

THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

131/2

PHYSICS 2

(For Both School and Private Candidates)

Time: 2:30 Hours

ANSWERS

Year: 2015

Instructions

1. This paper consists of section A, B and C with total of nine questions.
2. Answer five questions, choosing at least one question from each section.
3. Each question carries twenty marks.

maktaba.tetea.org



1. (a) (i) Write down the Bernoulli's equation for fluid flow in a pipe and indicate the term which will disappear when the fluid is stopped.

Bernoulli's equation states:

$$P + 0.5 \rho v^2 + \rho g h = \text{constant}$$

When the fluid stops flowing, velocity $v = 0$, so the term $0.5 \rho v^2$ disappears.

(ii) Name the principle on which the continuity equation is based.

The continuity equation is based on the principle of conservation of mass.

(iii) Basing on the applications of Bernoulli's principle, briefly explain why two ships which are moving parallel and close to each other experience an attractive force.

When two ships move close to each other, the water between them moves faster than the water outside. This decreases the pressure between the ships, while the higher pressure on the outer sides pushes them toward each other, causing an attractive force.

(b) (i) A sphere is dropped under gravity through a fluid of viscosity η . Taking average acceleration as half of the initial acceleration, show that the time taken to attain terminal velocity is independent of fluid density.

The equation of motion is given by:

$$m \frac{dv}{dt} = mg - F_b - F_v$$

where

m = mass of the sphere

g = acceleration due to gravity

F_b = buoyant force

F_v = viscous force

At terminal velocity v_t :

$$mg = \rho_f V g + 6 \pi \eta r v_t$$

Substituting $V = (4/3) \pi r^3$:

$$mg = \rho_f (4/3) \pi r^3 g + 6 \pi \eta r v_t$$

Solving for v_t :

$$v_t = (mg - \rho_f (4/3) \pi r^3 g) / (6 \pi \eta r)$$

Since the time taken to reach terminal velocity depends on acceleration a , and acceleration is determined from force balance independent of ρ_f , the time taken is independent of fluid density.

(ii) Water is flowing through a horizontal pipe having different cross-sections at two points A and B. The diameters of the pipe at A and B are 0.6m and 0.2m respectively. The pressure difference between points A and B is 1m column of water. Calculate the volume of water flowing per second.

Using continuity equation:

$$A_A v_A = A_B v_B$$

$$A_A = \pi (0.6/2)^2$$

$$A_A = \pi (0.3)^2$$

$$A_A = 0.283 \text{ m}^2$$

$$A_B = \pi (0.2/2)^2$$

$$A_B = \pi (0.1)^2$$

$$A_B = 0.0314 \text{ m}^2$$

Using Bernoulli's equation:

$$P_A + 0.5 \rho v_A^2 = P_B + 0.5 \rho v_B^2$$

$$P_A - P_B = \rho g h$$

$$P_A - P_B = (1000 \times 9.81 \times 1)$$

$$P_A - P_B = 9810 \text{ Pa}$$

Rearrange for v_A :

$$9810 = 0.5 \times 1000 \times v_B^2 - 0.5 \times 1000 \times v_A^2$$

Since v_A and v_B are related through continuity equation:

$$v_B = (A_A / A_B) v_A$$

Substituting and solving for v_A and v_B :

$$Q = A v$$

$$Q = 0.283 \times v_A$$

$$Q = 0.0314 \times v_B$$

Solving the equations gives

$$Q = 0.0847 \text{ m}^3/\text{s}$$

(c) (i) The flow rate of water from a tap of diameter 1.25 cm is 3 litres per minute. The coefficient of viscosity of water is $10^{-3} \text{ N}\cdot\text{s}/\text{m}^2$. Determine the Reynolds number and then state the type of flow of water.

$$Re = (\rho v d) / \eta$$

$$Q = 3 \text{ L/min} = 3 \times 10^{-3} \text{ m}^3 / 60 \text{ s}$$

$$Q = 5 \times 10^{-5} \text{ m}^3/\text{s}$$

$$A = \pi d^2 / 4$$

$$A = \pi (0.0125)^2 / 4$$

$$A = 1.227 \times 10^{-4} \text{ m}^2$$

$$v = Q / A$$

$$v = (5 \times 10^{-5}) / (1.227 \times 10^{-4})$$

$$v = 0.407 \text{ m/s}$$

$$Re = (1000 \times 0.407 \times 0.0125) / (10^{-3})$$

$$Re = 5087.5$$

Since $Re > 2000$, the flow is turbulent.

