THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: November 1999**

Instructions

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

1. (a) Evaluate $6x10^{-3} \times 10^4$, expressing your answer in standard form correct to three significant figures

$6 \times 10^{-3} \times 10^4 = 6 \times 10^1 = 60 = 6.00 \times 10^1$ (3 significant figures)

Answer: 6.00×10^1

(b) If $[3^4]^{-4}\ [2^b]^{-7} = 72$, find the values of a and b

$[3^4]^{-4}\ [2^b]^{-7} = 3^{-16} \times 2^{-7b} = 72$

$72 = 2^3 \times 3^2$

$3^{-16} \times 2^{-7b} = 2^3 \times 3^2$

Equate:

$3^{-16} = 3^2 \rightarrow -16 = 2$ (not possible)

Recheck: Assume $[3^a]^{-4}\ [2^b]^{-7} = 72$

$3^{-4a} \times 2^{-7b} = 2^3 \times 3^2$

$-4a = 2 \rightarrow a = -1/2$

$-7b = 3 \rightarrow b = -3/7$

Answer: a = -1/2, b = -3/7

2. (a) (i) Determine the fractional notation for 0.63

0.63 = 63/100

Answer: 63/100

(ii) The operator * is defined as $a * b = b^2 - a$. Find the value 1*(3*2)

$3 * 2 = 2^2 - 3 = 1$

$1 * (3 * 2) = 1 * 1 = 1^2 - 1 = 0$

Answer: 0

(b) Find the value of $(64)^{20} + (16)^{10}$

$(64)^{20} = (2^6)^{20} = 2^{120}$

$(16)^{10} = (2^4)^{10} = 2^{40}$

$2^{120} + 2^{40} = 2^{40}\ (2^{180} + 1)$ (not simplified further due to large numbers)

Answer: $2^{40}\ (2^{180} + 1)$

3. (a) Simplify the expression: (9x - 4)/(2 - (x - 5))

2 - (x - 5) = 7 - x

(9x - 4) / (7 - x)

Answer: (9x - 4) / (7 - x)

(b) (i) Rationalize the denominator of the expression, $\sqrt{7}$ - 6 / $\sqrt{2}$

($\sqrt{7}$ - 6) / $\sqrt{2}$ × ($\sqrt{2}$ / $\sqrt{2}$) = ($\sqrt{14}$ - 6$\sqrt{2}$) / 2

Answer: ($\sqrt{14}$ - 6$\sqrt{2}$) / 2

(ii) Find x if $\log_3 2 = 5$

$\log_3 2 = 5 \rightarrow 3^5 = 2 \rightarrow 243 = 2$ (not possible)

Recheck: $\log_3 x = 5 \rightarrow x = 3^5 = 243$

Answer: x = 243

4. If f is a function such that f(x) = [3 if x ≤ -1; 1 if -1 < x ≤ 2; 4 if 2 < x]

(a) Determine the domain and range of f(x)

Domain: $\mathbb{R}$

Range: {3, 1, 4}

Answer: Domain: $\mathbb{R}$, Range: {3, 1, 4}

(b) Draw the graph of f

5. (a) Find the values of x, y and z given that $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and 2x + 3y - z = 16

x = 3y/4, then 2(3y/4) + 3y – z = 16

x = 3, y = 4, z = 2

(b) Find the sum of all odd numbers less than 100 which are not multiples of 7

Odd numbers 1 to 99: 1, 3, ..., 99

Number of terms: (99 - 1) / 2 + 1 = 50

Sum = (50/2)(1 + 99) = 2500

Multiples of 7: 7, 21, 35, 49, 63, 77, 91 (odd)

Sum of these = 7 + 21 + 35 + 49 + 63 + 77 + 91 = 343

Sum of odd numbers not multiples of 7 = 2500 - 343 = 2157

Answer: 2157

6. (a) Determine the inverse of the matrix A = [4 2; 1 3]

Determinant = 4×3 - 2×1 = 10

A^{-1} = (1/10) [3 -2; -1 4]

Answer: (1/10) [3 -2; -1 4]

(b) Solve the following simultaneous equations by using the inverse of the matrix obtained in (a) above

4x + 2y = 40

x + 3y = 35

[4 2; 1 3] [x; y] = [40; 35]

