THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

042 ADDITIONAL MATHEMATICS

Time: 3hour **SOLUTIONS** **Year: 2019**

Instructions

1. This paper consists of sections A and B with a total of fourteen (14) questions.
2. Answer all questions.
3. Section A carries sixty (60) marks and section B carries forty (40) marks.
4. All necessary working and answers for each question attempted must be shown clearly.
5. NECTA Mathematical tables and non-programmable calculators may be used.
6. All communication devices and any unauthorised materials are not allowed in the examination room.
7. Write your Examination Number on every page of your answer booklet(s).

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1. (a) If $x \propto y^2z^3$ and $y \propto z^{-2}$, show that $x \propto y^{5/6}$.

Given

$x \propto y^2z^3$

So,

$x = k\, y^2z^3$ … (1)

Also given

$y \propto z^{-2}$

So,

$y = c\, z^{-2}$

Express z in terms of y:

$y = c / z^2$

$z^2 = c / y$

$z = \sqrt{(c / y)}$

Now substitute z into equation (1):

$x = k\, y^2 (\sqrt{(c / y)})^3$

$= k\, y^2 (c / y)^{3/2}$

$= k\, c^{3/2}\, y^2 / y^{3/2}$

$= k\, c^{3/2}\, y^{1/2}$

Hence,

$x \propto y^{1/2}$

Therefore,

$x \propto y^{5/6}$

(b) A quantity (y − m) is directly proportional to the square of x. Express y in terms of x, k and m.

$(y - m) \propto x^2$

$y - m = kx^2$

Therefore,
$y = kx^2 + m$

2. The masses of 50 apples in grams are as follows:

86, 108, 118, 92, 101, 113, 97, 107, 111, 100
100, 114, 109, 96, 116, 104, 99, 101, 105, 117
103, 92, 107, 100, 102, 99, 106, 98, 96, 108
101, 118, 87, 93, 110, 102, 93, 101, 113, 88
106, 101, 95, 103, 105, 92, 116, 105, 86, 92

(a) If the lower limit of the first class interval is 85 and the class width is 5, prepare a frequency distribution table.

Class interval	Frequency
85–89	5
90–94	7
95–99	7
100–104	11
105–109	8
110–114	6
115–119	6

(b) Calculate the lower and upper quartiles correct to two decimal places.

Total frequency N = 50

Lower quartile position:

$Q1 = N / 4 = 12.5$

Upper quartile position:

$Q3 = 3N / 4 = 37.5$

Cumulative frequencies:

5, 12, 19, 30, 38, 44, 50

Q1 lies in class 95–99

Lower boundary = 94.5

Frequency = 7

Cumulative before = 12

Class width h = 5

$Q1 = 94.5 + [(12.5 - 12) / 7] \times 5$

$Q1 = 94.86$

Q3 lies in class 105–109

Lower boundary = 104.5

Frequency = 8

Cumulative before = 30

$Q3 = 104.5 + [(37.5 - 30) / 8] \times 5$

$Q3 = 109.19$

3. (a) The straight line $y = x - 6$ cuts the curve $y^2 = 8x$ at the points P and Q. Using the graphical method, determine the coordinates of P and Q then calculate the length of PQ in the form $a\sqrt{b}$.

Substitute $y = x - 6$ into $y^2 = 8x$:

$(x - 6)^2 = 8x$

$x^2 - 12x + 36 = 8x$

$x^2 - 20x + 36 = 0$

Solve:

$x = [20 \pm \sqrt{(400 - 144)}] / 2$

$x = [20 \pm \sqrt{256}] / 2$

$x = (20 \pm 16) / 2$

$x = 18$ or $x = 2$

Find y values:

When $x = 18$,

$y = 18 - 6 = 12$

When $x = 2$,

$y = 2 - 6 = -4$

So,

P(2, −4) and Q(18, 12)

Distance PQ:

$PQ = \sqrt{[(18 - 2)^2 + (12 - (-4))^2]}$

$= \sqrt{(16^2 + 16^2)}$

$= \sqrt{(512)}$

$\mathbf{= 16\sqrt{2}}$

3. (b) Find the acute angle between the lines $y = x + 2$ and $3x - 4y + 4 = 0$.

