THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

042 ADITIONAL MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2015**

Instructions

1. This paper consists of SIXTEEN questions.
2. Answer all questions in section A and any FOUR questions in section B.

1. (a) Find the next three terms in each of the following sequences:

(i) 3/5, 10/8, 16/18, 16/34, ____, ____, ____

We analyze numerators and denominators separately:
Numerators: 3, 10, 16, 16
10 - 3 = 7, 16 - 10 = 6, 16 - 16 = 0 → no clear pattern
But if pattern repeats last value: next numerators may remain 16

Denominators: 5, 8, 18, 34
8 - 5 = 3, 18 - 8 = 10, 34 - 18 = 16
So difference increases by 7: next = 34 + 23 = 57, then 57 + 30 = 87

So next terms: 16/57, 16/87, 16/120

(ii) 1, 4, 9, 16, 25, ___, ___, ___

This is the sequence of perfect squares:
$1^2, 2^2, 3^2, 4^2, 5^2$ → next: $6^2 = 36$, $7^2 = 49$, $8^2 = 64$

(b) By rounding each term to 2 significant figures, find approximate value of M = (6.7782 + 2.974)/(7.332 - 2.422)

Rounding:
6.7782 ≈ 6.8, 2.974 ≈ 3.0
7.332 ≈ 7.3, 2.422 ≈ 2.4
M ≈ (6.8 + 3.0)/(7.3 - 2.4) = 9.8 / 4.9 = 2.0

2. If the sets μ = {1,2,3,4,5,6,7,8,9}, A = {1,2,3,4}, B = {2,4,6,8}, C = {3,4,5,6}

(a) A' = complement of A = elements in μ not in A = {5,6,7,8,9}

(b) (A ∩ C)'
A ∩ C = {3,4} → complement = {1,2,5,6,7,8,9}

(c) (B - C)'
B - C = elements in B not in C = {2,8}
Complement = {1,3,4,5,6,7,9}

3. (a) If α and β are roots of $x^2 - 2x - 4 = 0$, find $\alpha^2 + \beta^2$

$\alpha + \beta = 2$, $\alpha\beta = -4$
$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4 - (-8) = 12$

(b) Remainder when $P(x) = x^3 + 2x^2 - 4x + 1$ is divided by $D(x) = x - 3$

Use remainder theorem:
$R = P(3) = 27 + 18 - 12 + 1 = 34$

4. (a) Make t subject in $s = ut - \frac{1}{2}gt^2$
$s = ut - \frac{1}{2}gt^2 \rightarrow \frac{1}{2}gt^2 - ut + s = 0$
Use quadratic formula to solve for t

(b) Solve:
$xy = 10 \rightarrow y = 10/x$
$3x + 2y = 16$
Substitute: $3x + 2(10/x) = 16$
$3x + 20/x = 16 \rightarrow$ multiply by x: $3x^2 + 20 = 16x$
$3x^2 - 16x + 20 = 0 \rightarrow x = [16 \pm \sqrt{(256 - 240)}]/6 = [16 \pm \sqrt{16}]/6 = [16 \pm 4]/6$
$x = 20/6 = 10/3$ or $x = 12/6 = 2$
If $x = 2 \rightarrow y = 5$
If $x = 10/3 \rightarrow y = 3$
Solutions: (2,5), (10/3,3)

5. (a) Exterior angle of polygon with 12 sides = $360° / 12 = 30°$

(b) If sum of interior angles = 1520°
Sum = $(n - 2) \times 180 \rightarrow (n - 2) \times 180 = 1520$
$n - 2 = 1520 / 180 = 8.44 \rightarrow$ not a whole number
So check work:
$n = (1520 / 180) + 2 = 10.44 \rightarrow$ sum does not match exactly with any integer
Check if sum was 1620 $\rightarrow (n - 2) \times 180 = 1620 \rightarrow n = 11$

6. T varies jointly with $\sqrt{x}$ and inversely with y^2
$T = k\sqrt{x} / y^2$
When $x = 9$, $y = 8$, $T = 6 \rightarrow 6 = k \times 3 / 64 \rightarrow k = 128$

(a) Equation: $T = 128\sqrt{x} / y^2$

(b) When $x = 1/4$, $y = 1/6$
$\sqrt{x} = 1/2$, $y^2 = 1/36$
$T = 128 \times 1/2 \div (1/36) = 64 \times 36 = 2304$

7. Differentiate $y = 2\pi x - 3x^2$ using first principle

$f(x + h) = 2\pi(x + h) - 3(x + h)^2 = 2\pi x + 2\pi h - 3(x^2 + 2xh + h^2)$

$= 2\pi x + 2\pi h - 3x^2 - 6xh - 3h^2$
$f(x + h) - f(x) = 2\pi h - 6xh - 3h^2$
Divide by h:
$(2\pi - 6x - 3h)$
As $h \to 0 \to$ derivative $= 2\pi - 6x$

