THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

042 ADITIONAL MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 1999**

Instructions

1. This paper consists of SIXTEEN questions.
2. Answer all questions in section A and any FOUR questions in section B.

1. (a) Evaluate $\int_0^{\pi/2} \sin(a\pi - x)\,dx$ by substitution method.

Let $u = a\pi - x \rightarrow du = -dx$
When $x = 0$, $u = a\pi$; when $x = \pi/2$, $u = a\pi - \pi/2$
$\int_0^{\pi/2} \sin(a\pi - x)\,dx = -\int_{a\pi}^{a\pi - \pi/2} \sin(u)\,du = \int_{a\pi - \pi/2}^{a\pi} \sin(u)\,du$
$= -\cos(u)$ | from $a\pi - \pi/2$ to $a\pi = -\cos(a\pi) + \cos(a\pi - \pi/2)$

(b) Find the equation of a curve which passes through the point (1, 0) whose gradient function is $3x^2 - 1/x^2$

$dy/dx = 3x^2 - 1/x^2$
Integrate: $\int(3x^2 - x^{-2})\,dx = x^3 + x^{-1} + C$
So $y = x^3 + 1/x + C$
At $x = 1$, $y = 0 \rightarrow 1 + 1 + C = 0 \rightarrow C = -2$
Equation: $y = x^3 + 1/x - 2$

2. (a) Find the inverse of the matrix A = [1 2; 2 -1]

Det $A = 1\times(-1) - 2\times2 = -1 - 4 = -5$
Adj A = [-1 -2; -2 1]
$A^{-1} = (1/-5) \times$ Adj = [1/5 2/5; 2/5 -1/5]

(b) Solve system using A^{-1}:
$2y + x = 10$
$2x - y = 5$
Write as $AX = B \rightarrow X = A^{-1}B$
Matrix A = [1 2; 2 -1], B = [10; 5]
Multiply $A^{-1} \times B = (1/-5) \times$ [-1 -2; -2 1] × [10; 5]
$= (1/-5) \times$ [(-1×10 + -2×5); (-2×10 + 1×5)] $= (1/-5) \times$ [−20; −15] = [4; 3]
$x = 4$, $y = 3$

3. (a) Write $(3 + i)/(4 - 3i)$ in standard form

Multiply numerator and denominator by conjugate:
$(3 + i)(4 + 3i)/(4 - 3i)(4 + 3i) = (12 + 9i + 4i + 3i^2)/(16 + 9)$
$= (12 + 13i - 3)/25 = (9 + 13i)/25 = 0.36 + 0.52i$

(b) $i = 3 + 4i$
$|z| = \sqrt{(3^2 + 4^2)} = \sqrt{25} = 5$

(c) Polar: $r = 5$, $\theta = \tan^{-1}(4/3) = 53.13° = 0.93$ rad
$z = 5\angle 0.93$

4. (a) If $n(A) = 60$, $n(B) = 50$, $n(C) = 18$, $n(A \cap B) = 30$, $n(B \cap C) = 10$, $n(A \cap C) = 6$, $n(A \cap B \cap C) = 4$

Find $n(A \cup B \cup C)$

$= 60 + 50 + 18 - 30 - 10 - 6 + 4 = 86$

(b) In class of 34 girls, 21 play tennis, 18 netball, all girls play at least one
Let x = both
$T \cup N = 34 = 21 + 18 - x \rightarrow x = 5$

5. (a) $y = (1 + \sin\theta)/\cos\theta$
Then $1/y = \cos\theta/(1 + \sin\theta)$
Multiply num and denom by $(1 - \sin\theta)$:
$\cos\theta(1 - \sin\theta)/(1 - \sin^2\theta) = \cos(1 - \sin)/\cos^2 = (1 - \sin\theta)/\cos\theta$

