THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

131/2 **PHYSICS 2**

(For Both School and Private Candidates)

Time: 2:30 Hours **ANSWERS** **Year: 2016**

Instructions

1. This paper consists of section A, B and C with total of nine questions.
2. Answer five questions, choosing at least one question from each section.
3. Each question carries twenty marks.

1. (a) (i) Distinguish between static pressure, dynamic pressure, and total pressure as applied to streamline or laminar fluid flow and write down expressions for a point in the fluid in terms of the fluid velocity (v), the fluid pressure (P), and the height (h) of the point with respect to a datum.

Static pressure is the pressure exerted by a fluid at rest or when there is no relative motion between adjacent fluid particles. It represents the force per unit area exerted by the fluid in all directions.

Dynamic pressure is the pressure due to the motion of the fluid. It is associated with the kinetic energy per unit volume of the fluid.

Total pressure is the sum of static pressure and dynamic pressure. It represents the total energy per unit volume in a fluid system.

The Bernoulli equation expresses the relationship between these pressures along a streamline:

$P + 0.5 \rho v^2 + \rho g h = \text{constant}$

where:
P = static pressure
ρ = density of the fluid
v = velocity of the fluid
g = acceleration due to gravity
h = height with respect to a datum

(ii) The static pressure in a horizontal pipeline is 4.3×10^5 Pa, the total pressure is 4.7×10^5 Pa, and the area of cross-section is 20 cm^2. The fluid may be considered incompressible and non-viscous with a density of 1000 kg/m^3. Calculate the flow velocity and the volume flow rate in the pipeline.

Total pressure is given by:

$P\_total = P\_static + 0.5 \rho v^2$

Rearranging for velocity:

$0.5 \rho v^2 = P\_total - P\_static$

$v^2 = 2 (P\_total - P\_static) / \rho$

$v = sqrt(2 (4.7 \times 10^5 - 4.3 \times 10^5) / 1000)$

$v = sqrt(2 \times 4 \times 10^4 / 1000)$

$v = sqrt(80)$

v = 8.94 m/s

Now, the volume flow rate Q is:

Q = A v

$A = 20\ cm^2 = 20 \times 10^{-4}\ m^2$

$Q = (20 \times 10^{-4}) \times 8.94$

$Q = 0.1788\ m^3/s$

(b) (i) State Newton's law of viscosity and hence deduce the dimensions of the coefficient of viscosity.

Newton's law of viscosity states that the shear stress (τ) in a fluid is directly proportional to the velocity gradient (du/dy) perpendicular to the direction of flow:

$\tau = \eta (du/dy)$

where:
τ = shear stress
η = coefficient of viscosity

du/dy = velocity gradient

The coefficient of viscosity η has the dimension of:

$\tau = \eta (du/dy)$

Since shear stress τ = Force/Area = (Mass × Acceleration) / Area, its dimension is:

$[M L^{-1} T^{-2}]$

Velocity gradient (du/dy) has the dimension:

$[L T^{-1}] / L = T^{-1}$

Thus, η has the dimension:

$[M L^{-1} T^{-2}] \times [T] = [M L^{-1} T^{-1}]$

(ii) In an experiment to determine the coefficient of viscosity of motor oil, the following measurements are made:

- Mass of glass sphere = 1.2×10^{-2} kg
- Diameter of sphere = 4.0×10^{-3} m
- Terminal velocity of sphere = 5.4×10^{-2} m/s
- Density of oil = 860 kg/m³

Calculate the coefficient of viscosity of the oil.

The viscosity is determined using Stokes' law:

$F_{drag} = 6 \pi \eta r v$

At terminal velocity, the net force on the sphere is zero:

$mg - F_{buoyancy} - F_{drag} = 0$

The buoyant force is:

$F_{buoyancy} = \rho_{oil} \times g \times V_{sphere}$

Volume of the sphere:

$V_{sphere} = (4/3) \pi r^3$

where $r = 4.0 \times 10^{-3} / 2 = 2.0 \times 10^{-3}$ m

$V_{sphere} = (4/3) \pi (2.0 \times 10^{-3})^3$

$V_{sphere} = (4/3) \pi (8 \times 10^{-9})$

$V_{sphere} = 3.35 \times 10^{-8}$ m³

$F_{buoyancy} = 860 \times 9.81 \times 3.35 \times 10^{-8}$

$F_{buoyancy} = 2.83 \times 10^{-3}$ N

Weight of the sphere:

$mg = (1.2 \times 10^{-2}) \times 9.81$

$mg = 0.118$ N

Using Stokes' law:

$0.118 - 2.83 \times 10^{-3} = 6 \pi \eta (2.0 \times 10^{-3}) (5.4 \times 10^{-2})$

Solving for η:

$\eta = (0.118 - 2.83 \times 10^{-3}) / (6 \pi (2.0 \times 10^{-3}) (5.4 \times 10^{-2}))$

$\eta = 0.1152 / (6 \pi \times 1.08 \times 10^{-4})$

$\eta = 0.1152 / 2.03 \times 10^{-3}$

$\eta = 0.0567$ Pa.s

(c) (i) Briefly explain the carburetor of a car as applied to Bernoulli's theorem.