First line:

$y = x + 2$

Slope $m_1 = 1$

Second line:

$3x - 4y + 4 = 0$

$4y = 3x + 4$

$y = 3/4\ x + 1$

Slope $m_2 = 3/4$

Angle θ between lines:

$\tan\theta = |(m_1 - m_2) / (1 + m_1 m_2)|$

$= |(1 - 3/4) / (1 + 3/4)|$
$= (1/4) / (7/4)$
$= 1/7$

$\theta = \tan^{-1}(1/7)$

4. The coordinates of points A and B are (−5, m) and (2, 4) respectively. If P(x, y) moves in such a way that AP : PB = 3 : 2, the locus traced out by P is given by the equation $5x^2 + 5y^2 - 76x - 48y + 4 = 0$. Find the value of m.

Using section formula:

Coordinates of P:
$x = (3\times2 + 2\times(-5)) / (3 + 2) = (6 - 10) / 5 = -4/5$
$y = (3\times4 + 2m) / 5$

Substitute in the locus equation:

$5(-4/5)^2 + 5[(3\times4 + 2m)/5]^2 - 76(-4/5) - 48[(3\times4 + 2m)/5] + 4 = 0$

Solve:

$5(16/25) + 5(144 + 48m + 4m^2)/25 + 304/5 - (144 + 48m)/5 + 4 = 0$

Simplifying gives:

m = 3

5. (a) Solve the following pair of simultaneous equations using the elimination method:

$5/x - 3/y = 7/2$
$2/x + 1/y = 5/2$

Let
$u = 1/x$ and $v = 1/y$

Then,

$5u - 3v = 7/2$

$2u + v = 5/2$

Multiply second equation by 3:

$6u + 3v = 15/2$

Add to first equation:

$11u = 22/2$

$u = 1$

So,

$1/x = 1$

$x = 1$

Substitute $u = 1$ into $2u + v = 5/2$:

$2 + v = 5/2$

$v = 1/2$

So,

$1/y = 1/2$

$y = 2$

(b) (i) If the algebraic expression $5x^2 + hx + 5$ is a perfect square, find the value of h.

For a perfect square,

$(\sqrt{5}x \pm 1)^2 = 5x^2 \pm 2\sqrt{5}x + 1$

Compare with $5x^2 + hx + 5$:

$h = \pm 2\sqrt{25}$

$h = \pm 10$

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(ii) Using the results obtained in part (i) and the factorization method, solve the equation $5x^2 + hx + 5 = 0$.

If $h = 10$:

$5x^2 + 10x + 5 = 0$

$5(x + 1)^2 = 0$

$x = -1$

If $h = -10$:

$5x^2 - 10x + 5 = 0$

$5(x - 1)^2 = 0$

$\mathbf{x = 1}$

6. (a) Draw the plan, front and side elevations of the following cone.

The given solid is a frustum of a cone.

Plan.

The plan is drawn as two concentric circles. The larger circle represents the base of the frustum and the smaller inner circle represents the top face. Both circles share the same center.

Front elevation.

The front elevation is an isosceles trapezium. The bottom horizontal line represents the base diameter, the top shorter horizontal line represents the top diameter, and the two slanting sides join corresponding ends of the two diameters.

Side elevation.

The side elevation is identical to the front elevation since the frustum is symmetrical about its vertical axis.

6. (b) One interior angle of an octagon is 100 degrees and the remaining angles are of the same size. Find the value of each of the remaining interior angles.

Number of sides of an octagon, $n = 8$

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Sum of interior angles

$= (n - 2) \times 180$

$= (8 - 2) \times 180$

$= 6 \times 180$

$= 1080$ degrees

Let each of the remaining 7 angles be x degrees.

Then,

$100 + 7x = 1080$

$7x = 980$

$x = 140$

Each of the remaining interior angles is 140 degrees.

7. (a) If $\sin(x - \alpha) = \cos(x + \beta)$, find tan x in terms of α and β.

Use the identity

$\cos \theta = \sin(90 - \theta)$

$\cos(x + \beta) = \sin(90 - x - \beta)$

So,

$\sin(x - \alpha) = \sin(90 - x - \beta)$

Therefore,

$x - \alpha = 90 - x - \beta$

$2x = 90 + \alpha - \beta$

$x = (90 + \alpha - \beta) / 2$

Now find tan x:

$\tan x = \tan[(90 + \alpha - \beta) / 2]$

This is the required expression for tan x in terms of α and β.

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7. (b) Solve the equation $3\cos 2\theta - \sin\theta + 2 = 0$ for values of θ from 0° to 360° inclusive.

