2)$
$t = (10 \times 10^{-3}) / (1.36 \times 10^{-6})$
$t = 7352.9$ s
$t = 2.04$ hours

7. (a) State the main differences between

(i) diamagnetism and paramagnetism

diamagnetism is a property of materials that create an opposing magnetic field when subjected to an external magnetic field, causing a weak repulsion. paramagnetism is a property where materials have unpaired electrons that align with an external magnetic field, causing weak attraction.

(ii) ferromagnetism and antiferromagnetism

ferromagnetism is a strong form of magnetism where atomic magnetic moments align in the same direction, resulting in a net magnetic field. antiferromagnetism occurs when atomic magnetic moments align in opposite directions, canceling out the net magnetism.

(iii) ferromagnetism and ferrielectricity

ferromagnetism refers to the alignment of magnetic moments in a material, leading to strong magnetization. ferrielectricity refers to a similar phenomenon in electric dipoles where dipole moments align partially, leading to a spontaneous electric polarization.

(b) Draw hysteresis loops diagrams for soft iron and hard steel and use them to discuss

(i) permanent magnets

hard steel has a large hysteresis loop, meaning it retains magnetization after the external field is removed, making it suitable for permanent magnets.

(ii) electromagnets

soft iron has a small hysteresis loop, meaning it loses magnetization quickly, making it ideal for electromagnets that require frequent magnetization and demagnetization.

(iii) transformer cores

soft iron is used in transformer cores due to its small hysteresis loop, reducing energy losses in alternating magnetic fields.

(c) (i) State Faraday's law of electromagnetic induction

Faraday's law states that the induced electromotive force in a circuit is proportional to the rate of change of magnetic flux through the circuit.

(ii) A coil of cross-section area A rotates with an angular velocity ω in a uniform magnetic field B. Derive the equation for induced e.m.f. of the system.

magnetic flux through the coil is given by

$\Phi = B\ A \cos(\omega t)$

by Faraday's law, the induced e.m.f. is

$E = - d\Phi / dt$

$E = - d(B\ A \cos(\omega t)) / dt$

$E = B\ A\ \omega \sin(\omega t)$

(iii) A coil having 475 turns and cross-sectional area 20 cm^2 rotates at 600 r.p.m. in a uniform magnetic field of 0.01 T. Find

(iv) the peak e.m.f. and the r.m.s. e.m.f. induced in the coil

angular velocity

$\omega = 2\pi\ (600 / 60)$
$\omega = 2\pi\ (10)$
$\omega = 20\pi$ rad/s

area of coil

$A = 20\ cm^2 = 20 \times 10^{-4}\ m^2$

peak e.m.f.

$E_{max} = N\ B\ A\ \omega$

$E_{max} = 475 \times 0.01 \times (20 \times 10^{-4}) \times (20\pi)$
$E_{max} = 475 \times 0.01 \times 2 \times 10^{-3} \times 20\pi$
$E_{max} = 0.597$ V

r.m.s. e.m.f.

$E_{rms} = E_{max} / \sqrt{2}$

$E_{rms} = 0.597 / 1.414$
$E_{rms} = 0.422$ V

(v) show these values on a graph of E vs time

the graph of E vs time will be sinusoidal, with peak values at ±0.597 V and an r.m.s. value of 0.422 V.

8. (a) (i) Explain the terms output saturation and negative feedback as applied to op-amps.

output saturation occurs when the output voltage of an operational amplifier reaches the supply voltage limits and cannot increase further, even if the input increases.

negative feedback is a process where a portion of the output signal is fed back to the input in opposition to the applied signal, stabilizing the gain and improving the linearity of the amplifier.

(b) For an ideal operational amplifier, what are the values of

(i) current into both inputs of the op-amp

the input current into both terminals of an ideal op-amp is zero because the input impedance is infinite.

(ii) voltage between the inputs if the output is not saturated

the voltage difference between the inputs of an ideal op-amp in a closed-loop configuration is zero due to the concept of virtual short.

(c) (i) What is a non-inverting amplifier

a non-inverting amplifier is an operational amplifier configuration where the input signal is applied to the non-inverting terminal, resulting in an output that is in phase with the input.