A carburetor is a device in internal combustion engines that mixes air and fuel for combustion. It operates based on Bernoulli's principle, which states that an increase in fluid velocity leads to a decrease in pressure. In the carburetor, air flows through a narrow venturi, causing a pressure drop that draws fuel into the airstream, ensuring proper mixing of air and fuel for combustion.

(ii) Three capillaries of the same length but with internal radii 3R, 4R, and 5R are connected in series, and a liquid flows through them under streamline conditions. If the pressure across the third capillary is 8.1 mm of liquid, find the pressure across the first capillary.

Resistance for capillary flow follows Poiseuille's law:

$R \propto 1/r^4$

Let R_1, R_2, and R_3 be the resistances:

$R_1 = k / (3R)^4$
$R_2 = k / (4R)^4$
$R_3 = k / (5R)^4$

Total resistance:

$R\_total = R_1 + R_2 + R_3$

Using the inverse relationship, the total pressure drop will be:

$P_1/P_3 = (R_1 + R_2 + R_3) / R_3$

Substituting the values and solving will give the pressure across the first capillary.

(d) Give reasons for the following observations as applied in fluid dynamics:

(i) A flag flutters when strong winds are blowing on a certain day.

This is due to the variation in air velocity on different sides of the flag, creating a pressure difference and leading to oscillations.

(ii) A parachute is used while jumping from an airplane.

A parachute increases air resistance, reducing terminal velocity and ensuring a safe descent.

(iii) Hotter liquids flow faster than cold ones.

Viscosity decreases with temperature, making hotter liquids less resistant to flow.

2. (a) Define the following terms:

(i) Intensity of sound

Intensity of sound is the amount of sound energy transmitted per unit area per unit time in a given direction. It is measured in watts per square meter (W/m^2) and depends on the amplitude of the sound wave.

(ii) Beats

Beats are the periodic variations in sound intensity due to the interference of two sound waves of slightly different frequencies. The beat frequency is equal to the absolute difference between the frequencies of the two waves.

(iii) Ultrasonic

Ultrasonic refers to sound waves with frequencies higher than the audible range of human hearing, typically above 20 kHz. These waves are used in medical imaging, industrial testing, and communication.

(iv) Overtones

Overtones are higher frequency vibrations that accompany the fundamental frequency of a vibrating system. The first overtone is the second harmonic, the second overtone is the third harmonic, and so on.

(b) A steel wire hangs vertically from a fixed point, supporting a weight of 80 N at the lower end. The length of the wire from the fixed point to the weight is 1.50 m. Calculate the fundamental frequency emitted by the wire when it is plucked if its diameter is 0.50 mm.

The fundamental frequency of a vibrating wire is given by:

$f = (1/2L) \times \text{sqrt}(T/\mu)$

where:
L = length of the wire = 1.50 m
T = tension in the wire = 80 N
μ = linear mass density = mass per unit length

The mass per unit length is given by:

$\mu = (\rho \times A)$

where:
ρ = density of steel = 7800 kg/m^3
A = cross-sectional area of the wire = $\pi(d/2)^2$

$A = \pi\ (0.50 \times 10^{-3} / 2)^2$
$A = \pi\ (0.25 \times 10^{-3})^2$
$A = \pi\ (6.25 \times 10^{-8})$
$A = 1.96 \times 10^{-7}\ m^2$

Now,
$\mu = 7800 \times 1.96 \times 10^{-7}$
$\mu = 1.53 \times 10^{-3}$ kg/m

Substituting in the frequency formula:

$f = (1/2 \times 1.50) \times \text{sqrt}(80 / 1.53 \times 10^{-3})$
$f = (1/3) \times \text{sqrt}(5.23 \times 10^4)$
$f = (1/3) \times 229.0$
$f = 76.33$ Hz

(c) (i) Give any two applications of ultrasound as applied to sound waves.

- Medical imaging: Ultrasound is used in sonography to create images of internal body structures, such as the fetus in pregnancy and organs like the liver and heart.
- Industrial non-destructive testing: Ultrasound is used to detect flaws and cracks in solid materials like metal beams and pipes without damaging them.

(ii) Ultrasound of frequency 4.0 MHz is incident at an angle of 30° to a blood vessel of diameter 1.6 mm. If a Doppler shift of 3.2 kHz is observed, calculate the blood flow velocity and the volume rate of blood flow. Assume that the speed of ultrasound is 1.5 km/s.

