THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

042 ADITIONAL MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2000**

Instructions

1. This paper consists of SIXTEEN questions.
2. Answer all questions in section A and any FOUR questions in section B.

1. (a) Simplify

(i) $(A \cup B)' \cap A$

By De Morgan's law: $(A \cup B)' = A' \cap B'$

So: $(A' \cap B') \cap A = (A \cap A') \cap B' = \emptyset \cap B' = \emptyset$

(ii) $(A - B) \cup A$

$A - B = A \cap B'$

So: $(A \cap B') \cup A = A$

(b) In a class of twenty pupils, there are twelve who study English but not History, four who study History but not English and one who study neither English nor History.

How many pupils study History?

Let E = English, H = History

Total = 20

E only = 12, H only = 4, Neither = 1

So $(E \cup H)' = 1 \rightarrow (E \cup H) = 19$

Total with both = $19 - 12 - 4 = 3$

So students who study History = H only + both = $4 + 3 = 7$

2. (a) Given the three points A(4, 0), B(0, 2) and C(−2, −2), show that AB = BC

$AB^2 = (0 - 4)^2 + (2 - 0)^2 = 16 + 4 = 20$

$BC^2 = (-2 - 0)^2 + (-2 - 2)^2 = 4 + 16 = 20$

So $AB = BC = \sqrt{20}$

(b) A square ABCD is formed where A, B and C are points given in 2(a) above and D is the fourth point whose coordinates are not known. Calculate the coordinates of D.

Use vector approach: AB = $(-4, 2)$, BC = $(-2, -4)$

Then CD = AB = $(-4, 2) \rightarrow D = C + CD = (-2, -2) + (-4, 2) = (-6, 0)$

3. (a) (i) Prove that $(1 - \cos A)(1 + \sec A) = \sin A \tan A$

LHS:

$(1 - \cos A)(1 + 1/\cos A)$

$= (1 - \cos A)((\cos A + 1)/\cos A)$

$= [(1 - \cos A)(1 + \cos A)] / \cos A$

$= (1 - \cos^2 A)/\cos A = \sin^2 A / \cos A = \sin A \tan A$

(ii) Eliminate θ if:

$x = 4 \sec \theta$

$y = 4 \tan \theta$

$\sec^2 \theta - \tan^2 \theta = 1$

$(x/4)^2 - (y/4)^2 = 1 \rightarrow x^2/16 - y^2/16 = 1$

Multiply both sides by 16: $x^2 - y^2 = 16$

(b) Given that $f(x) = 2x + 1$ and $g(x) = x^2 - 2$, show that the composition function is not commutative

$f(g(x)) = f(x^2 - 2) = 2x^2 - 4 + 1 = 2x^2 - 3$

$g(f(x)) = g(2x + 1) = (2x + 1)^2 - 2 = 4x^2 + 4x + 1 - 2 = 4x^2 + 4x - 1$

So $f(g(x)) \neq g(f(x))$

4. (a) Given $y = (\sin x - \cos x)/(\sin x + \cos x)$, find dy/dx

Let $u = \sin x - \cos x$, $v = \sin x + \cos x$

$dy/dx = (v \cdot du/dx - u \cdot dv/dx)/v^2$

$= [(\sin x + \cos x)(\cos x + \sin x) - (\sin x - \cos x)(\cos x - \sin x)] / (\sin x + \cos x)^2$

(b) Differentiate $4x^7y + x + xy = 0$

Use implicit differentiation:

$d/dx(4x^7y) + d/dx(x) + d/dx(xy) = 0$

$\rightarrow 28x^6y + 4x^7(dy/dx) + 1 + x(dy/dx) + y = 0$

Group dy/dx terms:

$(4x^7 + x)(dy/dx) = -28x^6y - y - 1$

$dy/dx = (-28x^6y - y - 1)/(4x^7 + x)$

5. (a) $\int \cos^3 x \, dx$

$= \int \cos x(1 - \sin^2 x) \, dx$

Let $u = \sin x \rightarrow du = \cos x \, dx$

$= \int (1 - u^2) \, du = u - u^3/3 = \sin x - \sin^3 x/3 + C$

(b) $\int x^2(1 - x)^3 \, dx$

Expand $(1 - x)^3 = 1 - 3x + 3x^2 - x^3$

Then multiply by x^2:

$x^2 - 3x^3 + 3x^4 - x^5$

Integrate: $x^3/3 - 3x^4/4 + 3x^5/5 - x^6/6 + C$

6. (a) The displacement vector a maps A(3, −2) to B(−1, 3) and b maps A to C(4, 5).

a = B − A = (−1 − 3, 3 − (−2)) = (−4, 5)

b = C − A = (4 − 3, 5 − (−2)) = (1, 7)

a + b = (−3, 12)

(b) Show that the vector 3i + 2j − k is at right angles to the straight line

r = (i + 7j + 2k) + s(2i − 5j − 4k)

Direction vector of line = (2, −5, −4)

Dot product with (3, 2, −1):

= 3×2 + 2×(−5) + (−1)×(−4) = 6 − 10 + 4 = 0

So vectors are perpendicular

7. (a) (i) Find the value of 15! / (12!3!)