(ii) Air is moving fast horizontally past an airplane. The speed over the top surface is 60 m/s and under the bottom surface is 45 m/s. Calculate the difference in pressure.

$$\Delta P = 0.5 \rho (v_{\text{bottom}}^2 - v_{\text{top}}^2)$$

$$\rho_{\text{air}} = 1.225 \text{ kg/m}^3$$

$$\Delta P = 0.5 \times 1.225 \times (45^2 - 60^2)$$

$$\Delta P = 0.5 \times 1.225 \times (2025 - 3600)$$

$$\Delta P = 0.5 \times 1.225 \times (-1575)$$

$$\Delta P = -964.7 \text{ Pa}$$

The negative sign indicates lower pressure on the top surface.

2. (a) Define the following terms:

(i) Damped oscillations

Oscillations whose amplitude gradually decreases over time due to energy loss.

(ii) Forced oscillations

Oscillations maintained by an external periodic force.

(iii) Resonance

A condition where an external force matches the natural frequency of a system, increasing amplitude.

(b) (i) What is meant by the Doppler effect?

The apparent change in frequency due to the motion of the source or observer.

(ii) Write down three uses of the Doppler effect.

- Speed detection by police radar.
- Blood flow monitoring in medical ultrasound.
- Determining the motion of stars and galaxies.

(c) A source of sound emits waves of frequency f , and is moving with a speed of u_0 towards the listener and away from the listener. Derive an expression for apparent frequency f' of sound in each case if the velocity of sound wave in air is V .

For motion towards the listener:

$$f' = (V / (V - u_0)) f$$

For motion away from the listener:

$$f' = (V / (V + u_0)) f$$

(d) (i) A whistle emitting a sound of frequency 440 Hz is tied to a string of length 1.5 m and rotated with an angular velocity of 20 rad/s in the horizontal plane.

Velocity of whistle $v = \omega r$

$$v = 20 \times 1.5$$

$$v = 30 \text{ m/s}$$

Maximum frequency:

$$f_{\text{max}} = (V / (V - v)) f$$

$$f_{\text{max}} = (343 / (343 - 30)) \times 440$$

$$f_{\text{max}} = (343 / 313) \times 440$$

$$f_{\text{max}} = 482.2 \text{ Hz}$$

Minimum frequency:

$$f_{\min} = (V / (V + v)) f$$

$$f_{\min} = (343 / (343 + 30)) \times 440$$

$$f_{\min} = (343 / 373) \times 440$$

$$f_{\min} = 404.4 \text{ Hz}$$

Range of frequencies heard is 404.4 Hz to 482.2 Hz.

(ii) A police on duty detects a drop of 10% in the pitch of the horn of a moving car as it crosses him. Calculate the speed of the car.

$$f_{\text{approaching}} = f$$

$$f_{\text{receding}} = 0.9 f$$

Using Doppler formula for receding source:

$$0.9 f = (V / (V + u)) f$$

$$0.9 = 343 / (343 + u)$$

$$0.9 (343 + u) = 343$$

$$308.7 + 0.9 u = 343$$

$$0.9 u = 34.3$$

$$u = 38.1 \text{ m/s}$$

The speed of the car is 38.1 m/s.

3. (a) (i) What is meant by the statement that light is plane polarized?

Plane polarized light is light in which the vibrations of the electric field vector are confined to a single plane perpendicular to the direction of wave propagation.

(ii) State Brewster's law.

Brewster's law states that when unpolarized light is incident at a particular angle on a transparent surface, the reflected light is completely polarized perpendicular to the plane of incidence. The Brewster angle θ_B is given by:

$$\tan \theta_B = n_2 / n_1$$

where n_1 and n_2 are the refractive indices of the two media.

(iii) Sunlight is reflected from a calm lake. The reflected sunlight is totally polarized. What is the angle between the sun and the horizon?

The angle of reflection is equal to Brewster's angle:

$$\tan \theta_B = n_2 / n_1$$

For air and water,

$$\tan \theta_B = 1.33 / 1.00$$

$$\theta_B = \tan^{-1}(1.33)$$

$$\theta_B = 53.1^\circ$$

The angle between the sun and the horizon is:

$$90^\circ - 53.1^\circ = 36.9^\circ$$

(b) (i) State four conditions for sustained interference of light.

- The two sources of light must be coherent, meaning they must have a constant phase difference.
- The two sources must emit light of the same wavelength or frequency.
- The sources should have approximately the same amplitude for maximum contrast.
- The path difference between the interfering waves should be an integral multiple of the wavelength for constructive interference and a half-integral multiple for destructive interference.

(ii) In a Young's double slit experiment, the interval between the slits is 0.2 mm. For light of wavelength 6.0×10^{-7} m, interference fringes are formed on a screen at a distance of 0.8 m. Find the distance of the second dark fringe from the central fringe.

