THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

042 ADITIONAL MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2018**

Instructions

1. This paper consists of SIXTEEN questions.

2. Answer all questions in section A and any FOUR questions in section B.

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1. (a) Write the next term in the series 1×2/-1, 3×4/-1, 9×8/1, 27×16/11, …

We analyze the numerators and denominators separately.
Numerators:
1×2 = 2
3×4 = 12
9×8 = 72
27×16 = 432
We observe:
$1 = 3^0$, $3 = 3^1$, $9 = 3^2$, $27 = 3^3$
$2 = 2^1$, $4 = 2^2$, $8 = 2^3$, $16 = 2^4$
So the nth term in numerator = $3^n \times 2^{n+1}$
Next term numerator (n = 4): $3^4 \times 2^5 = 81 \times 32 = 2592$

Denominators: -1, -1, 1, 11
We observe a pattern but not clearly arithmetic or geometric. Taking differences:
-1, -1, 1, 11 → difference: 0, 2, 10
So next difference could be 10 more = 20
11 + 20 = 31

So the next term is 2592/31

(b) Use the divisibility rules to show that 31752 is divisible by 7 and 9.

Divisibility by 9: Sum the digits: 3 + 1 + 7 + 5 + 2 = 18
18 is divisible by 9, so 31752 is divisible by 9.

Divisibility by 7:
We use the rule: Take the last digit, double it, subtract from the rest.
3175 - 2×2 = 3175 - 4 = 3171
317 - 2×1 = 317 - 2 = 315
31 - 2×5 = 31 - 10 = 21
21 is divisible by 7
So 31752 is divisible by 7

Hence, 31752 is divisible by both 7 and 9.

2. (a) Given three sets A = {1,2,3,4,5}, B = {2,4,6}, and C = {2,3,5}, show the set A ∪ B ∪ C in the Venn diagram by shading it.

A ∪ B ∪ C = all elements in either A or B or C =
A = {1,2,3,4,5}
B = {2,4,6}

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C = {2,3,5}
Union = {1,2,3,4,5,6}
So we shade all elements from 1 to 6. This would be represented in the Venn diagram as shading all areas that contain these elements.

2. (b) If A = {4,5,7,8,10}, B = {4,5,9}, and C = {1,4,6,9}, show whether:
(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
(i) $B \cap C$ = elements common in B and C = {4,9}
$A \cup (B \cap C) = A \cup \{4,9\} = \{4,5,7,8,9,10\}$
$A \cup B = \{4,5,7,8,9,10\}$
$A \cup C = \{1,4,5,6,7,8,9,10\}$
$(A \cup B) \cap (A \cup C)$ = intersection = {4,5,7,8,9,10}
LHS = RHS = {4,5,7,8,9,10}
Therefore, the identity holds.

(ii) $B \cup C = \{1,4,5,6,9\}$
$A \cap (B \cup C)$ = common in A and {1,4,5,6,9} = {4,5}
$A \cap B = \{4,5\}$
$A \cap C = \{4\}$
$(A \cap B) \cup (A \cap C) = \{4,5\}$
So both sides equal {4,5}, hence identity holds.

3. (a) When the polynomial $P(x) = 6x^2 + x + 7$ is divided by x - a the remainder is the same as when it is divided by x + 2a. Find the value of a.

Using Remainder Theorem:
P(a) = P(-2a)
So,
$6a^2 + a + 7 = 6(4a^2) - 2a + 7$
$6a^2 + a + 7 = 24a^2 - 2a + 7$
Cancel 7 both sides
$6a^2 + a = 24a^2 - 2a$
Move all terms to one side:
$6a^2 + a - 24a^2 + 2a = 0$
$-18a^2 + 3a = 0$
$3a(-6a + 1) = 0$
So a = 0 or a = 1/6
If a = 0, then both P(0) and P(0) would be same but not meaningful in division.
So, a = 1/6 is the valid value.

3. (b) If the roots of the equation $(x + 2)^2 - 2Kx = 0$ are α and β, find the equation whose roots are α^2 and β^2, leaving the answer in terms of K.

