THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

042 **ADITIONAL MATHEMATICS**

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2006**

Instructions

1. This paper consists of SIXTEEN questions.

2. Answer all questions in section A and any FOUR questions in section B.

1. (a) Simplify using basic properties of sets:

(i) $(A' \cap B') \cup (A' \cap B) = A' \cap (B' \cup B) = A' \cap U = A'$

(ii) $(A - B)' \cap (B - A)' = (A \cap B')' \cap (B \cap A')' = (A' \cup B) \cap (B' \cup A) = (A \cup B)$

(b) In a class of 28 pupils:
Let C = chemistry, P = physics, B = biology
$C \cup P \cup B = 28$
$n(C \cap B') = 16$, $n(B \cap C') = ?$
Given: 16 study chemistry & physics, equal to those who study biology → $n(B) = 16$
2 study none, so:
$n(C \cup P \cup B) = 26$
Let:
$x = C \cap P \cap B$
From total equations and overlaps, apply Venn diagram and solve.
(i) Biology only = B - overlaps = 6
(ii) Chemistry only = C - overlaps = 5

2. (a) If p is the statement "$x = 1$" and q is "$x^2 = 1$"

(i) $x^2 \neq 1$ because $x \neq \pm 1$ → q is false because p is false
(ii) $x = 1 \Leftrightarrow x^2 = 1 \rightarrow p \leftrightarrow q$
(iii) If $x^2 \neq 1$, then $x \neq \pm 1 \rightarrow \sim q \longrightarrow \sim p$

(b) Use truth tables to show $p \longrightarrow (q \wedge r) \equiv (p \longrightarrow q) \wedge (p \longrightarrow r)$

Construct truth table to confirm equivalence

3. (a) (i) Inverse of M = [x y; x -y]
Det $M = -xy - x^2 = -x(y + x)$
M^{-1} = 1/det × adjugate = (1/(-x(y + x))) × [-y -y; -x   x]

(ii) Solve M [x; y] = [-2; 3]
Use inverse found to multiply right-hand side

(b) (i) For no solution: determinant = 0
Matrix: |2 k|
|3 k + 1|
Det $= 2(k + 1) - 3k = 2k + 2 - 3k = 2 - k$
Set $2 - k = 0 \rightarrow k = 2$

(ii) Use Cramer's Rule when $k = -3$

Solve by finding D, Dx, Dy using determinants

4. Cumulative frequency:
Total = 55
Last freq = 49 → missing X = 33 - 17 = 16

(a) Two classes with equal freq = 8 → find X = 25
(b) Complete table: class widths and frequencies
Calculate midpoint × frequency, sum and divide by total
(c) Modal class = class with highest frequency

5. (a) Show that $f(1/x) = 1/(x + 1) - 1 = (1 - x)/(x + 1) = f(x)$

(b)
(i) $g(x) = x / (2 + x^2)$
Domain: all real x
Range: use limits, find max/min

(ii) $h(x) = 1 / \sqrt{(x - 4)}$
Domain: $x > 4$
Range: $h(x) > 0$

6. (a) Prove $(\cos A + \cos B)^2 + (\sin A + \sin B)^2 = 2 + 2\cos(A - B)$
Use identity: $a^2 + b^2 + 2ab = (a + b)^2$
Apply trigonometric identities

(b) Solve for θ:
$6 \cos^2 \theta - \cos \theta - 1 = 0$
Use quadratic formula to find cos θ

(c) $A + B = 2\pi/3$, $A - B = \pi/3$
Solve for A and B → use to evaluate expression

7. (a) $P = 10V + 4000/V$
Differentiate: $dP/dV = 10 - 4000/V^2$
Set $dP/dV = 0 \rightarrow V^2 = 400 \rightarrow V = 20$

(b) $y = \sqrt{(1 - \sin x)}$
$dy/dx = 1/2\sqrt{(1 - \sin x)} \times (-\cos x) = -\cos x / (2\sqrt{(1 - \sin x)})$

8. (a) ∫ from π/4 to π of $\sin^2(2x) \cos(2x)\, dx$
Use substitution: let $u = \sin 2x \rightarrow du = 2\cos 2x\, dx$
Adjust integral