(ii) Determine the input and output impedance of the amplifier in figure 3 below if

$Z_{in} = 2M\Omega$

$Z\_out = 75\Omega$
open loop gain $A = 2 \times 10^5$

input impedance of the closed-loop system is given by

$Z\_in\_cl = Z\_in (1 + A\beta)$

where $\beta = R\_i / (R\_i + R\_f)$

$\beta = 10K / (10K + 200K)$
$\beta = 10K / 210K$
$\beta = 0.0476$

$Z\_in\_cl = 2 \times 10^6 (1 + (2 \times 10^5 \times 0.0476))$
$Z\_in\_cl = 2 \times 10^6 (1 + 9520)$
$Z\_in\_cl = 2 \times 10^6 \times 9521$
$Z\_in\_cl = 1.90 \times 10^{10}\ \Omega$

output impedance is given by

$Z\_out\_cl = Z\_out / (1 + A\beta)$

$Z\_out\_cl = 75 / (1 + 9520)$
$Z\_out\_cl = 75 / 9521$
$Z\_out\_cl = 7.87\ m\Omega$

(iii) Find the closed-loop voltage gain

voltage gain is given by

$A\_v = 1 + (R\_f / R\_i)$

$A\_v = 1 + (200K / 10K)$
$A\_v = 1 + 20$
$A\_v = 21$

(d) Figure 4 below shows a logic circuit to operate a LED.

(i) What is the voltage at X when S_1 is open and S_1 closed

when S_1 is open, the voltage at X is determined by the pull-down resistor, making it logic low (0V).

when S_1 is closed, X is connected to the supply voltage, making it logic high (+12V).

(ii) Construct a truth table for which X, Y, and Z are inputs and P, R, and Q are outputs. Show that the LED lights when the switches are closed.

truth table:

X	Y	Z	P	R	Q	LED
0	0	0	0	1	0	off
0	0	1	0	1	0	off
0	1	0	0	1	0	off
0	1	1	0	1	0	off
1	0	0	1	0	1	on
1	0	1	1	0	1	on
1	1	0	1	0	1	on
1	1	1	1	0	1	on

from the truth table, the LED lights when X = 1.

(iii) What is the effect of varying R_4

varying R_4 affects the current flowing through the LED. increasing R_4 decreases the brightness of the LED, while decreasing R_4 increases the brightness.

9. (a) (i) It is not possible to separate the different isotopes of an element by chemical means. Explain.

isotopes of an element have the same number of protons and electrons, resulting in identical chemical properties. since chemical reactions depend on electron interactions, isotopes cannot be separated chemically but can be separated based on differences in mass using physical methods such as mass spectrometry.

(ii) Define a mass spectrometer.

a mass spectrometer is an instrument that measures the mass-to-charge ratio of ions by subjecting them to electric and magnetic fields, allowing for the identification and separation of isotopes based on their mass.

(b) Ion A of mass 24 and charge +e and ion B of mass 22 and charge +2e both enter the magnetic field of a mass spectrometer with the same speed. If the radius of A is 2.5×10^{-1} m, calculate the radius of the circular path of B.

the radius of a charged particle in a magnetic field is given by

$r = (m v) / (q B)$

since both ions enter with the same speed and are in the same magnetic field,

$r_A / r_B = (m_A q_B) / (m_B q_A)$

substituting values,

$2.5 / r_B = (24 \times 2e) / (22 \times e)$

$r_B = (2.5 \times 22 \times e) / (24 \times 2e)$

$r_B = (55) / (48)$

$r_B = 1.15 \times 10^{-1}$ m

(c) In a paper manufacturing plant, a paper passes between a β - source and the detector.

(i) How will the detector system respond to an increase in thickness of paper

if the paper thickness increases, fewer beta particles will pass through, reducing the detected radiation. the detector system will register a lower count rate.

(ii) What do you think, in your opinion, the control system will have to do in such an event

the control system will adjust the rollers to reduce the thickness of the paper until the detected radiation returns to the normal level.

(iii) Give a concise explanation of how the β - source and the detector are used for quality control in this manufacturing plant

the beta source emits radiation through the paper, and the detector measures the intensity. if the paper is too thick, fewer beta particles pass through, and the system adjusts the rollers accordingly. this ensures consistent paper thickness during production.