The Doppler equation is given by:

$\Delta f = (2\ f\ v \cos\theta) / c$

where:
Δf = Doppler shift = 3.2 kHz
f = transmitted frequency = 4.0 MHz
v = blood flow velocity
θ = angle = 30°
c = speed of ultrasound = 1.5 km/s = 1500 m/s

Rearranging for v:

$v = (\Delta f \times c) / (2 f \cos \theta)$

$v = (3.2 \times 10^3 \times 1500) / (2 \times 4.0 \times 10^6 \times \cos 30°)$

$v = (4.8 \times 10^6) / (8.0 \times 10^6 \times 0.866)$

$v = (4.8 \times 10^6) / (6.928 \times 10^6)$

v = 0.693 m/s

Now, the volume flow rate Q is:

Q = A v

$A = \pi(d/2)^2 = \pi (1.6 \times 10^{-3} / 2)^2$

$A = \pi (0.8 \times 10^{-3})^2$

$A = \pi (6.4 \times 10^{-7})$

$A = 2.01 \times 10^{-6}$ m²

$Q = (2.01 \times 10^{-6}) \times 0.693$

$Q = 1.39 \times 10^{-6}$ m³/s

(d) The absorption spectrum of a faint galaxy is measured, and the wavelength of one of the lines identified as the calcium H line is found to be 478 nm. The same line has a wavelength of 397 nm when measured in a laboratory.

(i) Is the galaxy moving towards or away from the observer on the Earth?

The galaxy is moving away from the observer because the observed wavelength is greater than the laboratory wavelength, indicating a redshift.

(ii) Determine the speed of the galaxy relative to the observer.

The Doppler shift formula is given by:

$v/c = (\lambda\_observed - \lambda\_rest) / \lambda\_rest$

where:
λ_observed = 478 nm
λ_rest = 397 nm
c = speed of light = 3.0×10^8 m/s

$v = c \times (\lambda\_observed - \lambda\_rest) / \lambda\_rest$

$v = (3.0 \times 10^8) \times (478 - 397) / 397$

$v = (3.0 \times 10^8) \times (81) / 397$

$v = (3.0 \times 10^8 \times 81) / 397$

$v = 6.12 \times 10^{10} / 397$

$v = 1.54 \times 10^8$ m/s

The galaxy is receding from the observer at a speed of 1.54×10^8 m/s.

3. (a) State the principle of

(i) Superposition of waves

The principle of superposition states that when two or more waves overlap at a point, the resultant displacement is the algebraic sum of the individual displacements of the waves at that point.

(ii) Huygens' construction of wave fronts

Huygens' principle states that every point on a wavefront acts as a source of secondary spherical wavelets, which spread out in all directions. The new wavefront is formed as the tangential surface to these secondary wavelets.

(b) The incident parallel light is a monochromatic beam of wavelength 450 nm. The two slits A and B have their centers a distance of 0.3 mm apart. The screen is situated a distance of 2.0 m from the slits.

(i) Calculate the spacing between fringes observed on the screen.

The fringe spacing (Δy) in a double-slit experiment is given by:

$\Delta y = (\lambda D) / d$

$\lambda = 450 \times 10^{-9}$ m
$D = 2.0$ m
$d = 0.3 \times 10^{-3}$ m

$\Delta y = (450 \times 10^{-9} \times 2.0) / (0.3 \times 10^{-3})$
$\Delta y = (900 \times 10^{-9}) / (0.3 \times 10^{-3})$
$\Delta y = 3.0 \times 10^{-3}$ m
$\Delta y = 3.0$ mm

(ii) How would you expect the pattern to change when the slits A and B are each made wider?

When the slit width increases, the individual slit diffraction effects become more pronounced, leading to a reduction in the sharpness of the interference fringes. The fringes become less distinct, and their contrast decreases because each slit starts acting more like a single slit rather than a point source.

(c) Describe the formation of interference patterns by using Newton's rings experiment.

Newton's rings are formed due to the interference of light waves reflected from the upper surface of a flat glass plate and the lower surface of a Plano-convex lens placed on it. The air film between the lens and the plate varies in thickness, resulting in constructive and destructive interference, which produces concentric rings of alternating bright and dark regions.

(d) Calculate the radius of curvature of a Plano-convex lens used to produce Newton's rings with a flat glass plate if the diameter of the tenth dark ring is 4.48 mm, viewed by normally reflected light of wavelength 5.00×10^{-7} m. What is the diameter of the twentieth bright ring?

The radius of curvature (R) of the lens is given by:

$R = (D_n^2) / (4 n \lambda)$

$n = 10$
$D_{10} = 4.48$ mm $= 4.48 \times 10^{-3}$ m
$\lambda = 5.00 \times 10^{-7}$ m

$R = (4.48 \times 10^{-3})^2 / (4 \times 10 \times 5.00 \times 10^{-7})$
$R = (2.006 \times 10^{-5}) / (2.0 \times 10^{-5})$
$R = 1.003$ m

For the twentieth bright ring:

$D_{20} = \text{sqrt}(4 \text{ m R } \lambda)$
$D_{20} = \text{sqrt}(4 \times 20 \times 1.003 \times 5.00 \times 10^{-7})$
$D_{20} = \text{sqrt}(4.012 \times 10^{-5})$
$D_{20} = 6.34 \times 10^{-3}$ m
$D_{20} = 6.34$ mm

4. (a) Define the following terms:

(i) Free surface energy

Free surface energy is the work required to increase the surface area of a liquid by a unit amount. It is a measure of the cohesive forces at the surface of a liquid.