8. (a) Eliminate θ from $x = a \tan\theta$ and $y = b \cos\theta$

$\tan\theta = x/a \to \theta = \arctan(x/a)$
$\cos\theta = y/b$
Use identity: $\tan^2\theta + 1 = \sec^2\theta \to 1/\cos^2\theta = \tan^2\theta + 1$
$\cos\theta = y/b \to \cos^2\theta = y^2/b^2 = 1/(x^2/a^2 + 1)$
Solve accordingly: relation between x and y

(b) (i) Supplementary angles are two angles whose sum is 180°

(ii) If $2x - 40°$ and $80° - 2x$ are supplementary:
$2x - 40 + 80 - 2x = 180 \to 40 = 180 \to$ contradiction
Check setup again:
$(2x - 40) + (80 - 2x) = 40 + 40 = 80 \neq 180$
So must be:
$(2x - 40) + (80 - 2x) = 180 \to$ same as above $\to$ contradiction
Check again: maybe question miswritten

9. (a) Locus is the set of all points that satisfy a given condition

(b) Find equation of locus equidistant from point A(0,1) and line $x - y = 0$

Let point be (x,y)
Distance from (x,y) to point (0,1) $= \sqrt{[(x - 0)^2 + (y - 1)^2]}$
Distance to line $x - y = 0 \to |x - y| / \sqrt{2}$
Equating distances:
$\sqrt{(x^2 + (y - 1)^2)} = |x - y| / \sqrt{2}$
Square both sides and solve

10. (a) Define:
(i) Front elevation: view of object as seen from front
(ii) Plan view: top-down view of the object

(b) Draw plan, front and side views of cylinder with 1.5 cm diameter, 2 cm height
Plan: circle
Front and side: rectangle 1.5 cm wide and 2 cm high

11. (a) Point dividing segment A(5,8), B(-8,5) in ratio 3:2
$x = (3\times(-8) + 2\times5)/(3 + 2) = (-24 + 10)/5 = -14/5 = -2.8$
$y = (3\times5 + 2\times8)/5 = (15 + 16)/5 = 31/5 = 6.2$
Point = (-2.8, 6.2)

(b) Find tangents between $4x + 3y - 12 = 9$ and $y - 3x = 0$
Line 1: $4x + 3y = 21$
Line 2: $y = 3x$
Find angle using $\tan\theta = |(m1 - m2)/(1 + m1m2)|$
m1 = -4/3, m2 = 3
$\tan\theta = |(-4/3 - 3)/(1 + (-4/3)(3))| = |-13/3 / (1 - 4)| = |-13/3 / -3| = 13/9$
$\theta = \arctan(13/9)$

(c) Circle: $4x^2 + 4y^2 + 20x - 16y + 37 = 0$
Divide by 4: $x^2 + y^2 + 5x - 4y + 37/4 = 0$
Complete square:
$x^2 + 5x = (x + 2.5)^2 - 6.25$
$y^2 - 4y = (y - 2)^2 - 4$
So full equation: $(x + 2.5)^2 + (y - 2)^2 = 6.25 + 4 - 37/4 = 25/4$
Centre = (-2.5, 2), radius = $\sqrt{(25/4)} = 2.5$

12. (a) Rainfall:
Arrange values, get median = average of 8th and 9th
Sorted list:
25, 26, 26, 27, 27, 27, 28, 29, 32, 33, 34, 35, 38, 39, 42, 44, 45
Median = (29 + 32)/2 = 30.5
Range = max - min = 45 - 25 = 20

(b) Use frequency table:

Age	Freq	Midpoint	fx
52–48	4	50	200
47–43	6	45	270
42–38	7	40	280
37–33	11	35	385
32–28	9	30	270
27–23	8	25	200
22–18	5	20	100

Total frequency = 50, sum fx = 1705
Mean = 1705 / 50 = 34.1

To calculate the standard deviation:

Age Interval	Frequency (f)	Midpoint (x)	f × x	f × x^2
52–48	4	50	200	10000
47–43	6	45	270	12150
42–38	7	40	280	11200
37–33	11	35	385	13475
32–28	9	30	270	8100
27–23	8	25	200	5000
22–18	5	20	100	2000

Total f = 50
Total f × x = 1705
Mean = 1705 / 50 = 34.1
Total f × x^2 = 61725

Variance = ($\Sigma f \times x^2 / N$) - $(\text{mean})^2$
= 61725 / 50 - $(34.1)^2$ = 1234.5 - 1162.81 = 71.69

Standard deviation = $\sqrt{71.69}$ = 8.7

13. (a) Write the truth value of the statement: "If 2 is a prime number, then 2 is not an even number."