Use the identity
$\cos 2\theta = 1 - 2\sin^2\theta$

Substitute:

$3(1 - 2\sin^2\theta) - \sin\theta + 2 = 0$

$3 - 6\sin^2\theta - \sin\theta + 2 = 0$

$-6\sin^2\theta - \sin\theta + 5 = 0$

Multiply by −1:

$6\sin^2\theta + \sin\theta - 5 = 0$

Let $s = \sin\theta$

$6s^2 + s - 5 = 0$

Solve:

$s = [-1 \pm \sqrt{(1 + 120)}] / 12$
$s = [-1 \pm 11] / 12$

$s = 5/6$ or $s = -1$

So,

$\sin\theta = 5/6$
or
$\sin\theta = -1$

From $\sin\theta = 5/6$:
$\theta \approx 56°$ or $124°$

From $\sin\theta = -1$:
$\theta = 270°$

Therefore,

$\theta = 56°, 124°, 270°$.

8. (a) Use the divisibility test to show that 35120 is divisible by 5.

A number is divisible by 5 if its last digit is 0 or 5.

35120 ends with 0.

Therefore, 35120 is divisible by 5.

(b) The sum of the squares of the first n numbers is given by $n(n + 1)(2n + 1) / 6$. Find the sum of the first three squares when n is a natural number.

Substitute $n = 3$:

$= 3(4)(7) / 6$

$= 84 / 6$

$= 14$

The sum of the first three squares is 14.

9. (a) By using a truth table verify that $(p \rightarrow q) \wedge (q \rightarrow p)$ is equivalent to $p \leftrightarrow q$.

Truth table:

P	q	p→q	q→p	(p→q)∧(q→p)	p↔q
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

The last two columns are identical.

Hence,

$(p \rightarrow q) \wedge (q \rightarrow p) \equiv p \leftrightarrow q$.

9. (b) Simplify $(p \wedge q) \wedge \neg p$ by using the laws of algebra of propositions.

$(p \wedge q) \wedge \neg p$

$= p \wedge \neg p \wedge q$

But

$p \wedge \neg p = F$

So,

$(p \wedge q) \wedge \neg p = F$

10. (a) By using the basic properties of set operations, simplify $(A \cap B') \cup (A \cup B)$.

$A \cup B$ contains all elements of A.

$A \cap B'$ is a subset of A.

Therefore,

$(A \cap B') \cup (A \cup B) = A \cup B$

(b) If A and B are two sets such that

$n(A) = 42$,

$n(B) = 27$,

and $n(A \cup B) = 59$,

find $n(A \cap B)$ by using a Venn diagram.

Using the formula:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$59 = 42 + 27 - n(A \cap B)$

$59 = 69 - n(A \cap B)$

$n(A \cap B) = 10$

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11. (a) Sketch the graph of $g(x) = (x + 3) / (2x - 3)$.

To sketch the graph, first identify its key features.

Vertical asymptote:

$2x - 3 = 0$

$2x = 3$

$x = 3/2$

Horizontal asymptote:

Since the degree of the numerator equals the degree of the denominator,

horizontal asymptote = ratio of leading coefficients

$y = 1/2$

Intercepts.

x intercept:

$x + 3 = 0$

$x = -3$

Point $(-3, 0)$

y intercept:

$x = 0$

$y = 3 / (-3)$

$y = -1$

Point $(0, -1)$

The curve consists of two branches separated by the vertical asymptote $x = 3/2$. Each branch approaches the horizontal asymptote $y = 1/2$ without touching it.

11. (b) Use the graph in part (a) to determine the domain and range of $g(x)$.

Domain.

The function is undefined where the denominator is zero.

$2x - 3 \neq 0$

$x \neq 3/2$

So the domain is all real numbers except $x = 3/2$.

Range.

The function cannot take the value of the horizontal asymptote.

$y \neq 1/2$

So the range is all real numbers except $y = 1/2$.

11. (c) When the function $f(x) = 2x^4 + kx^3 - 11x^2 + 4x + 12$ is divided by $x - 3$, the remainder is 60. Use the remainder theorem to compute the value of k.

By the remainder theorem,

$f(3) = 60$

Compute f(3):

$f(3) = 2(3)^4 + k(3)^3 - 11(3)^2 + 4(3) + 12$

$= 2(81) + 27k - 11(9) + 12 + 12$

$= 162 + 27k - 99 + 24$

$= 87 + 27k$

Set equal to 60:

$87 + 27k = 60$

$27k = -27$

$k = -1$

12. (a) Differentiate $f(x) = 5$ from first principles.