(ii) $\int (1 - 3x / \sqrt{x})\, dx = \int (1 - 3x^{0.5})\, dx$
$= x - 2x^{1.5} + C$

(b) $dy/dx = 2x - 4$
At minimum, $dy/dx = 0 \rightarrow x = 2$
$y = 3$, so equation: $y = (x - 2)^2 + 3$
Expand to get full equation of curve

9. (a) Find the diameter of the circle
$x^2 + y^2 - 4x + 6y - 12 = 0$
Complete square:
$(x - 2)^2 + (y + 3)^2 = 25$
So radius $= \sqrt{25} = 5 \rightarrow$ diameter $= 10$

(b) Show $4x - 3y - 5 = 0$ is a chord of $x^2 + y^2 + 3x - y - 10 = 0$

Complete square:
$x^2 + 3x + y^2 - y = 10$
$(x + 1.5)^2 - 2.25 + (y - 0.5)^2 - 0.25 = 10$
$(x + 1.5)^2 + (y - 0.5)^2 = 12.5$
It is a circle, and $4x - 3y - 5$ intersects the circle $\rightarrow$ chord
Length $= 2\sqrt{(r^2 - d^2)}$, use geometry or simultaneous solution

10. (a) Name objects:
(i) Square pyramid
(ii) Hexagonal prism

(b) Sketch shows a solid with a semi-circular cut and flat top → draw front and plan with semi-arch

11. (a) Tens digits from {2, 3, 5}, unit digits from {5, 6}
All combinations: $3 \times 2 = 6$
Numbers: 25, 26, 35, 36, 55, 56
Greater than 30: 35, 36, 55, 56 → 4
$P = 4/6 = 2/3$

(b) 30 sheep: 15 white, 15 black
P(all 3 black) $= 15/30 \times 14/29 \times 13/28 = 1/2 \times 14/29 \times 13/28 = 91/1160$

12. (a) $a = 3i + 2j + 9k$, $b = i + pj + 3k$
(i) Parallel $\rightarrow a = kb$
So:
$3 = k$, $2 = kp \rightarrow k = 3 \rightarrow p = 2/3$

Not constant → not parallel

(ii) Perpendicular → a • b = 0
3×1 + 2×p + 9×3 = 3 + 2p + 27 = 0 → 2p = -30 → p = -15

(b) u = 3i + j - 2k, v = 2i + 3j - k
u × v = |i j k|
|3 1 -2|
|2 3 -1|
= i(1×-1 - (-2)×3) - j(3×-1 - (-2)×2) + k(3×3 - 1×2)
= i(-1 + 6) - j(-3 + 4) + k(9 - 2) = 5i - 1j + 7k
Magnitude = √(25 + 1 + 49) = √75
Unit vector = (1/√75)(5i - j + 7k)

(c) PQ = 2i + j, P = (3, 2) → Q = (5, 3)
R = (-2, 6)
OQ = vector from origin to Q = 5i + 3j
OR = -2i + 6j
Check if they are scalar multiples → they're not → not parallel

13. (a) A(0,3), B(1,1), C(3,2), D(4,5)
Transformation M = [1 -2; 1 2]
Apply to each point to find A', B', C', D'
Example: A' = [1 -2; 1 2] × [0;3] = [-6;6]

(b) T = [1 -1; 5 2], maps (x, y) to x', y'

(i) y = 2x and y = x - 3
Find equations of images by substituting
Example: (x, 2x) → x' = x - 2x = -x, y' = 5x + 4x = 13x → y' = -13x'

(ii) Line y = 3x + 1
x' = x - y, y' = 5x + 2y
Find new equation → check if still same form