(ii) Capillary action

Capillary action is the ability of a liquid to flow in narrow spaces without external force due to the combined effect of cohesion and adhesion.

(iii) Angle of contact

The angle of contact is the angle between the tangent to the liquid surface and the solid surface at the point of contact. It determines whether a liquid wets a surface or not.

(b) Briefly explain the following observations:

(i) Soap solution is a better cleansing agent than ordinary water.

Soap solution lowers the surface tension of water, allowing it to penetrate and emulsify grease and dirt, making cleaning more effective.

(ii) When a piece of chalk is put into water, it emits bubbles in all directions.

Chalk has tiny pores filled with air. When submerged in water, the air is displaced by the water, escaping as bubbles.

(c) (i) Two spherical soap bubbles are combined. If V is the change in volume of the contained air, A is the change in total surface area, show that 3P V + 4A T = 0, where T is the surface tension and P_a is the atmospheric pressure.

Using the relationship between pressure and surface tension in a bubble:

$\Delta P = 4T / r$

Since the pressure inside the bubble changes due to volume change, we apply the equation of state for the gas inside:

$P \, dV + V \, dP = 0$

Using surface tension contribution:

$dP = - 4T \, dA / 3V$

Multiplying by V gives:

$3 P V + 4 A T = 0$

(ii) There is a soap bubble of radius 3×10^{-3} m in an air cylinder which is originally at a pressure of 10^5 N/m². The air in the cylinder is now compressed isothermally until the radius of the bubble is halved. Calculate the pressure of air in the cylinder.

Using Boyle’s Law:

$P_1 V_1 = P_2 V_2$

Volume of a sphere is $V = (4/3) \pi r^3$
When the radius is halved:

$V_2 = (1/8) V_1$

$P_1 \times V_1 = P_2 \times (1/8) V_1$

$P_2 = 8 P_1$
$P_2 = 8 \times 10^5$
$P_2 = 8.0 \times 10^5 N/m^2$

(d) What is strain energy?

Strain energy is the energy stored in a material due to deformation under applied stress. It is given by:

U = (1/2) × stress × strain × volume

A piece of rod 1.05 m long whose weight is negligible is supported at its ends by wires Q and P of equal lengths as shown in Figure 1.

The cross-sectional area of P is 1 mm² and that of Q is 2 mm². At what point along the bar should the weight be suspended in order to produce:

(i) Equal stress of P and Q.

Stress is given by:

σ = Force / Area
For equal stress:

$F_P / A_P = F_Q / A_Q$
$F_P / 1 = F_Q / 2$
$F_P = 0.5 F_Q$

Total force balance:

$F_P + F_Q = W$
$0.5 F_Q + F_Q = W$
$1.5 F_Q = W$
$F_Q = 2W / 3, F_P = W / 3$

Using torque balance to find the position:

$(W \times x) = (F_Q \times 1.05)$
$x = (2/3) \times 1.05$
x = 0.70 m from the left end

(ii) Equal Strain in P and Q

Strain = Stress/Young's modulus. For equal strain:

$(Stress_P) / Y_P = (Stress_Q) / Y_Q$

$(F_P / A_P) / Y_P = (F_Q / A_Q) / Y_Q$

Substituting $Y_P = 2.4 \times 10^{11}$ N/m², $Y_Q = 1.6 \times 10^{11}$ N/m², $A_P = 1 \times 10^{-6}$ m², $A_Q = 2 \times 10^{-6}$ m²:

$(F_P / (1 \times 10^{-6})) / (2.4 \times 10^{11}) = (F_Q / (2 \times 10^{-6})) / (1.6 \times 10^{11})$
$F_P / (2.4 \times 10^5) = F_Q / (3.2 \times 10^5)$
$F_P = (2.4 / 3.2) F_Q$
$F_P = 0.75 F_Q$

Now, $W = F_P + F_Q = 0.75 F_Q + F_Q = 1.75 F_Q$. Taking moments about P:

$W \times x = F_Q \times 1.05$

$1.75 F_Q \times x = F_Q \times 1.05$

$1.75x = 1.05$

$x = 1.05 / 1.75$

$x = 0.6$ m

The weight should be suspended 0.6 m from P (or 0.45 m from the center).

(i) 0.7 m from P for equal stress.
(ii) 0.6 m from P for equal strain.

5. (a) (i) State Coulomb's law of electrostatics.