The distance of a dark fringe is given by:

$$y_n = (n + 0.5) \lambda L / d$$

where

$n = 2$ (for the second dark fringe)

$\lambda = 6.0 \times 10^{-7}$ m

$L = 0.8$ m

$d = 0.2 \times 10^{-3}$ m

$$y_2 = (2 + 0.5) \times (6.0 \times 10^{-7}) \times (0.8) / (0.2 \times 10^{-3})$$

$$y_2 = (2.5 \times 6.0 \times 10^{-7} \times 0.8) / (0.2 \times 10^{-3})$$

$$y_2 = (1.2 \times 10^{-6}) / (0.2 \times 10^{-3})$$

$$y_2 = 6.0 \times 10^{-3} \text{ m}$$

$$y_2 = 6.0 \text{ mm}$$

(c) (i) Distinguish between diffraction and diffraction grating.

Diffraction is the bending of light around obstacles or the spreading of light when it passes through a narrow aperture.

A diffraction grating is an optical component with multiple slits or grooves that diffracts light into several orders, producing a spectrum of colors.

(ii) A parallel beam of monochromatic light is incident normally on a diffraction grating. The angle between the two first-order spectra on either side of the normal is 30° . Assume that the wavelength of light is 5893×10^{-10} m. Find the number of rulings per mm on the grating and the greatest number of bright images obtained.

The diffraction grating equation is:

$$n \lambda = d \sin \theta$$

For first-order diffraction ($n = 1$):

$$d = \lambda / \sin \theta$$

$$\lambda = 5893 \times 10^{-10} \text{ m}$$

$$\theta = 30^\circ$$

$$d = (5893 \times 10^{-10}) / \sin(30^\circ)$$

$$d = (5893 \times 10^{-10}) / 0.5$$

$$d = 1.1786 \times 10^{-6} \text{ m}$$

Number of rulings per mm:

$$N = 1 / d$$

$$N = 1 / (1.1786 \times 10^{-6})$$

$$N = 8.48 \times 10^5 \text{ lines per meter}$$

$$N = 848 \text{ lines per mm}$$

The maximum number of bright images is given by:

$$n_{\text{max}} = d / \lambda$$

$$n_{\text{max}} = (1.1786 \times 10^{-6}) / (5893 \times 10^{-10})$$

$$n_{\text{max}} = 2$$

4. (a) Define the following materials as classified on the basis of elastic properties.

(i) Ductile materials

Ductile materials can be stretched into thin wires without breaking. Examples include copper and aluminum.

(ii) Brittle materials

Brittle materials break or shatter without significant deformation when subjected to stress. Examples include glass and ceramics.

(iii) Elastomers

Elastomers are materials that can undergo large deformations and return to their original shape when the stress is removed. Examples include rubber and silicone.

(b) (i) Briefly explain why the stretching of a coil spring is determined by its shear modulus.

A coil spring deforms by shear rather than direct tensile or compressive stress. The applied force causes adjacent coils to slide over each other, making the deformation primarily a result of shear stress. The shear modulus (G) determines the material's resistance to shear deformation, which in turn governs the elasticity of the spring.

(ii) A copper wire of negligible mass, 1m long and cross-sectional area 10^{-6} m^2 is kept on a smooth horizontal table with one end fixed. A ball of 1kg is attached to the other end. The wire and the ball are rotating with an angular velocity of 35 rad/s. If the elongation of the wire is 10^{-3} m , find Young's modulus of the wire.

The centripetal force exerted by the rotating ball is given by:

$$F = m \omega^2 r$$

where

$$m = 1 \text{ kg}$$

$$\omega = 35 \text{ rad/s}$$

$$r = 1 \text{ m}$$

$$F = (1) \times (35)^2 \times (1)$$

$$F = 1225 \text{ N}$$

Using Hooke's Law:

$$Y = (F L) / (A \Delta L)$$

$$L = 1 \text{ m}$$

$$A = 10^{-6} \text{ m}^2$$

$$\Delta L = 10^{-3} \text{ m}$$

$$Y = (1225 \times 1) / (10^{-6} \times 10^{-3})$$

$$Y = (1225) / (10^{-9})$$

$$Y = 1.225 \times 10^{12} \text{ N/m}^2$$

If the angular velocity is increased to 100 rad/s,

$$F_{\text{new}} = 1 \times (100)^2 \times 1$$

$$F_{\text{new}} = 10000 \text{ N}$$

The breaking stress is:

$$\sigma = F / A$$

$$\sigma = (10000) / (10^{-6})$$

$$\sigma = 1.0 \times 10^{10} \text{ N/m}^2$$

(c) (i) Differentiate bulk modulus from shear modulus.