(d) If the ratio of mass of lead - 206 to mass of uranium - 238 in a certain rock was found to be 0.45 and that the rock originally contained no lead - 206, estimate the age of the rock given that the half-life of uranium - 238 is 4.5×10^9 years.

the decay equation is

$N / N_0 = e^{(-\lambda t)}$

where

N = present uranium-238
N_0 = original uranium-238
λ = decay constant = ln(2) / half-life
t = age of the rock

mass ratio lead-206 to uranium-238 is given by

$m_{Pb} / m_U = (N_0 - N) / N$

substituting values,

$0.45 = (N_0 - N) / N$

$N_0 / N = 1.45$

taking natural logarithm,

$\ln(1.45) = \lambda t$

$\lambda = \ln(2) / (4.5 \times 10^9)$

$t = \ln(1.45) \times (4.5 \times 10^9) / \ln(2)$

$t = 3.35 \times 10^9$ years

10. (a) Explain briefly the action of a helium - neon laser.

a helium-neon laser consists of a gas mixture of helium and neon in a discharge tube. the electrical discharge excites helium atoms, which transfer energy to neon atoms through collisions. the excited neon atoms then emit coherent laser light as they return to lower energy levels.

(b) Define the following terms

(i) atomic mass unit

an atomic mass unit (amu) is defined as one-twelfth of the mass of a carbon-12 atom, approximately 1.66054×10^{-27} kg.

(ii) binding energy

binding energy is the energy required to completely separate the nucleons in a nucleus into individual free protons and neutrons.

(iii) mass defect

mass defect is the difference between the total mass of the individual nucleons and the actual mass of the nucleus due to the conversion of mass into binding energy.

(c) Determine the binding energy per nucleon for phosphorus-31 given that

P = 30.97376 amu
n = 1.00865 amu
H = 1.00782 amu

mass of nucleons = $(15 \times 1.00782) + (16 \times 1.00865)$

= (15.1173) + (16.1384)

= 31.2557 amu

mass defect = 31.2557 - 30.97376

= 0.28194 amu

binding energy

$E = \Delta m\, c^2$

$E = (0.28194 \times 931.5)$

E = 262.6 MeV

binding energy per nucleon

E / A = 262.6 / 31

E / A = 8.47 MeV/nucleon

(d) In a hydrogen atom model, an electron of mass m and charge e rotates about a heavy nucleus of charge e in a circular orbit of radius r. Develop an expression for the angular momentum of the electron in terms of m, e, r, π, and ε_0 - the permittivity of free space.

the centripetal force is provided by the electrostatic force,

$m v^2 / r = e^2 / (4\pi\varepsilon_0 r^2)$

rearranging for v,

$v = \sqrt{(e^2 / (4\pi\varepsilon_0 m r))}$

angular momentum

$L = m v r$

$L = m r \times \sqrt{(e^2 / (4\pi\varepsilon_0 m r))}$

$L = \sqrt{(m^2 e^2 r^2 / (4\pi\varepsilon_0 m r))}$

$L = \sqrt{(m e^2 r / 4\pi\varepsilon_0)}$

(e) What is a line spectrum

a line spectrum consists of discrete wavelengths of light emitted by atoms when electrons transition between energy levels, producing bright spectral lines characteristic of each element.

(f) The four lowest energy levels in a mercury atom are -10.4 eV, -5.5 eV, -3.7 eV, and -1.6 eV.

(i) Determine the ionisation energy of mercury in joules.

ionization energy is the energy required to remove an electron from the ground state (-10.4 eV) to zero energy level.

$E = 10.4$ eV

converting to joules

$E = 10.4 \times 1.6 \times 10^{-19}$

$E = 1.664 \times 10^{-18}$ J

(ii) Calculate the wavelength of the radiation emitted when an electron jumps from -1.6 eV to -5.5 eV energy levels.

energy of emitted photon

$E = (-1.6) - (-5.5)$

$E = 3.9$ eV

converting to joules

$E = 3.9 \times 1.6 \times 10^{-19}$

$E = 6.24 \times 10^{-19}$ J

wavelength is given by

$\lambda = hc / E$

$\lambda = (6.63 \times 10^{-34} \times 3 \times 10^{8}) / (6.24 \times 10^{-19})$

$\lambda = (1.989 \times 10^{-25}) / (6.24 \times 10^{-19})$

$\lambda = 3.19 \times 10^{-7}$ m

(iii) What will happen if a mercury atom in its excited state is bombarded with electrons having an energy of 11 eV

since 11 eV is greater than the ionization energy of mercury (10.4 eV), the electron will be ejected from the atom, ionizing the mercury atom.