= (15×14×13) / (3×2×1) = 2730

(ii) How many arrangements can be made of the letters in the word TROTTING?

TROTTING has 9 letters, T appears 3 times

Arrangements = 9! / 3! = 362880 / 6 = 60480

(b) (i) Write down the statement which is true among:

If P = 0, then P ⊂ 0 — False

If PQ = 0, then P = 0 or Q = 0 — True

If P = 0 or Q = 0, then PQ = 0 — Also True

So the second is definitely true.

(ii) Solve using matrix method:

$2x = 5 + y \rightarrow 2x - y = 5$

$3 + 2y + 3x = 0 \rightarrow 3x + 2y = -3$

Now solve:

From (1): $y = 2x - 5$

Substitute into (2):

$3x + 2(2x - 5) = -3$

$3x + 4x - 10 = -3 \rightarrow 7x = 7 \rightarrow x = 1$

Then $y = 2(1) - 5 = -3$

8. (a) In a geometric progression, sum of 2nd and 3rd terms is 6, sum of 3rd and 4th is −12

Let a = first term, r = common ratio

2nd term = ar, 3rd = ar^2, 4th = ar^3

$ar + ar^2 = 6 \rightarrow ar(1 + r) = 6$

$ar^2 + ar^3 = -12 \rightarrow ar^2(1 + r) = -12$

Divide:

$(ar^2(1 + r)) / (ar(1 + r)) = -12 / 6$

$\rightarrow r = -2$

Substitute into first: $ar(1 - 2) = 6 \rightarrow ar(-1) = 6 \rightarrow ar = -6$

So $a = -6/r = -6/-2 = 3$

$a = 3, r = -2$

(b) Obtain the first four terms in the expansion of $(1 - 2x)^6$

Use binomial theorem:

$= 1 - 12x + 60x^2 - 160x^3 + \ldots$

9. (a) Construct frequency distribution for leaves using class intervals 118−126, 127−135, 136−144 etc.

Class	Tally	Frequency
118–126	|||| ||	6
127–135	|||| |||| |	11
136–144	|||| |||	8
145–153	|||| ||||	10
154–162	|||| |	5

(b) Determine mean using

$\bar{X} = A + (\Sigma fd)/N$

Let A = 136 (mid of 136–144)

Compute midpoints, deviations (d), fd, then plug into formula

Total frequency N = 40

(c) Calculate median:

Class with 20th item (N/2 = 20) is 127–135

Use:

$\text{Median} = L + [(N/2 - F)/f] \times h$

Use L = 126.5, F = 6, f = 11, h = 9

$\text{Median} = 126.5 + [(20-6)/11]\times 9 \approx 126.5 + 11.45 \approx 137.95$ mm

10. (a) Two events A and B are such that $P(A) = 1/2$, $P(B) = 1/3$ and $P(A|B) = 1/4$

Evaluate:

(i) $P(A \cap B)$

By definition: $P(A \cap B) = P(A|B) \times P(B) = 1/4 \times 1/3 = 1/12$

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 1/2 + 1/3 - 1/12$

$= (6 + 4 - 1)/12 = 9/12 = 3/4$

(iii) $P(A' \cap B') = 1 - P(A \cup B) = 1 - 3/4 = 1/4$

Another event C is such that A and C are independent and

$P(A \cap C) = 1/12$, $P(B \cup C) = 1/2$

$P(A) = 1/2 \rightarrow P(A \cap C) = P(A) \times P(C) \rightarrow 1/12 = 1/2 \times P(C) \rightarrow P(C) = 1/6$

$P(B \cup C) = P(B) + P(C) - P(B \cap C)$

$1/2 = 1/3 + 1/6 - P(B \cap C) \rightarrow P(B \cap C) = 1/2 - 1/3 - 1/6 = 0$

So B and C are mutually exclusive

(b) Two balls are taken at random from a bag containing 6 red and 2 blue balls

Find the probability that just one ball is blue

Total = 8 balls

P(one blue and one red) = $(6C1 \times 2C1)/8C2 = (6\times2)/28 = 12/28 = 3/7$

11. (a) Verify that $(p \wedge q) \longrightarrow (p \vee q)$ is a tautology

Use truth table:

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

All cases true → tautology

(b) Let p = “Juma reads Daily News”, q = “Juma reads Mzalendo”, r = “Juma reads Nipashe”

(i) $(p \vee q) \wedge \sim r$:

Juma reads either Daily News or Mzalendo and does not read Nipashe

(ii) $(p \wedge q) \vee \sim(p \wedge r)$:

Either Juma reads both Daily News and Mzalendo, or he doesn’t read both Daily News and Nipashe

(iii) $\sim(p \wedge \sim r)$:

It is not true that Juma reads Daily News and does not read Nipashe

(c) Construct a compound statement that corresponds to the network:

From diagram: $(p \rightarrow q) \wedge (r \rightarrow s)$

12. (a) Find the smallest root of $x^2 - 4x + 2 = 0$ using

$x_{n+1} = \frac{1}{4}(x_n^2 + 2)$, starting with $x_0 = 0.5$

$x_1 = \frac{1}{4}(0.5^2 + 2) = \frac{1}{4}(0.25 + 2) = \frac{1}{4}(2.25) = 0.5625$

$x_2 = \frac{1}{4}(0.5625^2 + 2) = \frac{1}{4}(0.3164 + 2) = 0.5791$

(b) Stop when $|x_{n+1} - x_n|$ is very small, e.g., less than 0.001 or when values stabilize to same 2 decimal places

13. (a) If $z = 3 - i$, express $z + 1/z$ in form $a + bi$

$1/z = 1/(3 - i) = (3 + i)/(3^2 + 1) = (3 + i)/10 = 0.3 + 0.1i$

$z + 1/z = (3 - i) + (0.3 + 0.1i) = 3.3 - 0.9i$

(b) If $z = x + yi$, $\bar{z} = x - yi$

Given $1/z + 2/\bar{z} = 1 + i$

$1/z = 1/(x + yi) = (x - yi)/(x^2 + y^2)$

$2/\bar{z} = 2/(x - yi) = 2(x + yi)/(x^2 + y^2)$

Sum:

$[(x - yi) + 2(x + yi)]/(x^2 + y^2) = 1 + i$

Numerator: $x - yi + 2x + 2yi = 3x + yi$

So: $(3x + yi)/(x^2 + y^2) = 1 + i$

Equating real and imaginary:

$3x/(x^2 + y^2) = 1 \rightarrow 3x = x^2 + y^2$

$y/(x^2 + y^2) = 1 \rightarrow y = x^2 + y^2$

Substitute $y = x^2 + y^2$ into $3x = x^2 + y^2$

$\rightarrow 3x = y$

But $y = x^2 + y^2 \rightarrow 3x = x^2 + y^2$

So $x^2 + y^2 - 3x = 0$

Substitute $y = 3x$

$x^2 + 9x^2 - 3x = 10x^2 - 3x = 0 \rightarrow x(10x - 3) = 0$

$x = 0$ or $x = 3/10$

If $x = 0 \rightarrow y = 0 \rightarrow$ contradiction

$x = 3/10 \rightarrow y = 9/10$

(c) Given $2 + 3i$ is root of $z^2 - 6z + 21z - 26 = 0$

Let the equation be $z^3 - 6z^2 + 21z - 26 = 0$

If $z = 2 + 3i$ is root, then $2 - 3i$ is also root

So $(z - (2 + 3i))(z - (2 - 3i)) = (z - 2 - 3i)(z - 2 + 3i) = z^2 - 4z + 13$

Divide polynomial by $(z^2 - 4z + 13)$ to get remaining root

Use polynomial division or compare

Remaining root is $z = 2$

14. (a) Radius $r = 5$ cm, increase by 0.01 cm

$A = \pi r^2$

$dA = d/dr\ (\pi r^2) \times \Delta r = 2\pi r \times \Delta r = 2\pi \times 5 \times 0.01 = 0.1\pi \approx 0.314\ cm^2$

(b) $S(t) = 9t^2 - 2t^3$

Velocity $v = dS/dt = 18t - 6t^2$

Acceleration $a = d^2S/dt^2 = 18 - 12t$

At $t = 3$:

$v = 18\times 3 - 6\times 9 = 54 - 54 = 0$

$a = 18 - 36 = -18 \rightarrow$ particle is turning at $t = 3$

At $t = 4$:

$S = 9\times16 - 2\times64 = 144 - 128 = 16$

Particle is 16 m from O, and

$v = 72 - 96 = -24 \rightarrow$ negative velocity $\rightarrow$ moving towards O