2 is a prime number — True
2 is not an even number — False
If $p \longrightarrow q$ is False only when p is true and q is false
So this statement is False

(b) Construct the truth table for proposition $p \wedge (q \vee r)$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

(c) Test the validity of the argument:

Let

p: Tanzania is making a new constitution
q: Tanzania is editing the constitution
r: Tanzania has a constitution

Premises:
1. p
2. $q \lor p$
3. $p \longrightarrow r$
Conclusion: $r \land p$

From 1 and 3, p is true, $p \longrightarrow r$ implies r is true
So both r and p are true → $r \land p$ is valid
Hence, the argument is valid

14. (a) Three unbiased coins are tossed once

(i) Probability tree diagram has 3 levels (H/T each) → total 8 outcomes
HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

(ii) At most two heads = 0, 1, or 2 heads
Outcomes: TTT, HTT, THT, TTH, HHT, HTH, THH → 7 outcomes
P = 7/8

(b) Number of ways to select 9 balls: 3 red, 3 white, 3 blue
Ways = $C(6,3) \times C(5,3) \times C(5,3) = 20 \times 10 \times 10 = 2000$

15. (a) a = i + j + k
b = i - j - k
c = i - 2j + 3k
Find $a \times (b \times c)$

First find $b \times c$:
|i j   k|
|1 -1 -1|
|1 -2 3| = $i(-1\times3 + 1\times2) - j(1\times3 - (-1\times1)) + k(1\times-2 - (-1\times1))$
$= i(-3 + 2) - j(3 + 1) + k(-2 + 1) = -i - 4j - k$

Then $a \times (b \times c)$:
a = i + j + k
$\times$ (-i - 4j - k)
Use determinant:
= |i j k|
|1 1 1|

|-1 -4 -1| = i(1×-1 - 1×-4) - j(1×-1 - 1×-1) + k(1×-4 - 1×-1)
= i(-1 + 4) - j(-1 + 1) + k(-4 + 1) = 3i + 0j -3k

So result is 3i - 3k

(b) Solve system using substitution:

3y + 2x = z + 1 → z = 3y + 2x - 1
Substitute into:
3x + 2z = 8 - 5y
→ 3x + 2(3y + 2x - 1) = 8 - 5y
3x + 6y + 4x - 2 = 8 - 5y
7x + 6y = 10 - 5y → 7x + 11y = 10

Also: 3z - 1 = x - 2y
z = 3y + 2x - 1
So 3(3y + 2x - 1) - 1 = x - 2y
9y + 6x - 3 - 1 = x - 2y
6x + 9y - 4 = x - 2y → 5x + 11y = 4

Now solve:
7x + 11y = 10
5x + 11y = 4
Subtract: 2x = 6 → x = 3
Then 7(3) + 11y = 10 → 21 + 11y = 10 → y = -1
Then z = 3(-1) + 2(3) - 1 = -3 + 6 - 1 = 2
Solution: x = 3, y = -1, z = 2

(c) Transformation:
Matrix: [2 0; 0 -2], point R(1,1)
x' = 2×1 + 0 = 2
y' = 0 + (-2×1) = -2
Image: (2, -2)

16. (a) $dy/dx = 3x^4 - 4x^2 + 5x + 1/x^2$
Integrate:
$\int(3x^4 - 4x^2 + 5x + x^{-2})\,dx = (3/5)x^5 - (4/3)x^3 + (5/2)x^2 - x^{-1} + C$
So $y = (3/5)x^5 - (4/3)x^3 + (5/2)x^2 - 1/x + C$

(b) $\int_0$^π (cos x + 2cos 2x) dx = ∫ cos x dx + 2∫ cos 2x dx
= sin x + sin 2x from 0 to π
= sin π + sin 2π - sin 0 - sin 0 = 0

(c) Derivative of $(2 - x^2)(3x + x^2)$

Use product rule:

$u = 2 - x^2$, $v = 3x + x^2$

$u' = -2x$, $v' = 3 + 2x$

$dy/dx = u'v + uv' = (-2x)(3x + x^2) + (2 - x^2)(3 + 2x)$

$= -2x(3x + x^2) + (2 - x^2)(3 + 2x)$

$= -6x^2 - 2x^3 + (2\times3 + 2\times2x - 3x^2 - 2x^3) = -6x^2 - 2x^3 + 6 + 4x - 3x^2 - 2x^3$

$= -4x^3 - 9x^2 + 4x + 6$