The bulk modulus (K) measures a material's resistance to uniform compression, while the shear modulus (G) measures its resistance to shape deformation without volume change.

(ii) Two wires, one of steel and one of phosphor bronze, each 1.5m long and 2mm in diameter, are joined end to end as a composite wire of length 3m. What tension in the composite wire will produce total extension of 0.064cm?

Total extension is given by:

$$\Delta L = (F L_1) / (A Y_1) + (F L_2) / (A Y_2)$$

where

$$L_1 = L_2 = 1.5 \text{ m}$$

$$A = \pi (0.002)^2 / 4$$

$$Y_1 (\text{steel}) = 2 \times 10^{11} \text{ N/m}^2$$

$$Y_2 (\text{phosphor bronze}) = 1 \times 10^{11} \text{ N/m}^2$$

$$\Delta L = 0.064 \times 10^{-2} \text{ m}$$

Substituting values and solving for F gives:

$$F = 160 \text{ N}$$

5. (a) (i) Differentiate electric potential from electric potential difference.

Electric potential at a point is the work done in bringing a unit positive charge from infinity to that point in an electric field.

Electric potential difference between two points is the work done in moving a unit positive charge from one point to another in an electric field.

(ii) Sketch a graph of variation of electrical potential from the centre of a hollow charged conducting sphere of radius r , up to infinity. Explain the shape of the graph.

For a hollow charged conducting sphere:

- Inside the sphere ($r < R$), the potential is constant and equal to the potential on the surface.
- On the surface ($r = R$), the potential is given by $V = kQ / R$.
- Outside the sphere ($r > R$), the potential follows an inverse relation, decreasing as $V = kQ / r$.

The graph is constant from $r = 0$ to $r = R$ and then decreases as $1/r$ for $r > R$.

(b) Two bodies A and B are 0.1 m apart. A point charge of 3×10^{-6} C is placed at A and a point charge of -1×10^{-6} C is placed at B. C is the point on the straight line between A and B where the electric potential is zero. Calculate the distance between A and C.

The electric potential at any point due to a charge Q is given by:

$$V = k Q / r$$

For point C, the sum of potentials due to A and B must be zero:

$$(k \times 3 \times 10^{-6}) / x = (k \times 1 \times 10^{-6}) / (0.1 - x)$$

Cancel k :

$$(3 \times 10^{-6}) / x = (1 \times 10^{-6}) / (0.1 - x)$$

Cross multiply:

$$3 (0.1 - x) = x$$

$$0.3 - 3x = x$$

$$0.3 = 4x$$

$$x = 0.075 \text{ m}$$

The distance between A and C is 0.075 m.

(c) A square ABCD has each side of 100 cm. Four point charges of $+0.04 \mu\text{C}$, $-0.05 \mu\text{C}$, $+0.06 \mu\text{C}$, and $+0.05 \mu\text{C}$ are placed at A, B, C, and D respectively. Calculate the electric potential at the centre of the square.

Electric potential at the center due to a charge Q at a distance r is:

$$V = k Q / r$$

The distance from each corner to the center is:

$$r = (\sqrt{2} / 2) \times \text{side}$$

$$r = (\sqrt{2} / 2) \times 1$$

$$r = 0.707 \text{ m}$$

Total potential at the center:

$$V_{\text{total}} = k (Q_A + Q_B + Q_C + Q_D) / r$$

$$V_{\text{total}} = (9 \times 10^9) \times (0.04 - 0.05 + 0.06 + 0.05) \times 10^{-6} / 0.707$$

$$V_{\text{total}} = (9 \times 10^9) \times (0.10) \times 10^{-6} / 0.707$$

$$V_{\text{total}} = (9 \times 10^3) / 0.707$$

$$V_{\text{total}} = 12.73 \text{ kV}$$

6. (a) (i) What do you understand by dielectric constant?

Dielectric constant (relative permittivity) is the ratio of the capacitance of a capacitor with a dielectric to the capacitance of the same capacitor in a vacuum.

(ii) When are the capacitors said to be connected in parallel?

Capacitors are connected in parallel when their positive plates are connected to one terminal and their negative plates to another, so they share the same voltage. The total capacitance is given by:

$$C_{\text{total}} = C_1 + C_2 + \dots + C_n$$

(b) The parallel plate capacitor consisting of two metal plates each of area 20 cm^2 placed at 1 cm apart are connected to the terminals of an electrostatic voltmeter. The system is charged to give a reading of 120V on the voltmeter scale. When the space between the plates is filled with a glass of dielectric constant of 5, the voltmeter reading falls to 50V. What is the capacitance of the voltmeter?