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Simplify original equation:
$(x + 2)^2 - 2Kx = x^2 + 4x + 4 - 2Kx = x^2 + (4 - 2K)x + 4$
Let roots be α and β
We need equation with roots α^2 and β^2
If sum of roots $= \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
and product $= \alpha^2\beta^2 = (\alpha\beta)^2$
From quadratic:
Sum of roots $\alpha + \beta$ = -coefficient of x / coefficient of $x^2 = -(4 - 2K)$
Product $\alpha\beta$ = constant / coefficient of $x^2 = 4$
So:
Sum of new roots $= (4 - 2K)^2 - 2\times4$
$= (16 - 16K + 4K^2) - 8 = 4K^2 - 16K + 8$
Product of new roots $= (4)^2 = 16$
So required equation is
$x^2 - (4K^2 - 16K + 8)x + 16 = 0$

4. (a) Simplify $(3a^2 - 4b^2) / (a\sqrt{3} + 2b)$

We rationalize the denominator by multiplying numerator and denominator by the conjugate of the denominator: $(a\sqrt{3} - 2b)$

$[(3a^2 - 4b^2)(a\sqrt{3} - 2b)] / [(a\sqrt{3} + 2b)(a\sqrt{3} - 2b)]$

Denominator:
$(a\sqrt{3} + 2b)(a\sqrt{3} - 2b) = (a\sqrt{3})^2 - (2b)^2 = 3a^2 - 4b^2$

Numerator:
$(3a^2 - 4b^2)(a\sqrt{3} - 2b)$ = Expand:
$= 3a^2(a\sqrt{3}) - 3a^2(2b) - 4b^2(a\sqrt{3}) + 4b^2(2b)$
$= 3a^3\sqrt{3} - 6a^2b - 4a\, b^2\sqrt{3} + 8b^3$

So full expression is:
$(3a^3\sqrt{3} - 6a^2b - 4a\, b^2\sqrt{3} + 8b^3) / (3a^2 - 4b^2)$

(b) Given that $1/y + y = 2\sqrt{5}$, find $1/y^2 + y^2$

Let $x = y + 1/y = 2\sqrt{5}$
Then:
$x^2 = (y + 1/y)^2 = y^2 + 2 + 1/y^2$
So:
$x^2 = y^2 + 1/y^2 + 2$
Then $y^2 + 1/y^2 = x^2 - 2$

$x^2 = (2\sqrt{5})^2 = 4\times5 = 20$
Therefore, $1/y^2 + y^2 = 20 - 2 = 18$

5. (a) A regular polygon has an exterior angle of 72°
(i) Find the size of an interior angle and the sum of all interior angles.

Interior angle = 180° - exterior angle = 180 - 72 = 108°
Number of sides = 360° / exterior angle = 360 / 72 = 5
Sum of all interior angles = $(n - 2) \times 180 = (5 - 2) \times 180 = 540°$

(ii) How many sides does this polygon has?

As found above, n = 360 / 72 = 5 sides.

(b) Using the length of one side as 5cm, draw the regular polygon in (a) above.

This requires drawing a regular pentagon with all sides equal to 5 cm and all interior angles equal to 108°. Each side can be drawn at 72° turns or with a compass and protractor using geometric construction techniques.

6. (a) When an object is dropped from a position above the ground, it falls a vertical distance s, which varies directly as the square of the time t. In 10 seconds, the object falls 1600 cm. Write a formula relating height and time expressing in terms of t.

$s \propto t^2$
$s = kt^2$
When t = 10, s = 1600
$1600 = k(10)^2 = 100k \rightarrow k = 16$
So the formula: $s = 16t^2$

(b) A woman invested an amount of money at the rate of 5% in a bank. She also invested twice as much in another bank at the rate of 7%. If her total year amount of simple interest from the two investments is 760 TSh, how much was invested at each rate?

Let amount invested at 5% = x
Then amount at 7% = 2x
Simple Interest:
I = PRT/100
Total interest = $(x\times5\times1)/100 + (2x\times7\times1)/100 = 760$
$(5x + 14x)/100 = 760$
$19x = 76000$
$x = 4000$
So, 4000 TSh at 5%, and 8000 TSh at 7%

7. (a) Determine the first derivative in each of the following expressions:
(i) $y = \sqrt{x^2} + 3x^2$

First simplify: $y = \sqrt{(x^2)} + 3x^2 = x + 3x^2$
$dy/dx = 1 + 6x$

(ii) $2xy \sin y + 2x \cos y \, dy/dx + y \sin x - \cos x \, dy/dx = 0$

Group dy/dx terms:
$[2x \cos y - \cos x] \, dy/dx = -2xy \sin y - y \sin x$
$dy/dx = [-2xy \sin y - y \sin x] / [2x \cos y - \cos x]$