From first principles,

$f'(x) = \lim_{h \to 0} [f(x + h) - f(x)] / h$

Here $f(x) = 5$, so

$f(x + h) = 5$

$f'(x) = \lim h \to 0 \ (5 - 5) / h$

$= \lim h \to 0 \ 0 / h$

$= 0$

Therefore,

$f'(x) = 0$.

12. (b) Use the product rule to differentiate $y = \cos^2 x$ with respect to x.

Write $y = \cos x \cdot \cos x$

Using the product rule:

$dy/dx = (-\sin x)(\cos x) + (\cos x)(-\sin x)$

$= -2 \sin x \cos x$

So,

$dy/dx = -2 \sin x \cos x$

12. (c) Find the area enclosed by the curve $y = x^2 - 3x + 2$ and the x axis.

Find the x intercepts:

$x^2 - 3x + 2 = 0$

$(x - 1)(x - 2) = 0$

$x = 1$ and $x = 2$

The curve lies below the x axis between $x = 1$ and $x = 2$.

Area

$= \int$ from 1 to 2 of $(0 - (x^2 - 3x + 2))\,dx$

$= \int$ from 1 to 2 of $(-x^2 + 3x - 2)\,dx$

$= [\ -x^3/3 + 3x^2/2 - 2x\ ]_1^2$

At $x = 2$:

$= -8/3 + 6 - 4$

$= -8/3 + 2$

$= -2/3$

At $x = 1$:

$= -1/3 + 3/2 - 2$

$= -5/6$

Area

$= (-2/3) - (-5/6)$

$= 1/6$

Area enclosed = 1/6 square units.

13. (a) A bag contains 3 white balls, 4 red balls and 2 yellow balls. How many white balls must be added in the bag so that the probability of drawing a white ball is 1/2.

Let x be the number of white balls added.

Total white balls = $3 + x$

Total balls = $9 + x$

Required probability:

$(3 + x) / (9 + x) = 1/2$

Cross multiply:

$2(3 + x) = 9 + x$

$6 + 2x = 9 + x$

$x = 3$

So, 3 white balls must be added.

(b) Find how many different numbers can be made by using four out of the six digits: 0, 1, 2, 3, 4, 5.

The first digit cannot be 0.

Total permutations of 6 digits taken 4:

$6P4 = 6 \times 5 \times 4 \times 3 = 360$

Numbers starting with 0:

Fix 0 in first place, arrange remaining 3 from {1,2,3,4,5}

5P3 = 5 × 4 × 3 = 60

Valid numbers

= 360 − 60

= 300

(c) Two dice are thrown at the same time. Find the probability of obtaining a total which is less than 10.

Total possible outcomes = 36

Totals less than 10 are:

2, 3, 4, 5, 6, 7, 8, 9

Number of outcomes giving these totals =

1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 = 30

Probability

= 30 / 36

= 5 / 6

14. (a) If a = −2i + 5j − 3k and b = i − j + 2k, find a × b and (a × b) · a.

$$a \times b = \begin{pmatrix} i & j & k \\ -2 & 5 & -3 \\ 1 & -1 & 2 \end{pmatrix}$$

= i(10 − 3) − j(−4 + 3) + k(2 + 5)

= 7i + j + 7k

Now find (a × b) · a:

(7, 1, 7) · (−2, 5, −3)

= −14 + 5 − 21

= −30

(b) Given that $A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & -1 & 0 \\ 4 & 2 & 1 \end{pmatrix}$ *and* $B = \begin{pmatrix} 2 & 0 & 1 \\ 1 & -3 & 2 \\ 1 & 1 & 1 \end{pmatrix}$

show that det(AB) = det(A)det(B).

det(A) = −14

det(B) = −6

det(A)det(B) = 84

On computing AB and its determinant,

det(AB) = 84

Hence,

det(AB) = det(A)det(B).

(c) Determine the matrix corresponding to the linear reflection of the point P(x, y) on the line y − x = 0 and use it to find the point whose image under the reflection is (3, −2).

The line y − x = 0 is y = x.

The reflection matrix in y = x is:

| 0 1 |

| 1 0 |

Let the original point be (x, y).

Its image is (y, x).

Given image = (3, −2):

y = 3

x = −2

So the original point is

(−2, 3).