Coulomb's law states that the electrostatic force between two point charges is directly proportional to the product of their magnitudes and inversely proportional to the square of the distance between them.

$F = (k q_1 q_2) / r^2$

(ii) Define electric field strength, E, at any point.

Electric field strength at a point is defined as the force experienced per unit positive charge placed at that point.

$E = F / q$

(iii) Mention two common properties of electric field lines.

1. Electric field lines start from positive charges and terminate at negative charges.
2. Electric field lines never intersect.

(b) Two identical balls, each of mass 0.8 kg, carry identical charge and are separated by a thread of equal length. At equilibrium, they position themselves at a distance of 1.2 cm apart as shown in Figure 2. Calculate the charge on either ball.

The forces acting on each ball are:
- Electrostatic force (F_e)
- Gravitational force (mg)
- Tension (T) in the thread

Using Coulomb's law:

$F_e = (k q^2) / r^2$

where
$k = 8.99 \times 10^9$ N·m²/C²
$r = 1.2 \times 10^{-2}$ m

$F_e = (8.99 \times 10^9 \times q^2) / (1.2 \times 10^{-2})^2$

$F_e = (8.99 \times 10^9 \times q^2) / (1.44 \times 10^{-4})$

$F_e = (6.243 \times 10^{13} \times q^2)$

The angle θ of the string is 60° from the vertical, so the horizontal component of tension balances the electrostatic force.

$T \sin(\theta) = F_e$

The vertical component of tension balances the weight.

$T \cos(\theta) = mg$

Dividing both equations:

$\tan(\theta) = F_e / mg$

$q^2 = (mg \tan(60°)) / (6.243 \times 10^{13})$

$q = \text{sqrt}((0.8 \times 9.81 \times 1.732) / (6.243 \times 10^{13}))$

$q = \text{sqrt}((13.55) / (6.243 \times 10^{13}))$

$q = \text{sqrt}(2.17 \times 10^{-13})$

$q = 1.47 \times 10^{-7}$ C

(c) Two capacitors C_1 and C_2, each of area 36 cm², separated by 4 cm, have capacitances of 6 μC and 8 μC respectively. The capacitor C_1 is charged to a potential difference of 110V, whereas the capacitor C_2 is charged to a potential difference of 140V. The capacitors are now joined with plates of like charges connected together.

(i) What will be the loss of energy transferred to heat in the connecting wires?

Energy stored in a capacitor:

$U = 0.5\ C\ V^2$

$U_1 = 0.5 \times 6 \times 10^{-6} \times (110)^2$

$U_1 = 0.5 \times 6 \times 10^{-6} \times 12100$

$U_1 = 3.63 \times 10^{-2}$ J

$U_2 = 0.5 \times 8 \times 10^{-6} \times (140)^2$

$U_2 = 0.5 \times 8 \times 10^{-6} \times 19600$

$U_2 = 7.84 \times 10^{-2}$ J

Total initial energy:

$U_{initial} = U_1 + U_2$

$U_{initial} = 3.63 \times 10^{-2} + 7.84 \times 10^{-2}$

$U_{initial} = 1.147 \times 10^{-1}$ J

Final potential difference:

$V_{final} = (C_1 V_1 + C_2 V_2) / (C_1 + C_2)$

$V_{final} = (6 \times 10^{-6} \times 110 + 8 \times 10^{-6} \times 140) / (6 \times 10^{-6} + 8 \times 10^{-6})$

$V_{final} = (6.6 \times 10^{-4} + 1.12 \times 10^{-3}) / 1.4 \times 10^{-5}$

$V_{final} = 1.78 \times 10^{-3} / 1.4 \times 10^{-5}$

$V_{final} = 127.14$ V

Final energy:

$U_{final} = 0.5\ (C_1 + C_2)\ V_{final}^2$

$U_{final} = 0.5 \times 1.4 \times 10^{-5} \times (127.14)^2$

$U_{final} = 0.5 \times 1.4 \times 10^{-5} \times 16169$

$U_{final} = 1.132 \times 10^{-1}$ J

Energy lost as heat:

$U\_lost = U\_initial - U\_final$

$U\_lost = 1.147 \times 10^{-1} - 1.132 \times 10^{-1}$

$U\_lost = 1.5 \times 10^{-3}$ J

(ii) What will be the loss of energy per unit volume transferred to heat in the connecting wires?

Energy loss per unit volume is:

U_loss / volume

Volume = A × d

$A = 36\ cm^2 = 36 \times 10^{-4}\ m^2$
$d = 4\ cm = 4 \times 10^{-2}\ m$

$Volume = (36 \times 10^{-4}) \times (4 \times 10^{-2})$

$Volume = 1.44 \times 10^{-3}\ m^3$

Energy loss per unit volume:

$(1.5 \times 10^{-3}) / (1.44 \times 10^{-3})$

$1.04\ J/m^3$

6. (a) Define the following terms:

(i) Capacitance

Capacitance is the ability of a system to store charge per unit potential difference.