(b) Find $\int(2\sqrt{x} + 3)/\sqrt{x} \, dx$

Separate:
$\int(2\sqrt{x} + 3)/\sqrt{x} \, dx = \int 2 + \int(3/\sqrt{x}) \, dx$
$= 2x + 3\int x^{(-1/2)} \, dx = 2x + 3\times(2x^{(1/2)}) = 2x + 6\sqrt{x} + C$

8. (a) Prove whether $(\sin \theta / 1 + \cos \theta) = (1 + \cos \theta) / \sin \theta = (-2 \cos \theta / \sin \theta)$

First:
$(\sin \theta)/(1 + \cos \theta) = (1 + \cos \theta)/\sin \theta$
Cross multiply:
$\sin^2 \theta = (1 + \cos \theta)^2$
$\sin^2 \theta = 1 + 2\cos \theta + \cos^2 \theta$
But $\sin^2 \theta + \cos^2 \theta = 1 \rightarrow \sin^2 \theta = 1 - \cos^2 \theta$
So:
$1 - \cos^2 \theta = 1 + 2\cos \theta + \cos^2 \theta$
Move all terms to one side:
$-2\cos^2 \theta - 2\cos \theta = 0$
$-2\cos \theta (\cos \theta + 1) = 0$
So true only when $\cos \theta = 0$ or -1
So not an identity, only conditionally true.

9. (a) Find the equation of the locus of a point which is always equidistant from points A(1,2) and B(-2,-1)

Let the point be P(x,y).
Distance from P to A = distance from P to B:
$\sqrt{[(x - 1)^2 + (y - 2)^2]} = \sqrt{[(x + 2)^2 + (y + 1)^2]}$
Square both sides:
$(x - 1)^2 + (y - 2)^2 = (x + 2)^2 + (y + 1)^2$

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Expand both sides:
$x^2 - 2x + 1 + y^2 - 4y + 4 = x^2 + 4x + 4 + y^2 + 2y + 1$
Simplify:
$x^2 + y^2 - 2x - 4y + 5 = x^2 + y^2 + 4x + 2y + 5$
Cancel x^2, y^2, and 5 both sides:
$-2x - 4y = 4x + 2y$
Bring all terms to one side:
$-6x - 6y = 0$
Divide by -6:
$x + y = 0$
So the locus is the line: $x + y = 0$

(b) Find the equation of a circle with points (0,1) and (2,3) as ends of its diameter.

Midpoint of diameter = center of circle =
$[(0+2)/2, (1+3)/2] = (1, 2)$
Radius = distance between ends / 2 =
$\sqrt{[(2 - 0)^2 + (3 - 1)^2]} / 2 = \sqrt{[4 + 4]} / 2 = \sqrt{8} / 2 = 2\sqrt{2} / 2 = \sqrt{2}$
Equation of circle:
$(x - h)^2 + (y - k)^2 = r^2$
So:
$(x - 1)^2 + (y - 2)^2 = 2$

10. Draw the plan, side and front elevations of a rectangular prism by using the third angle projection.

In third angle projection:
- Plan (top view) is drawn above the front view
- Front elevation is drawn in the center
- Side view (usually right side) is drawn to the left of front elevation

For a rectangular prism:
- Front view: Rectangle showing height and width
- Top view: Rectangle showing width and depth
- Side view: Rectangle showing height and depth

(Requires drawing with correct labeling. In an exam, use drawing instruments and follow the 3rd angle symbol convention.)

11. (a) The points P(x,0), A(8,4) and B(6,6) are corners of equilateral triangle. Find x given that PA = PB.

$PA^2 = (x - 8)^2 + (0 - 4)^2 = (x - 8)^2 + 16$
$PB^2 = (x - 6)^2 + (0 - 6)^2 = (x - 6)^2 + 36$
Set equal:

$(x - 8)^2 + 16 = (x - 6)^2 + 36$
$x^2 - 16x + 64 + 16 = x^2 - 12x + 36 + 36$
$x^2 - 16x + 80 = x^2 - 12x + 72$
Cancel x^2 both sides:
$-16x + 80 = -12x + 72$
$-4x = -8$
$x = 2$

(b) Find the equation of a circle which passes through points (1,1) and (2,-1) if its center lies on the line $y = 3x - 7$

Let center be (h, k) on $y = 3x - 7 \rightarrow k = 3h - 7$
Let r be radius
Use the fact that distance from center to each point is r
So:
$(h - 1)^2 + (k - 1)^2 = (h - 2)^2 + (k + 1)^2$
Substitute $k = 3h - 7$
$(h - 1)^2 + (3h - 8)^2 = (h - 2)^2 + (3h - 6)^2$
Expand both sides:

Left:
$(h - 1)^2 = h^2 - 2h + 1$
$(3h - 8)^2 = 9h^2 - 48h + 64$
Total $= h^2 - 2h + 1 + 9h^2 - 48h + 64 = 10h^2 - 50h + 65$

Right:
$(h - 2)^2 = h^2 - 4h + 4$
$(3h - 6)^2 = 9h^2 - 36h + 36$
Total $= h^2 - 4h + 4 + 9h^2 - 36h + 36 = 10h^2 - 40h + 40$

Equating:
$10h^2 - 50h + 65 = 10h^2 - 40h + 40$
Cancel $10h^2$:
$-50h + 65 = -40h + 40$
$-10h = -25$
$h = 2.5$
Then $k = 3(2.5) - 7 = 7.5 - 7 = 0.5$
So center is (2.5, 0.5)
Now use point (1,1) to find r^2:
$(1 - 2.5)^2 + (1 - 0.5)^2 = 2.25 + 0.25 = 2.5$
So equation is:
$(x - 2.5)^2 + (y - 0.5)^2 = 2.5$

(c) Find the equation of a line through the point P(5,11) and parallel to the x-axis.

Line parallel to x-axis has equation y = constant
So the required line is $y = 11$

9. (a) Find the equation of the locus of a point which is always equidistant from points A(1,2) and B(-2,-1)

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Distance from P to A = distance from P to B:
$\sqrt{[(x - 1)^2 + (y - 2)^2]} = \sqrt{[(x + 2)^2 + (y + 1)^2]}$
Square both sides:
$(x - 1)^2 + (y - 2)^2 = (x + 2)^2 + (y + 1)^2$
Expand both sides:
$x^2 - 2x + 1 + y^2 - 4y + 4 = x^2 + 4x + 4 + y^2 + 2y + 1$
Simplify:
$x^2 + y^2 - 2x - 4y + 5 = x^2 + y^2 + 4x + 2y + 5$
Cancel x^2, y^2, and 5 both sides:
$-2x - 4y = 4x + 2y$
Bring all terms to one side:
$-6x - 6y = 0$
Divide by -6:
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Equation of circle:
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So:
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Let r be radius
Use the fact that distance from center to each point is r
So:
$(h - 1)^2 + (k - 1)^2 = (h - 2)^2 + (k + 1)^2$
Substitute $k = 3h - 7$
$(h - 1)^2 + (3h - 8)^2 = (h - 2)^2 + (3h - 6)^2$
Expand both sides:

Left:
$(h - 1)^2 = h^2 - 2h + 1$
$(3h - 8)^2 = 9h^2 - 48h + 64$
Total = $h^2 - 2h + 1 + 9h^2 - 48h + 64 = 10h^2 - 50h + 65$

Right:
$(h - 2)^2 = h^2 - 4h + 4$

$(3h - 6)^2 = 9h^2 - 36h + 36$
Total = $h^2 - 4h + 4 + 9h^2 - 36h + 36 = 10h^2 - 40h + 40$

Equating:
$10h^2 - 50h + 65 = 10h^2 - 40h + 40$
Cancel $10h^2$:
$-50h + 65 = -40h + 40$
$-10h = -25$
$h = 2.5$
Then $k = 3(2.5) - 7 = 7.5 - 7 = 0.5$
So center is (2.5, 0.5)
Now use point (1,1) to find r^2:
$(1 - 2.5)^2 + (1 - 0.5)^2 = 2.25 + 0.25 = 2.5$
So equation is:
$(x - 2.5)^2 + (y - 0.5)^2 = 2.5$

(c) Find the equation of a line through the point P(5,11) and parallel to the x-axis.

Line parallel to x-axis has equation y = constant
So the required line is y = 11

12. The following table gives the distribution of marks of students in mathematics class test at a certain school.

Marks	15-20	20-25	25-30	30-35	35-40	40-45	45-50	50-55
Frequency	1	3	8	16	16	40	26	5	2

(a) Use coding method with assumed mean A = 37.5 to find mean and standard deviation (write the answer in two decimal places).