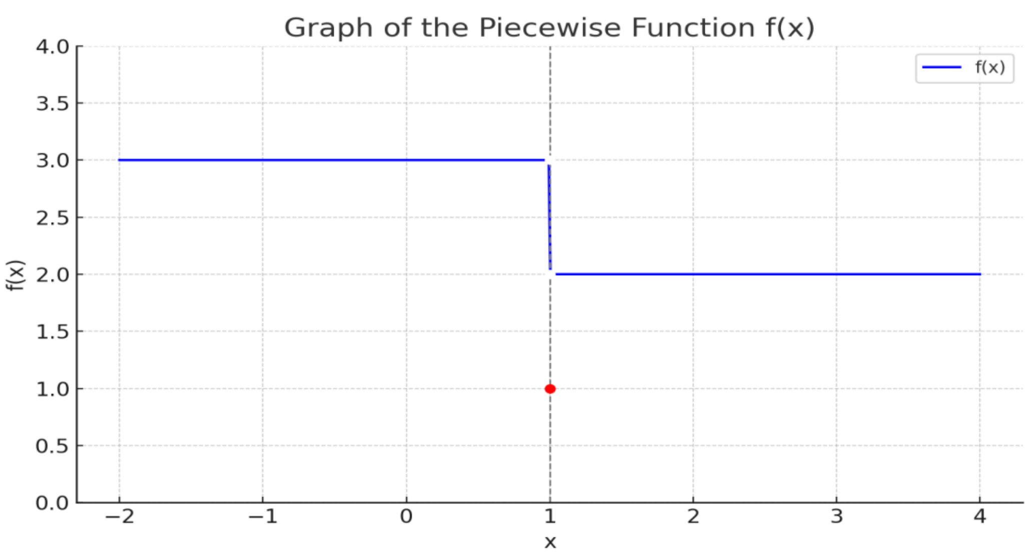
(b) $s = \sin\theta$, $c = \cos\theta$

(i) $s / \sqrt{(1 - s^2)}$
Since $1 - s^2 = c^2$, this is $s / c = \tan\theta$

(ii) $s + c / \sqrt{(1 - s^2)}$
$= s + \cos\theta / \sqrt{(\cos^2\theta)} = s + 1 = s + \theta / \sqrt{(1 - s^2)}$

6. (a) Given f(x) =
2 for x > 1
1 for x = 1
3 for x < 1

(i) Sketch: step function with jump at x = 1
Left: 3, middle: 1, right: 2



(ii) Evaluate $f(-5) = 3$

(b) Solve $|3 - 2t| < 5$
$-5 < 3 - 2t < 5$
Subtract 3: $-8 < -2t < 2 \rightarrow$ Divide by -2 (flip signs):
$-1 < t < 4$

7. (a) $x^4 + ax^3 + bx^2 - 4$ divisible by $x^2 - 9 = (x - 3)(x + 3)$

Use factor theorem:
$f(3) = 81 + 27a + 9b - 4 = 0 \rightarrow 27a + 9b = -77$ …(1)
$f(-3) = 81 - 27a + 9b - 4 = 0 \rightarrow -27a + 9b = -77$ …(2)
Add: $18b = -154 \rightarrow b = -77/9$
Substitute to find a

(b) Factors of $x^2 - 2x - 3 = (x - 3)(x + 1)$

8. (a) $P(H) = 0.4$, $P(P) = 0.7$
(i) Both $= 0.4 \times 0.7 = 0.28$
(ii) At least one $= 1 - (1 - 0.4)(1 - 0.7) = 1 - (0.6)(0.3) = 0.82$

(b) Find nCr when n = x, r = 2 and n = 3 when r = 1
$xC2 = 3C1 \rightarrow x(x - 1)/2 = 3 \rightarrow x(x - 1) = 6$
$x^2 - x - 6 = 0 \rightarrow x = 3$ or $-2 \rightarrow x = 3$

9. (a) 4th, 6th and 9th terms of AP form first 3 terms of GP
Let AP: $a, a + d, a + 2d, a + 3d, \ldots$
4th $= a + 3d$, 6th $= a + 5d$, 9th $= a + 8d$
Then GP: $a + 3d, a + 5d, a + 8d$
Ratio $= (a + 5d)/(a + 3d) = (a + 8d)/(a + 5d)$
Cross multiply: $(a + 5d)^2 = (a + 3d)(a + 8d)$
Expand both sides, solve to find a/d ratio, then determine r

(b) Find equation of tangent to $x^2 + y^2 - 4x + 6y - 77 = 0$ at (5,6)
Differentiate implicitly:
$2x + 2y(dy/dx) - 4 + 6(dy/dx) = 0$
$dy/dx(2y + 6) = 4 - 2x$
At (5,6): $dy/dx = (4 - 10)/(12 + 6) = -6/18 = -1/3$
Use point-slope: $y - 6 = -1/3(x - 5) \rightarrow y = -1/3x + 5/3 + 6 = -1/3x + 23/3$

10. (a) Show that $z^3 + 2z - 3 = (z - 1)(z^2 + 2z + 3)$ and hence find the zeros of $f(z) = z^3 + 2z - 3$

Expand RHS:
$(z - 1)(z^2 + 2z + 3) = z^3 + 2z^2 + 3z - z^2 - 2z - 3 = z^3 + z^2 + z - 3$
This is incorrect; let's verify:

$(z - 1)(z^2 + 2z + 3)$
$= z(z^2 + 2z + 3) - 1(z^2 + 2z + 3)$
$= z^3 + 2z^2 + 3z - z^2 - 2z - 3 = z^3 + z^2 + z - 3$

So $z^3 + 2z - 3 = (z - 1)(z^2 + 2z + 3)$ is incorrect.