The capacitance of a parallel plate capacitor is given by:

$$C = (\epsilon_0 \epsilon_r A) / d$$

Initial capacitance (without dielectric):

$$C_0 = (8.85 \times 10^{-12} \times 1 \times 20 \times 10^{-4}) / (1 \times 10^{-2})$$

$$C_0 = (1.77 \times 10^{-12}) / (1 \times 10^{-2})$$

$$C_0 = 1.77 \times 10^{-10} \text{ F}$$

Final capacitance with dielectric:

$$C = k C_0$$

$$C = 5 \times (1.77 \times 10^{-10})$$

$$C = 8.85 \times 10^{-10} \text{ F}$$

(c) A $4.0 \mu\text{F}$ capacitor is charged by a 12V supply and is then discharged through a $1.5 \text{ M}\Omega$ resistor.

(i) Obtain the time constant.

The time constant is given by:

$$\tau = RC$$

$$\tau = (1.5 \times 10^6) \times (4.0 \times 10^{-6})$$

$$\tau = 6.0 \text{ s}$$

(ii) Calculate the charge on the capacitor at the start of the discharge.

$$Q_0 = C V$$

$$Q_0 = (4.0 \times 10^{-6}) \times (12)$$

$$Q_0 = 4.8 \times 10^{-5} \text{ C}$$

(iii) What will be the value of the charge on the capacitor, the potential difference across the capacitor, and the current in the circuit 2 seconds after the discharge starts?

Charge on the capacitor:

$$Q = Q_0 e^{-(t/\tau)}$$

$$Q = (4.8 \times 10^{-5}) e^{-2/6}$$

$$Q = (4.8 \times 10^{-5}) e^{-0.333}$$

$$Q = (4.8 \times 10^{-5}) \times 0.717$$

$$Q = 3.44 \times 10^{-5} \text{ C}$$

Potential difference:

$$V = Q / C$$

$$V = (3.44 \times 10^{-5}) / (4.0 \times 10^{-6})$$

$$V = 8.6 \text{ V}$$

Current:

$$I = V / R$$

$$I = (8.6) / (1.5 \times 10^6)$$

$$I = 5.73 \times 10^{-6} \text{ A}$$

The charge on the capacitor after 2 seconds is $3.44 \times 10^{-5} \text{ C}$, the potential difference is 8.6 V, and the current in the circuit is $5.73 \mu\text{A}$.

7. (a) (i) Distinguish between self-inductance and mutual inductance.

Self-inductance is the property of a coil by which it opposes the change in current flowing through it by inducing an electromotive force (emf) in itself. It is given by

$$\varepsilon = -L (dI/dt)$$

where L is the self-inductance of the coil.

Mutual inductance is the property of two coils in which a change in current in one coil induces an emf in the other coil due to the magnetic flux linkage between them. It is given by

$$\varepsilon = -M (dI/dt)$$

where M is the mutual inductance between the two coils.

(ii) A horizontal straight wire 0.05m long weighing 2.4 g m^{-1} is placed perpendicular to a uniform horizontal magnetic field of flux density 0.8T. If the resistance of the wire is 7.62 m^{-1} , calculate the potential difference that has to be applied between the ends of the wire to make it just self-supporting.

The force per unit length due to the magnetic field is given by

$$F = B I L$$

For the wire to be self-supporting, the magnetic force must balance the gravitational force:

$$B I L = mg$$

Rearrange to find the current I:

$$I = mg / (B L)$$

The mass per unit length is $2.4\text{g/m} = 2.4 \times 10^{-3} \text{ kg/m}$, and $g = 9.81 \text{ m/s}^2$.

$$I = (2.4 \times 10^{-3} \times 9.81) / (0.8 \times 0.05)$$

$$I = (2.3544 \times 10^{-2}) / (0.04)$$

$$I = 0.5886 \text{ A}$$

The potential difference is given by

$$V = I R L$$

$$V = (0.5886) \times (7.62) \times (0.05)$$

$$V = (0.5886 \times 0.381)$$

$$V = 0.224 \text{ V}$$

(b) Two very long wires made of copper and of equal lengths are placed parallel to each other in such a way that they are 10 cm apart. If the total power dissipated in the two wires is 75W, find the force between them if the resistivity of the copper wire is $1.69 \times 10^{-8} \Omega\text{m}$ and of diameter 2mm.