$C = Q / V$

(ii) Charge density

Charge density is the amount of charge per unit area.

$\sigma = Q / A$

(iii) Equipotential surface

An equipotential surface is a surface where the electric potential is constant at every point.

(b) By using Coulomb's law of electrostatics, derive an expression for the electric field strength E, due to a point charge if the material is surrounded by a material of permittivity ε, and hence show how it relates with charge density σ.

Coulomb's force:

$F = (1 / 4\pi\varepsilon) \times (Qq / r^2)$

Electric field:

$E = F / q$

$E = (1 / 4\pi\varepsilon) \times (Q / r^2)$

Charge density:

$\sigma = Q / A$

Surface area of a sphere:

$A = 4\pi r^2$

$E = \sigma / \varepsilon$

(c) Describe the structure and the mode of action of a simplified version of the Van de Graaff generator.

The Van de Graaff generator consists of a moving belt, rollers, and a metal dome.

- Charge is transferred by the lower roller to the belt.
- The belt carries charge to the upper roller near the dome.
- The dome collects charge, creating high voltage.
- The generator is used for experiments requiring high electrostatic potential.

(d) A proton of mass 16.7×10^{-27} kg falls through a distance of 2.5 cm in a uniform electric field of magnitude 2.65×10^4 V/m. Determine the time of fall if the air resistance and the acceleration due to gravity, g, are neglected.

The force acting on the proton due to the electric field is given by

$F = qE$

Charge of a proton:

$q = 1.6 \times 10^{-19}$ C

$F = (1.6 \times 10^{-19}) \times (2.65 \times 10^4)$

$F = 4.24 \times 10^{-15}$ N

Using Newton's second law:

$F = ma$

$a = F / m$

$a = (4.24 \times 10^{-15}) / (16.7 \times 10^{-27})$

$a = 2.54 \times 10^{12}$ m/s²

Using the kinematic equation:

$s = ut + 0.5 a t^2$

Initial velocity u = 0, so

$s = 0.5 a t^2$

Rearrange for t:

$t^2 = (2s) / a$

$t = \text{sqrt}((2 \times 2.5 \times 10^{-2}) / (2.54 \times 10^{12}))$

$t = \text{sqrt}(5.0 \times 10^{-2} / 2.54 \times 10^{12})$

$t = \text{sqrt}(1.97 \times 10^{-14})$

$t = 1.4 \times 10^{-7}$ s

The time of fall is 1.4×10^{-7} seconds.

(ii) A parallel plate capacitor is made of a paper 40 mm wide and 3.0×10^{-2} mm thick. Determine the length of the paper sheet required to construct a capacitance of 15 μF, if its relative permittivity is 2.5.

The capacitance of a parallel plate capacitor is given by

$C = (\varepsilon_0 \varepsilon_r A) / d$

where
$C = 15 \times 10^{-6}$ F
$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m
$\varepsilon_r = 2.5$
A = width × length = $40 \times 10^{-3} \times L$
$d = 3.0 \times 10^{-2} \times 10^{-3}$ m

Rearrange for L:

$L = (C d) / (\varepsilon_0 \varepsilon_r w)$

$L = (15 \times 10^{-6} \times 3.0 \times 10^{-2} \times 10^{-3}) / (8.85 \times 10^{-12} \times 2.5 \times 40 \times 10^{-3})$

$L = (4.5 \times 10^{-10}) / (8.85 \times 10^{-12} \times 10^{-1})$

$L = (4.5 \times 10^{-10}) / (8.85 \times 10^{-13})$

$L = 5.08 \times 10^{2}$ mm

$L = 0.508$ m

The required length of the paper sheet is 0.508 meters.

7. (a) (i) State any three magnetic components of the earth's magnetic field.

1. The horizontal component of the earth's magnetic field.
2. The vertical component of the earth's magnetic field.
3. The total intensity of the earth's magnetic field.

(ii) The horizontal and vertical components of the earth's magnetic field at a certain location are 2.7×10^{-5} T and 2.0×10^{-5} T respectively. Determine the earth's magnetic field at the location and its angle of inclination L.

The total magnetic field strength B is given by

$B = \sqrt{B_h^2 + B_v^2}$

$B = \sqrt{(2.7 \times 10^{-5})^2 + (2.0 \times 10^{-5})^2}$

$B = \sqrt{7.29 \times 10^{-10} + 4.00 \times 10^{-10}}$

$B = \sqrt{1.129 \times 10^{-9}}$

$B = 3.36 \times 10^{-5}$ T

The angle of inclination L is given by

$\tan L = B_v / B_h$

$L = \tan^{-1}(2.0 \times 10^{-5} / 2.7 \times 10^{-5})$

$L = \tan^{-1}(0.741)$

$L = 36.6°$

(b) State the following laws or theorems as applied in magnetism.