14. Let x units of A, y of B
Continuing and completing question 14:

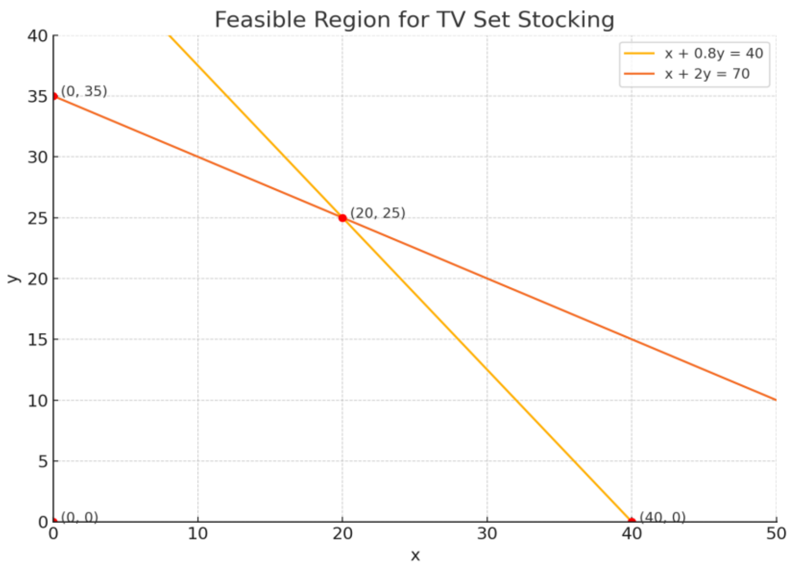
14. A retailer wants to stock TV sets A and B.
Set A costs 100,000 and uses 1 unit space
Set B costs 80,000 and uses 2 units space
Money constraint: 100000x + 80000y ≤ 4,000,000
Volume constraint: x + 2y ≤ 70

Profit: 10,000x + 15,000y
x, y ≥ 0

Simplify constraints:
Money: x + 0.8y ≤ 40
Volume: x + 2y ≤ 70

Now solve the system:

1) x + 0.8y ≤ 40
2) x + 2y ≤ 70
3) x ≥ 0, y ≥ 0



Find intersection of constraints to identify feasible region.
From (1) and (2):
x + 0.8y = 40
x + 2y = 70

Subtract:
(x + 2y) - (x + 0.8y) = 70 - 40
1.2y = 30 → y = 25
Then x = 40 - 0.8×25 = 20

Feasible points:
(0,0), (40,0), (0,35), (20,25)

Evaluate profit at each:
(0,0) → 0
(40,0) → 10,000×40 = 400,000

(0,35) → 15,000×35 = 525,000
(20,25) → 10,000×20 + 15,000×25 = 200,000 + 375,000 = 575,000

Maximum profit is 575,000 when x = 20 and y = 25.

15. (a) Box M: {1,3,5,7,9}, Box P: {1,4,9}

(i) Same value: only common = 1, 9 → P = 2/5 × 2/3 = 4/15

(ii) Product divisible by 3: check all combinations → count favorable / total 15
Favorable: (3,1), (3,4), (3,9), (1,9), (9,1), (9,4), (9,9) → total = 7
P = 7/15

(b) Given:
P(A) = 2/5, P(A ∪ B) = 3/8, P(B|A) = 1/2
(i) P(A ∩ B) = P(B|A) × P(A) = 1/2 × 2/5 = 1/5
Then P(A ∪ B) = P(A) + P(B) - P(A ∩ B) → 3/8 = 2/5 + P(B) - 1/5 → P(B) = 3/8 - 1/5
Convert and solve

(ii) P(A' ∩ B') = 1 - P(A ∪ B) = 1 - 3/8 = 5/8

(c) Choose 5 from 9 people = C(9,5) = 126 ways

15. (a) Box M contains 5 pieces of paper numbered 1, 3, 5, 7, 9.
Box P contains 3 pieces of paper numbered 1, 4, 9.
One piece of paper is removed at random from each box.
Calculate the probability that the two numbers obtained have:

(i) the same value
(ii) a product that is exactly divisible by 3.

(i) the same value
Common numbers in both boxes = {1, 9}
Probability of choosing 1 from M and 1 from P = (1/5) × (1/3) = 1/15
Probability of choosing 9 from M and 9 from P = (1/5) × (1/3) = 1/15
Total probability = 1/15 + 1/15 = 2/15

(ii) a product that is exactly divisible by 3
Products divisible by 3 occur if at least one number is divisible by 3
From M: numbers divisible by 3 = 3, 9
From P: numbers divisible by 3 = 9
Favorable combinations:

- (3 from M, 1 from P): (1/5)(1/3) = 1/15
- (3 from M, 4 from P): (1/5)(1/3) = 1/15
- (3 from M, 9 from P): (1/5)(1/3) = 1/15
- (9 from M, 1 from P): (1/5)(1/3) = 1/15
- (9 from M, 4 from P): (1/5)(1/3) = 1/15
- (9 from M, 9 from P): (1/5)(1/3) = 1/15
- (1 from M, 9 from P): (1/5)(1/3) = 1/15
- (5 from M, 9 from P): (1/5)(1/3) = 1/15
- (7 from M, 9 from P): (1/5)(1/3) = 1/15

Total favorable outcomes = 9
So probability = 9/15 = 3/5

16. (a) Find the coordinates of the turning points of the curve $y = (2x - 3)(x^2 - 6)$ and determine the nature of each point.