Resistance of a wire is given by

$$R = \rho L / A$$

where

$$\rho = 1.69 \times 10^{-8} \Omega\text{m}$$

$$A = \pi (d/2)^2 = \pi (1 \times 10^{-3})^2$$

$$A = \pi \times 10^{-6}$$

$$A = 3.1416 \times 10^{-6} \text{ m}^2$$

$$R = (1.69 \times 10^{-8} \times L) / (3.1416 \times 10^{-6})$$

$$R = (5.38 \times 10^{-3} L)$$

Using $P = I^2 R$,

$$I = \sqrt{P / R}$$

$$I = \sqrt{75 / (5.38 \times 10^{-3} L)}$$

Force per unit length between two parallel current-carrying wires is

$$F/L = (\mu_0 I^2) / (2\pi d)$$

where

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$d = 0.1 \text{ m}$$

$$F/L = (4\pi \times 10^{-7} \times (75 / (5.38 \times 10^{-3} \text{ L}))^2) / (2\pi \times 0.1)$$

$$F/L = 2.09 \times 10^{-4} \text{ N/m}$$

(c) Explain the statement that a sinusoidal current of peak value 5A passed through an AC ammeter reads $5 / \sqrt{2}$ A.

The AC ammeter measures the root mean square (RMS) value of current, which is given by

$$I_{\text{rms}} = I_{\text{peak}} / \sqrt{2}$$

$$\text{For } I_{\text{peak}} = 5\text{A},$$

$$I_{\text{rms}} = 5 / \sqrt{2}$$

$$I_{\text{rms}} = 5 / 1.414$$

$$I_{\text{rms}} = 3.54 \text{ A}$$

Thus, the ammeter reads 3.54 A instead of the peak value.

(d) (i) Show that the average power transferred to an AC circuit is given by $P = E_{\text{rms}} I_{\text{rms}} (R / Z)$, where R is the resistance in the circuit and Z is the impedance.

The instantaneous power in an AC circuit is given by

$$P = VI \cos(\theta)$$

Since V and I are sinusoidal, their RMS values are used:

$$P_{\text{avg}} = V_{\text{rms}} I_{\text{rms}} \cos(\theta)$$

Impedance is given by

$$\cos(\theta) = R / Z$$

Substituting:

$$P = E_{\text{rms}} I_{\text{rms}} (R / Z)$$

(ii) A coil which has an inductance of 0.2H and negligible resistance is in series with a resistor whose resistance is 60Ω. The pair is connected across a 50V RMS supply alternating at 100/π Hz. Calculate the total impedance of the circuit and its power factor.

The inductive reactance is given by

$$X_L = 2\pi f L$$

$$f = 100/\pi \text{ Hz}$$

$$X_L = 2\pi (100/\pi) \times 0.2$$

$$X_L = 2 \times 100 \times 0.2$$

$$X_L = 40\Omega$$

The impedance is given by

$$Z = \sqrt{R^2 + X_L^2}$$

$$Z = \sqrt{60^2 + 40^2}$$

$$Z = \sqrt{3600 + 1600}$$

$$Z = \sqrt{5200}$$

$$Z = 72.1 \Omega$$

Power factor is given by

$$\cos(\theta) = R / Z$$

$$\cos(\theta) = 60 / 72.1$$

$$\cos(\theta) = 0.832$$

The total impedance of the circuit is 72.1 Ω and the power factor is 0.832.

8. (a) (i) Show that the de Broglie hypothesis of matter waves is in agreement with Bohr's theory. Bohr's theory postulates that an electron in an atom moves in circular orbits and that its angular momentum is quantized as

$$mvr = n(h/2\pi)$$

where m is the electron's mass, v is its velocity, r is the radius of the orbit, h is Planck's constant, and n is the principal quantum number.

The de Broglie hypothesis states that matter has a wave nature with a wavelength given by

$$\lambda = h/p = h/(mv)$$

For an electron to be in a stable orbit, its wavelength must fit into the circumference of the orbit as

$$2\pi r = n\lambda$$

Substituting $\lambda = h/(mv)$, we get

$$2\pi r = n(h/mv)$$

Rearranging, we obtain

$$mvr = n(h/2\pi)$$

which is exactly Bohr's quantization condition, confirming that the de Broglie hypothesis is consistent with Bohr's theory.

(ii) A 10 kg satellite circles the Earth once every 2 hours in an orbit having a radius of 8000 km. Assuming Bohr's angular momentum postulate applies to the satellite just as it does to an electron in the hydrogen atom, find the quantum number of the orbit of the satellite.

Answer:

Given:

Mass of satellite, $m = 10 \text{ kg}$

Orbital radius, $r = 8000 \text{ km} = 8.0 \times 10^6 \text{ m}$

Orbital period, $T = 2 \text{ hours} = 2 \times 3600 \text{ s} = 7200 \text{ s}$

The velocity of the satellite is given by

$$\begin{aligned} v &= 2\pi r/T \\ &= (2 \times 3.1416 \times 8.0 \times 10^6) / 7200 \\ &= 698.13 \text{ m/s} \end{aligned}$$

Using Bohr's quantization condition:

$$mvr = n(h/2\pi)$$

Solving for n :

$$\begin{aligned} n &= (mvr) / (h/2\pi) \\ &= (10 \times 698.13 \times 8.0 \times 10^6) / (6.626 \times 10^{-34} / 2\pi) \end{aligned}$$

Computing the value,

$$n \approx 8.46 \times 10^{39}$$

So the quantum number of the satellite's orbit is approximately 8.46×10^{39} .

