THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

042 **ADDITIONAL MATHEMATICS**
(For Both School and Private Candidates)

Time: 3 Hours 15 November 2001 p.m.

Instructions

1. This paper consists of sections A and B.
2. Answer ALL questions in section A and FOUR (4) questions from section B.
3. Write all the answers in the answer booklet(s) provided.
4. Mathematical tables, mathematical formulae and slide rules may be used.
5. Write your Examination Number on every page of your answer booklet(s).

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SECTION A (60 marks)

Answer ALL questions in this section. All working for each question must be shown clearly.

1. (a) The functions f and g are defined by

$f : x \rightarrow 5x + 4$

$g : x \rightarrow 6x - k$

where $x \in R$ and k is a constant.
Find the value of k for which fog(x) = gof(x). (3 marks)

(b) The expression $3x^3 + 2x^2 - bx + a$ is divisible by $(x - 1)$ and it leaves a remainder of [illegible] when divided by $x + 1$. Find the values of a and b. (3 marks)

2. (a) Prove that $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) = \sin^3\theta + \cos^3\theta$. (3 marks)

(b) Find the value of $\sin^2 A \operatorname{cosec}(\frac{\pi}{2} - A) - \cot^2(\frac{\pi}{2} - A)\cos A$ (3 marks)

3. (a) If $\underline{u} = 2\underline{i} - \underline{j}$, $\underline{v} = 6\underline{i} - 3\underline{j}$

(i) find $\underline{u} + \underline{v}$

(ii) show that $|\underline{u} + \underline{v}| = |\underline{u}| + |\underline{v}|$

(3 marks)

(b) In a triangle OAB, $\overrightarrow{OA} = \underline{a}$ nd $\overrightarrow{OB} = \underline{b}$. Given that P and Q are the midpoints of OA and OB respectively, express $\overrightarrow{PQ}$ and $\overrightarrow{AB}$ in terms of $\underline{a}$ and $\underline{b}$. State the geometrical relationship between $\overrightarrow{PQ}$ and $\overrightarrow{AB}$. (3 marks)

4. (a) Find the inverse A^{-1} of the matrix $A = \begin{pmatrix} 3 & -8 \\ 7 & 5 \end{pmatrix}$ (3 marks)

(b) Using the result of (a) above, and not otherwise solve the following system of simultaneous equations:

$$\begin{cases} 3x - 1 - 8y = 0 \\ 7x = 26 - 5y \end{cases}$$

(3 marks)

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5. The following table summarises the masses measured to the nearest gram of 200 animals from the same species:

Mass (g)	Frequency
75 - 79	7
80 - 84	30
85 - 89	66
90 - 94	57
95 - 99	27
100 - 104	13

Calculate the median and upper quartile of the distribution. (6 marks)

6. A social committee is to transport 20 boys and 32 girls to a place for a picnic. The committee can hire either a taxi which can carry 2 boys and 1 girl or a mini bus which can carry 2 boys and 4 girls. It costs sh.4000 to hire a taxi and sh 3000 for a mini bus. Find the cheapest means of transport.

(6 marks)

7. (a) Differentiate with respect to x

(i) $\frac{\sin x}{1+\tan x}$ (ii) $\sqrt{x^2+2x}$

(4 marks)

(b) Evaluate

$$\int_0^{\pi/2}(2\cos^2\theta + 3\sin^2\theta)d\theta$$ (2 marks)

8. (a) Show how the biconditional $p \leftrightarrow q$ can be written in terms of the original three connectives $\vee$, $\wedge$ and ~. (2 marks)

(b) Using a truth table verify that

(i) $\sim(p \rightarrow q) \equiv p \wedge \sim q$

(ii) $\sim(p \leftrightarrow q) \equiv \sim p \leftrightarrow q$

(4 marks)

(3 marks)

9. (a) $\int e^x \cos 3x dx$

(b) Find the value of p and q such that

$$\int_0^{3x}(pt-q)dt = 9x^2 + 9x$$ (3 marks)

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10. Solve the following system of simultaneous equations:

$$\begin{cases} 2x-5y+2z=14 \\ 9x+3y-4z=13 \\ 7x+3y-2z=3 \end{cases}$$

(6 marks)

SECTION B (40 marks)

Answer **FOUR (4)** questions from this section. All workings must be shown clearly.

11. (a) Find the 2 x 2 matrix that will transform the point (1, 2) to (3, 3) and the point (–1, 1) to (–3, 3). (5 marks)

(b) All the points on the line $y = 2x - 3$ have been transformed by the matrix

$$\begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}$$

Find the equation of the image line. (5 marks)

12. Find the equation of the circle which circumscribes the triangle with vertices (1, 0), (2, 1) and (0, 2). (10 marks)

13. (a) A company that manufactures cattle food wishes to pack the food in closed cylindrical tins. What should be the dimensions of each tin, if each tin is to have a volume of $128\pi \text{ cm}^3$ and a minimum possible area? (5 marks)

(b) The area of the segment cut off by $y = 5$ from the curve $y = x^2 + 1$ is rotated about $y = 5$. Find the volume generated. (5 marks)

14. (a) If A and B are independent events such that $P(A) = \frac{5}{8}$ and $P(B/A) = \frac{3}{7}$, find $P(A \cap B)$. (3 marks)

(b) In a class of 30 boys, 15 have bicycles, 10 have motorbikes and 4 have both. If a student is picked at random, what is the possibility that (7 marks)

(i) he has neither a bicycle nor a motorbike

(ii) he has a bicycle but no a motorbike.

15. (a) Find the modulus and argument of $z = \frac{1}{2} - \frac{\sqrt{3}}{2} i$ (2 marks)

(b) Given that $z = 1 + i$, show that $z^3 = -2 + 2i$. For this value of z, the real numbers p and q are such that $\frac{p}{1+2} = \frac{q}{1+z^3} = 2i$. Find the values of p and q. (5 marks)

(c) The complex number z satisfies the equation $2z\bar{z} - 4z = 3 - 6i$ where $\bar{z}$ is the conjugate of z. Find the two possible values of z in the form x + iy. (3 marks)

16. Show that the equation $x^3 - x - 6 = 0$ has a root between 1 and 2. Using Newton– Raphson method with starting point 1.6 determine in two iterations a better root giving your answer to two decimal places. (10 marks)

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