Expand the function:
$y = (2x - 3)(x^2 - 6) = 2x^3 - 12x - 3x^2 + 18 = 2x^3 - 3x^2 - 12x + 18$

Differentiate:
$dy/dx = 6x^2 - 6x - 12$
Set $dy/dx = 0$:
$6x^2 - 6x - 12 = 0 \rightarrow x^2 - x - 2 = 0 \rightarrow (x - 2)(x + 1) = 0$
x = 2 or x = -1

Find y at x = 2:
y = 2(8) - 3(4) - 12(2) + 18 = 16 - 12 - 24 + 18 = -2
At x = -1: $y = 2(-1)^3 - 3(-1)^2 - 12(-1) + 18 = -2 - 3 + 12 + 18 = 25$

Turning points: (2, -2) and (-1, 25)

Second derivative: $d^2y/dx^2 = 12x - 6$
At $x = 2 \rightarrow 24 - 6 = 18 > 0 \rightarrow$ minimum
At $x = -1 \rightarrow -12 - 6 = -18 < 0 \rightarrow$ maximum

(b) A cylindrical container opened at one end, has height h cm and base radius r cm.

(i) Write the expression for total surface area of the container s cm^2 and the volume V cm^3 in terms of h and r.

Since it's open at one end:
$s = \pi r^2 + 2\pi rh$
$V = \pi r^2 h$

(ii) Given that s has the value 3π, from (i) above show that $V = \frac{1}{2} \pi r [3 - r]$

$s = \pi r^2 + 2\pi rh = 3\pi$
Divide both sides by π:
$r^2 + 2rh = 3 \rightarrow 2rh = 3 - r^2$
So $h = (3 - r^2)/(2r)$

Now, $V = \pi r^2 h$
Substitute h:
$V = \pi r^2 \times (3 - r^2)/(2r) = (\pi r(3 - r^2))/2 = \frac{1}{2} \pi r(3 - r^2)$

(iii) Hence, find the value of r and the corresponding value of h which make V a maximum.

$V = \frac{1}{2} \pi r(3 - r^2)$
Differentiate: $dV/dr = \frac{1}{2} \pi(3 - r^2 - 2r^2) = \frac{1}{2} \pi(3 - 3r^2) = (3\pi/2)(1 - r^2)$
Set derivative $= 0 \rightarrow 1 - r^2 = 0 \rightarrow r = 1$
Then $h = (3 - 1)/(2\times1) = 1$

Maximum volume when r = 1 cm and h = 1 cm

(c) The curve $y = x^2 - 6x + 5$ cuts the y-axis at A and the x-axis at B and C as shown in the figure below.

Find the coordinates of A, B and C and calculate the total area of the shaded parts.

y-intercept: $x = 0 \rightarrow y = 0 - 0 + 5 = 5 \rightarrow A = (0, 5)$

x-intercepts: $x^2 - 6x + 5 = 0$
$x = [6 \pm \sqrt{(36 - 20)}]/2 = [6 \pm \sqrt{16}]/2 = [6 \pm 4]/2 \rightarrow x = 1$ or $x = 5$
So B = (1, 0), C = (5, 0)

Area under curve from 1 to 5:
$\int_1^5 (x^2 - 6x + 5)\,dx = [x^3/3 - 3x^2 + 5x]$ from 1 to 5
At x = 5: $(125/3 - 75 + 25) = 125/3 - 50$
At x = 1: $(1/3 - 3 + 5) = 1/3 + 2 = 7/3$
Area $= (125/3 - 50) - 7/3 = (125 - 150 - 7)/3 = -32/3$
Area = 32/3 units^2 (taking absolute value)

Total area of shaded region = 32/3 square units