(b) A muon is a particle which has the same charge as an electron but its mass is 207 times the mass of an electron. An unusual atom similar to hydrogen has been created consisting of a muon orbiting a single proton. An energy level diagram for this atom is shown in Figure 1.

(i) Determine the ionization energy of this atom in joules.

Answer:

Ionization energy is the energy required to remove the muon from the ground state, which is given as -2810 eV.

Converting to joules:

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$\begin{aligned} \text{Ionization energy} &= 2810 \times 1.602 \times 10^{-19} \\ &= 4.5 \times 10^{-16} \text{ J} \end{aligned}$$

(ii) Calculate the maximum possible wavelength of a photon which when absorbed will be able to ionize this atom.

Answer:

Using the energy-wavelength relation:

$$E = hc/\lambda$$

Solving for λ :

$$\lambda = hc/E$$

where

$$h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$$

$$c = 3.0 \times 10^8 \text{ m/s}$$

$$E = 4.5 \times 10^{-16} \text{ J}$$

$$\lambda = (6.626 \times 10^{-34} \times 3.0 \times 10^8) / (4.5 \times 10^{-16})$$

Computing the value,

$$\lambda \approx 4.42 \times 10^{-7} \text{ m or } 442 \text{ nm}$$

(iii) Calculate the de Broglie wavelength of muon traveling at 11 percent of the speed of light.

Answer:

Given:

$$\text{Velocity of muon, } v = 0.11c = 0.11 \times 3.0 \times 10^8 = 3.3 \times 10^7 \text{ m/s}$$

$$\text{Mass of muon, } m = 207 \times 9.11 \times 10^{-31} \text{ kg} = 1.89 \times 10^{-28} \text{ kg}$$

Using the de Broglie equation:

$$\lambda = h / (mv)$$

$$\lambda = (6.626 \times 10^{-34}) / (1.89 \times 10^{-28} \times 3.3 \times 10^7)$$

Computing the value,

$$\lambda \approx 1.05 \times 10^{-14} \text{ m}$$

(c) Why are the energy levels labeled with negative energies?

Energy levels are labeled with negative values because the electron (or in this case, the muon) is bound to the nucleus. The energy required to remove the particle from the atom (ionization energy) must be positive, and thus bound states have negative total energy relative to a free state. When the muon moves to higher energy levels, it requires energy input, making its energy less negative.

(d) Ultraviolet light of wavelength $3600 \times 10^{-10} \text{ m}$ is made to fall on a smooth surface of potassium. Determine:

(i) The maximum energy of emitted photoelectrons.

Using the photon energy equation:

$$E = hc/\lambda$$

where

$$h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$$

$$c = 3.0 \times 10^8 \text{ m/s}$$

$$\lambda = 3600 \times 10^{-10} \text{ m}$$

$$E = (6.626 \times 10^{-34} \times 3.0 \times 10^8) / (3600 \times 10^{-10})$$

Computing the value,

$$E \approx 5.52 \times 10^{-19} \text{ J}$$

Converting to eV:

$$E \approx (5.52 \times 10^{-19}) / (1.602 \times 10^{-19})$$

$$E \approx 3.45 \text{ eV}$$

(ii) The stopping potential.

The stopping potential is given by:

$$V = E/q$$

Since E is the energy of the emitted electrons and $q = e$ (electron charge),

$$V = 3.45 \text{ V}$$

(iii) The velocity of the most energetic photoelectrons given that work function for potassium is 2 eV.

The kinetic energy of the photoelectrons is given by:

$$KE = E - \text{work function}$$

$$KE = 3.45 - 2$$

$$KE = 1.45 \text{ eV}$$

Converting to joules:

$$KE = 1.45 \times 1.602 \times 10^{-19} \text{ J}$$

$$KE = 2.32 \times 10^{-19} \text{ J}$$

Using the kinetic energy formula:

$$KE = 0.5 mv^2$$

Solving for v:

$$v = \sqrt{(2KE/m)}$$

where

$$m = 9.11 \times 10^{-31} \text{ kg}$$

$$v = \sqrt{((2 \times 2.32 \times 10^{-19}) / (9.11 \times 10^{-31}))}$$

Computing the value,

$$v \approx 7.13 \times 10^5 \text{ m/s}$$

9. (a) (i) Define activity and half-life.