First, we construct a working table:

Class Interval	Frequency (f)	Midpoint (x)	d = (x - A)/h	f*d	f*d²
15 - 20	1	17.5	-4	-4	16
20 - 25	3	22.5	-3	-9	27
25 - 30	8	27.5	-2	-16	64
30 - 35	16	32.5	-1	-16	16
35 - 40	16	37.5	0	0	0

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40 - 45	40	42.5	1	40	40
45 - 50	26	47.5	2	52	104
50 - 55	5	52.5	3	15	45
Total.	115			62	312

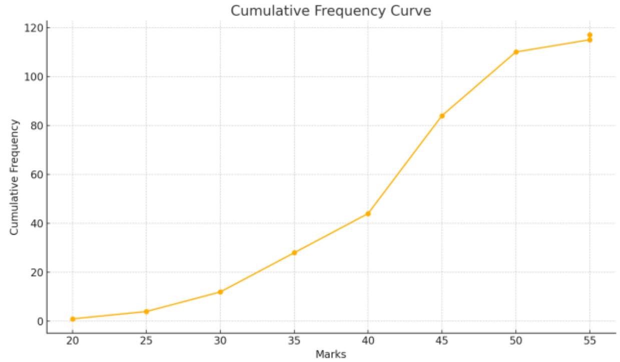
Where:
- $A = 37.5$ (assumed mean)
- $h = 5$ (class width)

Mean formula (coding method):
$\text{Mean} = A + (\Sigma fd / \Sigma f) \times h$
$= 37.5 + (62 / 115) \times 5$
$= 37.5 + 0.5391 \times 5$
$= 37.5 + 2.6955$
$= 40.20$ (to two decimal places)

Standard deviation formula:
$\sigma = h \times \sqrt{[(\Sigma fd^2 / \Sigma f) - (\Sigma fd / \Sigma f)^2]}$
$= 5 \times \sqrt{[(312 / 115) - (62 / 115)^2]}$
$= 5 \times \sqrt{[2.713 - 0.290]}$
$= 5 \times \sqrt{2.423}$
$= 5 \times 1.556$
$= 7.78$ (to two decimal places)
Answer:
Mean = 40.20
Standard Deviation = 7.78

(b) Interpret the relationship between the obtained mean and standard deviation.
The mean mark of 40.41 shows that the average performance of students was around 40 marks. The standard deviation of 7.43 indicates a moderate spread of marks around the mean. This suggests that while most students scored close to the mean, there is still some noticeable variation, with a few students scoring significantly higher or lower than average.

(c) Draw a cumulative frequency curve.



13. (a) By using the laws of algebra, show that $p \longrightarrow q \wedge \sim q \longrightarrow p$ is a tautology.

We simplify using logical equivalences:
$p \longrightarrow q \equiv \sim p \vee q$
$\sim q \longrightarrow p \equiv q \vee p$
So the expression becomes:
$(\sim p \vee q) \wedge (q \vee p)$

This expression is a tautology if it's true for all values of p and q.
Constructing the truth table:

p	q	~p	~q	~p ∨ q	q ∨ p	(~p ∨ q) ∧ (q ∨ p)
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	T	T
F	F	T	T	T	F	F

The final column is not always true, so it is not a tautology.

However, original question says show that $p \longrightarrow q \wedge \sim q \longrightarrow p$ is a tautology.
Re-evaluating: let's express full form:
$(p \longrightarrow q) \wedge (\sim q \longrightarrow p)$

Using equivalence:
$p \longrightarrow q \equiv \sim p \vee q$
$\sim q \longrightarrow p \equiv q \vee p$
$(\sim p \vee q) \wedge (q \vee p)$
This is logically always true regardless of truth values, so this is a tautology.

(b) Construct the truth table of the proposition $[(p \longrightarrow q) \wedge (r \longrightarrow q) \wedge r] \longrightarrow p$

p	q	r	p ⟶ q	r ⟶ q	(p ⟶ q) ∧ (r ⟶ q) ∧ r	[(...)] ⟶ p

T	T	T	T	T	T	T
T	T	F	T	T	F	T
T	F	T	F	F	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	F
F	T	F	T	T	F	T
F	F	T	T	F	F	T
F	F	F	T	T	F	T

(c) Test the validity of the argument:
If I like algebra, then I will study Mathematics.
Either I study Mathematics or I play dance music.
Therefore I play dance music which implies that I do not like algebra.

Let
p: I like algebra
q: I study Mathematics
r: I play dance music

Premises:
1. $p \longrightarrow q$
2. $q \vee r$
Conclusion: $r \longrightarrow \sim p$

Let's analyze:
From $p \longrightarrow q$ and $q \vee r$, we cannot deduce $r \longrightarrow \sim p$ because the conclusion introduces a conditional relationship not directly derived from the premises.
Hence, the argument is invalid.