(i) Biot-Savart law

Biot-Savart law states that the magnetic field dB at a point due to a small current element Idl is directly proportional to the current, the length of the element, and the sine of the angle between the element and the point, and inversely proportional to the square of the distance between them.

(ii) Ampere's theorem

Ampere's theorem states that the line integral of the magnetic field around a closed loop is equal to the permeability of free space multiplied by the total current enclosed by the loop.

$\oint B \cdot dl = \mu_0 I$

(c) (i) Draw hysteresis loops diagram for soft iron and hard steel and use them to discuss permanent magnets.

Soft iron has a narrow hysteresis loop, meaning it requires less energy to magnetize and demagnetize. It is used in electromagnets. Hard steel has a wider hysteresis loop, meaning it retains magnetism and is used in permanent magnets.

(ii) Define permeability constant.

Permeability constant (μ_0) is a measure of the ability of a material to support the formation of a magnetic field within itself. Its value in free space is $4\pi \times 10^{-7}$ H/m.

(iii) Derive an expression for the magnetic flux density B at the centre of the circular coil of radius r and N turns placed in air carrying a current I.

By Biot-Savart law, the magnetic field at the center of a circular coil is

$dB = (\mu_0 I \, dl \sin \theta) / (4\pi r^2)$

For a full circular loop,

$B = (\mu_0 I) / (2r)$

For N turns,

$B = (\mu_0 N I) / (2r)$

(d) The diameter of a 40-turn circular coil is 16 cm and it has a current of 5A.

(i) The magnetic induction at the centre of the coil.

$B = (\mu_0 N I) / (2r)$

r = 16 cm / 2 = 8 cm = 0.08 m

$B = (4\pi \times 10^{-7} \times 40 \times 5) / (2 \times 0.08)$

$B = (8\pi \times 10^{-5}) / 0.16$

$B = 1.57 \times 10^{-3}$ T

(ii) The magnetic moment of the coil.

Magnetic moment M is given by

$M = N I A$

$A = \pi r^2$

$A = \pi (0.08)^2$

$A = 2.01 \times 10^{-2}$ m²

$M = 40 \times 5 \times 2.01 \times 10^{-2}$

$M = 4.02$ A·m²

(iii) The torque acting on the coil if it is suspended in a uniform magnetic field of 0.76T such that its plane is parallel to the field.

Torque τ is given by

$\tau = M B \sin \theta$

Since the plane is parallel, $\theta = 90°$, so $\sin 90° = 1$

$\tau = 4.02 \times 0.76 \times 1$

$\tau = 3.06$ N·m

8. (a) (i) Briefly explain the production of X-rays.

X-rays are produced when high-energy electrons strike a metal target, causing sudden deceleration. This deceleration produces electromagnetic radiation in the X-ray region.

(ii) List down any three uses of X-rays.

1. Medical imaging for diagnosing bone fractures and internal conditions.
2. Security scanning at airports to inspect luggage.
3. Industrial testing for detecting structural defects in metals.

(iii) How are the intensity and penetrating power of an X-ray beam controlled?

The intensity of X-rays is controlled by adjusting the filament current, which controls the number of electrons produced. The penetrating power is controlled by varying the accelerating voltage, which determines the energy of the X-rays.

(b) An X-ray tube, operated at a d.c. potential difference of 60 kV, produces heat at the rate of 840 W. Assuming 0.65% of the energy of the incident electrons is converted into X-radiation, calculate:

(i) The number of electrons per second striking the target.

Power $P = VI$

$I = P / V$

$I = 840 / 60000$

$I = 0.014$ A

Number of electrons per second:

$n = I / e$

$n = 0.014 / (1.6 \times 10^{-19})$

$n = 8.75 \times 10^{16}$ electrons per second

(ii) The velocity of the incident electrons.

Using kinetic energy equation:

$eV = 0.5 m v^2$

$v = \text{sqrt}(2 eV / m)$

$v = \text{sqrt}((2 \times 1.6 \times 10^{-19} \times 60000) / (9.11 \times 10^{-31}))$

$v = \text{sqrt}((1.92 \times 10^{-14}) / (9.11 \times 10^{-31}))$

$v = \text{sqrt}(2.11 \times 10^{16})$

$v = 4.59 \times 10^{8}$ m/s

(iii) The energy of incident electrons.

$E = eV$

$E = (1.6 \times 10^{-19}) \times (60000)$

$E = 9.6 \times 10^{-15}$ J

(c)(i) Show that the possible energy levels (in joules) for the hydrogen atom are given by the formula

$E_n = -k \times (2\pi^2 m e^4) / (h^2 n^2)$

where m is the mass of the electron, e is the electronic charge, h is Planck's constant, $k = 1 / 4\pi\varepsilon_0$, and ε_0 is the permittivity of vacuum.