Activity is the rate at which a radioactive substance decays, measured in becquerels (Bq) or curies (Ci).

Half-life is the time taken for half of the radioactive substance to decay.

(ii) Give any four uses of LASER light.

- In surgery for precision cutting and eye correction
- In communication through optical fibers
- In industrial cutting and welding
- In barcode scanners and CD/DVD reading

(iii) The isotope ^{40}K with a half-life of 1.37×10^9 years decays to ^{40}Ar , which is stable. Moon rocks from the sea of tranquility show that the ratio of these potassium atoms to argon atoms is 1/7. Determine the age of the moon rock.

The ratio of parent to daughter atoms is given by

$$N/N_0 = 1/(1 + 7) = 1/8$$

Using the decay equation

$$N/N_0 = (1/2)^{(t/T)}$$

$$1/8 = (1/2)^{(t/1.37 \times 10^9)}$$

Taking logarithm

$$\log(1/8) = (t/1.37 \times 10^9) \log(1/2)$$

$$-0.903 = (t/1.37 \times 10^9) (-0.301)$$

$$t = (0.903 \times 1.37 \times 10^9) / 0.301$$

$$t \approx 4.11 \times 10^9 \text{ years}$$

(iv) The half-life of a radioactive substance is 1 hour. How long will it take for 60% of the substance to decay?

$$\text{Remaining fraction} = 100\% - 60\% = 40\% = 0.4$$

Using decay equation

$$N/N_0 = (1/2)^{(t/T)}$$

$$0.4 = (1/2)^{(t/1)}$$

Taking logarithm

$$\log 0.4 = (t/1) \log (1/2)$$

$$-0.398 = t (-0.301)$$

$$t = 0.398 / 0.301$$

$$t \approx 1.32 \text{ hours}$$

(b) (i) What is a nuclear reactor?

A nuclear reactor is a system that sustains and controls a nuclear fission chain reaction to generate heat, which is used to produce electricity.

(ii) Briefly explain any three main components in a nuclear reactor.

- Fuel rods: Contain fissile material such as uranium-235 or plutonium-239, which undergo fission.
- Moderator: Slows down neutrons to sustain the chain reaction, commonly water or graphite.
- Control rods: Absorb excess neutrons to regulate the reaction, usually made of cadmium or boron.

(c) (i) Sketch the binding energy curve.

(A sketch of the binding energy per nucleon vs. mass number curve should be drawn, showing a peak around iron-56.)

(ii) State any two conclusions that can be drawn from the curve in 9 (c) (i).

- Light nuclei can undergo fusion to release energy due to an increase in binding energy per nucleon.
- Heavy nuclei can undergo fission to release energy because splitting increases binding energy per nucleon.

(d)

If the mass of deuterium nucleus is 2.015 a.m.u, that of one isotope of helium is 3.017 a.m.u and that of neutron is 1.009 a.m.u, calculate the energy released by the fusion of 1kg of deuterium. Suppose 50% of this energy was used to produce 1MW of electricity, for how many days would be able to function.

Mass defect per reaction

$$\Delta m = 2(2.015) - (3.017 + 1.009)$$

$$= 4.030 - 4.026$$

$$= 0.004 \text{ a.m.u}$$

Energy released per reaction

$$E = \Delta m \times 931 \text{ MeV}$$

$$= 0.004 \times 931$$

$$= 3.724 \text{ MeV}$$

Number of deuterium nuclei in 1kg

$$\text{Number of nuclei} = (1\text{kg} \times \text{Avogadro's number}) / \text{Molar mass of deuterium}$$

$$= (1000\text{g} \times 6.022 \times 10^{23}) / 2$$

$$= 3.011 \times 10^{26} \text{ nuclei}$$

Total energy released

$$= 3.011 \times 10^{26} \times 3.724 \text{ MeV}$$

$$= 1.12 \times 10^{37} \text{ eV}$$

Converting to joules

$$1\text{eV} = 1.602 \times 10^{-19} \text{ J}$$

$$E = 1.12 \times 10^{37} \times 1.602 \times 10^{-19}$$

$$= 1.8 \times 10^{18} \text{ J}$$

Useful energy (50%)

$$= 0.5 \times 1.8 \times 10^{18}$$

$$= 9 \times 10^{17} \text{ J}$$

Operating time at 1MW (1MW = 10^6 J/s)

$$t = 9 \times 10^{17} \text{ J} / 10^6 \text{ J/s}$$

$$t = 9 \times 10^{11} \text{ s}$$

Converting to days

$$t = (9 \times 10^{11}) / (86400)$$

$$t \approx 1.04 \times 10^7 \text{ days}$$