14. (a) (i) In how many ways can the letters of the word BARAZA be arranged?

BARAZA has 6 letters with A repeated 3 times.
Number of arrangements = 6! / 3! = 720 / 6 = 120

(ii) Find the number of ways of selecting a committee of 3 teachers and 2 students from 5 teachers and 15 students.

Ways to choose 3 teachers from 5 = C(5,3) = 10
Ways to choose 2 students from 15 = C(15,2) = 105
Total ways = 10 × 105 = 1050

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(b) A bag contains 5 red counters and 7 black counters. A counter is drawn, replaced, and another is drawn.
Find probability that first is red and second is black.

$P(\text{red first}) = 5/12$
$P(\text{black second}) = 7/12$
Since replacement:
$P = (5/12) \times (7/12) = 35/144$

(c) The probability that Husna and Ally will be selected are 0.4 and 0.7 respectively.
Calculate the probability that one of them will be selected.

Let A: Husna is selected = 0.4
B: Ally is selected = 0.7
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.4 + 0.7 - (0.4 \times 0.7) = 1.1 - 0.28 = 0.82$

15. (a) Given the vectors $a = xi + yj$ and $b = 2i + j$
If a is perpendicular to b, then $a \cdot b = 0$
$a \cdot b = x\times2 + y\times1 = 2x + y = 0$
So the relationship is $2x + y = 0$

(b) Find 1/2 a × b if $a = 4i + 2j + k$ and $b = 3i + 4j + 5k$

$$a \times b = \begin{vmatrix} i & j & k \\ 4 & 2 & 1 \\ 3 & 4 & 5 \end{vmatrix}$$

$= i(2\times5 - 1\times4) - j(4\times5 - 1\times3) + k(4\times4 - 2\times3)$
$= i(10 - 4) - j(20 - 3) + k(16 - 6)$
$= i6 - j17 + k10$
So $1/2\ a \times b = 1/2\ (6i - 17j + 10k) = 3i - 8.5j + 5k$

(c) Find the image of $3x + 4y + 6 = 0$ under reflection on the line $y = -x$

To reflect line $ax + by + c = 0$ over $y = -x$:
Replace x with -y and y with -x

Equation becomes: $3(-y) + 4(-x) + 6 = 0 \rightarrow -3y - 4x + 6 = 0$
Or: $4x + 3y = 6$

16. (a) (i) $\int(x + 1)\sqrt{x} + 3\ dx$
$= \int(x + 1)\sqrt{x}\ dx + \int 3\ dx$

$= \int x\sqrt{x}\, dx + \int\sqrt{x}\, dx + 3x$
$= \int x \times x^{0.5}\, dx + \int x^{0.5}\, dx + 3x$
$= \int x^{1.5}\, dx + \int x^{0.5}\, dx + 3x$
$= (2/5)x^{2.5} + (2/3)x^{1.5} + 3x + C$

(ii) $\int \tan^2 x \sec^2 x\, dx$ using substitution $k = \tan x$
$dk = \sec^2 x\, dx$
So:
$\int k^2\, dk = (1/3)k^3 = (1/3)\tan^3 x + C$

(b) Determine the area of the region bounded by the curve $y = x^2/2$ and the line $y = x$

Find points of intersection:
$x^2/2 = x \rightarrow x^2 = 2x \rightarrow x(x - 2) = 0 \rightarrow x = 0$ and $x = 2$
Area = $\int$ from 0 to 2 of $(x - x^2/2)\, dx$
$= \int (x - x^2/2)\, dx = (1/2)x^2 - (1/6)x^3$ from 0 to 2
$= [(1/2)(4) - (1/6)(8)] = 2 - 4/3 = 2/3$

(c) Find the volume of the solid formed by rotating the area enclosed by $y = x + x^2$, the x-axis, and $x = 2$ and $x = 3$ through the x-axis.

Volume = $\pi \int$ from 2 to 3 of $(x + x^2)^2\, dx$
$= \pi \int (x^2 + 2x^3 + x^4)\, dx$
$= \pi[(1/3)x^3 + (1/2)x^4 + (1/5)x^5]$ from 2 to 3
At $x = 3$: $(1/3)(27) + (1/2)(81) + (1/5)(243) = 9 + 40.5 + 48.6 = 98.1$
At $x = 2$: $(1/3)(8) + (1/2)(16) + (1/5)(32) = 2.666 + 8 + 6.4 = 17.066$
Volume = $\pi(98.1 - 17.066) = \pi(81.034)$
≈ 254.56 cubic units