Using the Bohr model,

$E_n = - (m e^4) / (8 \varepsilon_0^2 h^2 n^2)$

Rewriting in terms of k,

$k = 1 / 4\pi\varepsilon_0$

$E_n = - k (2\pi^2 m e^4) / (h^2 n^2)$

(ii) What does the negative sign signify in the formula for E_n in (c) above?

The negative sign signifies that the electron is bound to the nucleus and requires energy to be freed.

(iii) The first member of the Balmer series of the hydrogen spectrum has a wavelength of 6563×10^{-10} m. Calculate the wavelength of its second member.

Using the Balmer formula:

$1/\lambda = R\_H (1/2^2 - 1/n^2)$

For the second member, n = 4
$1/\lambda_2 = R\_H (1/4 - 1/16)$
$R\_H = 1.097 \times 10^7 \text{ m}^{-1}$
$\lambda_2 = 1 / (1.097 \times 10^7 \times (0.1875))$
$\lambda_2 = 486.1 \times 10^{-9}$ m

The wavelength of the second member is 486.1 nm.

9. (a) (i) Differentiate natural radioactivity from artificial radioactivity.

Natural radioactivity occurs when unstable atomic nuclei spontaneously emit radiation in the form of alpha, beta, or gamma rays without any human intervention. It is commonly found in naturally occurring elements such as uranium and radium.

Artificial radioactivity, also known as induced radioactivity, is when stable nuclei are made radioactive by exposure to artificial nuclear reactions, such as neutron bombardment. This process is used in nuclear reactors and medical applications.

(ii) Name three applications of radioisotopes in medicine.

- Cancer treatment using radiotherapy with isotopes like cobalt-60.
- Medical imaging using technetium-99m for diagnosing internal body structures.
- Sterilization of medical equipment using gamma radiation from isotopes like cesium-137.

(iii) State two conditions for the stability of nuclides referring to light nuclides and heavy nuclides.

1. Light nuclides are stable when the neutron-to-proton (n/p) ratio is close to 1:1.

2. Heavy nuclides are stable when the neutron-to-proton ratio is higher, typically around 1.5:1, to counteract the increased electrostatic repulsion among protons.

(b) (i) Derive an expression for the half-life using the radioactive decay law.

The radioactive decay law states that the number of undecayed nuclei at any time t is given by

$N = N_0 e^{-(\lambda t)}$

where
N = number of undecayed nuclei at time t
N_0 = initial number of nuclei
λ = decay constant
t = time

At half-life $t = T_1/_2$, the number of undecayed nuclei is half of the initial value:

$N_0 / 2 = N_0 e^{-(\lambda T_1/_2)}$

Dividing both sides by N_0:

$1 / 2 = e^{-(\lambda T_1/_2)}$

Taking the natural logarithm on both sides:

$\ln(1/2) = -\lambda T_1/_2$

$\ln(2) = \lambda T_1/_2$

$T_1/_2 = \ln(2) / \lambda$

$T_1/_2 = 0.693 / \lambda$

(ii) What is carbon-14? Explain its production and how it is used in the dating process.

Carbon-14 is a radioactive isotope of carbon with a half-life of 5568 years. It is produced in the atmosphere when cosmic rays interact with nitrogen-14 nuclei:

$n + {}^{14}N \rightarrow {}^{14}C + p$

Carbon-14 is absorbed by living organisms through the carbon cycle. When an organism dies, it stops absorbing carbon-14, and the existing carbon-14 decays. By measuring the remaining activity of carbon-14 in a sample, scientists can estimate the time since the organism's death using the radioactive decay formula.

(c) Living wood has an activity of 16.0 counts per minute per gram of carbon. A certain sample of dead wood is found to have an activity of 18.4 counts per minute for 4.0 grams. Calculate the age of the sample of dead wood. Assume the half-life of carbon-14 is 5568 years.

Initial activity per gram:

A_0 = 16.0 counts/min per gram

Activity of dead wood sample:

A = 18.4 counts/min for 4.0 g

Activity per gram:

A' = A / mass

A' = 18.4 / 4.0

A' = 4.6 counts/min per gram

Using the decay formula:

$A = A_0 e^{-(\lambda t)}$

$4.6 = 16.0 e^{-(\lambda t)}$

Dividing both sides by 16.0:

$4.6 / 16.0 = e^{-(\lambda t)}$

$0.2875 = e^{-(\lambda t)}$

Taking the natural logarithm:

$\ln(0.2875) = -\lambda t$

$\lambda = \ln(2) / T_{1/2}$

$\lambda = 0.693 / 5568$

$\lambda = 1.245 \times 10^{-4}\ yr^{-1}$

Substituting values:

$\ln(0.2875) = -(1.245 \times 10^{-4})\ t$

$-1.247 = -(1.245 \times 10^{-4})\ t$

$t = 1.247 / 1.245 \times 10^{-4}$

t = 10,010 years

The age of the dead wood sample is 10,010 years.