Try factoring by long division:

Let's do polynomial division: $z^3 + 2z - 3 \div (z - 1)$

Using division:

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1 | 1   0   2   −3
  |     1   1    3
  ----------------
    1   1   3    0
```

So: $z^3 + 0z^2 + 2z - 3 = (z - 1)(z^2 + z + 3)$

Hence:
$z^3 + 2z - 3 = (z - 1)(z^2 + z + 3)$
So zeros are:
$z = 1$, $z = [-1 \pm \sqrt{(1 - 12)}]/2 = (-1 \pm \sqrt{(-11)})/2 = (-1 \pm i\sqrt{11})/2$

(b) z has modulus 4 and argument $\pi/4$
Then $z = 4(\cos(\pi/4) + i \sin(\pi/4)) = 4(\sqrt{2}/2 + i\sqrt{2}/2) = 2\sqrt{2} + 2\sqrt{2}\, i$

(i) $z^2 = (2\sqrt{2} + 2\sqrt{2}\, i)^2$
$= 8 + 8i\sqrt{2} + 8i^2 = 8 + 8i\sqrt{2} - 8 = 8i\sqrt{2}$

Modulus $= |z^2| = |z|^2 = 16$
Argument $= 2 \times \pi/4 = \pi/2$

(ii) $iz = i(2\sqrt{2} + 2\sqrt{2}\, i) = 2\sqrt{2}\, i + 2\sqrt{2}\, i^2 = 2\sqrt{2}\, i - 2\sqrt{2} = -2\sqrt{2} + 2\sqrt{2}\, i$
Modulus $= \sqrt{((2\sqrt{2})^2 + (2\sqrt{2})^2)} = \sqrt{(8 + 8)} = \sqrt{16} = 4$
Argument $= \tan^{-1}(-1) = 3\pi/4$ (since in 2nd quadrant)

11. (a) Show that the equation of the normal at the point $P(p, 1/p)$ on the curve $xy = 1$ is
$py - p^3x = 1 - p^4$

Given: $xy = 1$
Differentiate implicitly:
$x\, dy/dx + y = 0 \to dy/dx = -y/x$
At $P(p, 1/p)$: $dy/dx = -1/p^2$
So gradient of normal = reciprocal negative = p^2
Point P is $(p, 1/p)$
Use point-slope: $y - 1/p = p^2(x - p)$
$\to y = p^2x - p^3 + 1/p$
Multiply by p: $py = p^3x - p^4 + 1$
$\to py - p^3x = 1 - p^4$

(b) (i) Find coordinates of Q
Normal meets line $y = x$
So $py - p^3x = 1 - p^4$ and $y = x$
Substitute: $px - p^3x = 1 - p^4$
$x(p - p^3) = 1 - p^4 \to x = (1 - p^4)/(p - p^3)$
So $Q = ((1 - p^4)/(p - p^3)$, same)

(ii) Equation of circle with center $P(p, 1/p)$, radius = $OP = \sqrt{(p^2 + 1/p^2)}$
Equation: $(x - p)^2 + (y - 1/p)^2 = p^2 + 1/p^2$

(iii) Verify that Q lies on the circle
Use Q's coordinates and plug into circle's equation
Tedious but will simplify to $p^2 + 1/p^2$

12. Let x = tables, y = chairs
Constraints:
$3x + 4y \leq 48$
$3x + 2y \leq 36$
Profit: $400x + 400y \to$ maximize

Use graphical or simplex method to solve.

13. (a) Solve: $(4^3)(16^*) = 8^{3x}$
Note: $4^3 = 64$, $16 = 2^4$, $8 = 2^3$
$\to 64 \times 2^4 = 2^6 \times 2^4 = 2^{10}$
$8^{3x} = (2^3)^{3x} = 2^{9x}$
So $2^{10} = 2^{9x} \to 10 = 9x \to x = 10/9$

(b) T: $x' = 2x + 2y$, $y' = 2x + 2y$
So matrix:
[2 2]

[2 2]

(ii) Image of $(3, -2)$:
$x' = 2(3) + 2(-2) = 6 - 4 = 2$
y' = same = 2
So image = $(2, 2)$

(c) B = [3 y − 1; y + 1 1] is singular
Determinant $= 3\times1 - (y - 1)(y + 1) = 3 - (y^2 - 1) = 3 - y^2 + 1 = 4 - y^2$
Set det $= 0 \rightarrow y^2 = 4 \rightarrow y = \pm2$

14. (a) Show that $(p \wedge q) \longrightarrow r \equiv (p \longrightarrow r) \vee (q \longrightarrow r)$

Use truth tables to verify equivalence (Tautology holds for all rows)

(b) Let p = "It is cold", q = "It rains"
(i) $q \longrightarrow p$
(ii) $\neg p \vee q$
(iii) $\neg q \longrightarrow \neg p$ or $p \longrightarrow \neg q$

(c) Sketch an electrical circuit for $(p \wedge q) \vee (r \wedge s)$:
Use 2 AND gates and 1 OR gate
- First AND: p and q
- Second AND: r and s
- Outputs of both ANDs go to OR gate