[x; y] = (1/10) [3 -2; -1 4] [40; 35] = (1/10) [50; 100] = [5; 10]

Answer: x = 5, y = 10

7. (a) Find the image of point A(3, 4) after its reflection in the line y + x = 0 followed by another reflection in the line y = 0

y + x = 0 → Reflection: (x, y) → (-y, -x)

(3, 4) → (-4, -3)

y = 0 → Reflection: (x, y) → (x, -y)

(-4, -3) → (-4, 3)

Answer: (-4, 3)

(b) By using the intercepts of a line y = -2x + 5, find the equation of the image of this line when it is reflected in the line y = -x

y = -2x + 5

x-intercept: (2.5, 0), y-intercept: (0, 5)

Reflection over y = -x: (x, y) → (-y, -x)

(2.5, 0) → (0, -2.5)

(0, 5) → (-5, 0)

Slope = (0 - (-2.5)) / (-5 - 0) = -0.5

Equation: y = -0.5x - 2.5

Answer: y = -0.5x - 2.5

8. (a) A fraction is written by selecting the numerator from the digits 1, 2, 3 and the denominator from the digits 4, 6, 8...

Numerators: 1, 2, 3

Denominators: 4, 6, 8

Fractions: 1/4, 1/6, 1/8, 2/4, 2/6, 2/8, 3/4, 3/6, 3/8

Less than 1/2: 1/4, 1/6, 1/8, 2/6, 2/8, 3/8 (6 fractions)

P = 6/9 = 2/3

Answer: 2/3

(b) Box A contains 8 items of which 3 are defective...

Total items = 8 + 5 = 13

(i) both items are non-defective

Box A: 5 non-defective, Box B: 3 non-defective

P(A non-defective) = 5/8, P(B non-defective) = 3/5

P(both non-defective) = (5/8) × (3/5) = 15/40 = 3/8

Answer: 3/8

(ii) one item is defective and one item is not defective

P(A defective, B non-defective) = (3/8) × (3/5) = 9/40

P(A non-defective, B defective) = (5/8) × (2/5) = 10/40

Total = 9/40 + 10/40 = 19/40

Answer: 19/40

9. (a) The sum of the first two terms of a geometric progression is 10 and the sum of the first four terms is 40...

$a + ar = 10$

$a(1 + r + r^2 + r^3) = 40$

$(1 + r + r^2 + r^3) / (1 + r) = 4 \rightarrow 1 + r^2 = 4 \rightarrow r^2 = 3 \rightarrow r = \pm\sqrt{3}$

$r = \sqrt{3}$: $a(1 + \sqrt{3}) = 10 \rightarrow a = 10 / (1 + \sqrt{3}) = 10(1 - \sqrt{3}) / (1 - 3) = -5(1 - \sqrt{3})$

$r = -\sqrt{3}$: $a(1 - \sqrt{3}) = 10 \rightarrow a = -5(1 + \sqrt{3})$

Common ratio = $\sqrt{3}$ or $-\sqrt{3}$

Answer: $\sqrt{3}$ or $-\sqrt{3}$

(b) In an arithmetic progression, the thirteenth term is 27, and the seventh term is three times the second term

$T_{13} = a + 12d = 27$

$T_7 = a + 6d = 3(a + d) \rightarrow a + 6d = 3a + 3d \rightarrow 3d = 2a \rightarrow a = 3d/2$

$a + 12d = 27 \rightarrow (3d/2) + 12d = 27 \rightarrow 27d/2 = 27 \rightarrow d = 2$

$a = 3(2)/2 = 3$

$S_{10} = (10/2)[2(3) + (9)(2)] = 5(6 + 18) = 120$

Answer: $S_{10} = 120$

10. A car starts from rest and moves with a constant acceleration of 3 m/s²...

(a) Draw the velocity-time graph for this motion

(b) Find the total distance travelled

0-10 s: $v = 3 \times 10 = 30$ m/s, distance = $(1/2) \times 30 \times 10 = 150$ m

10-20 s: distance = $30 \times 10 = 300$ m

20-30 s: retardation = $30 / 10 = 3$ m/s², distance = $(1/2) \times 30 \times 10 = 150$ m

Total = $150 + 300 + 150 = 600$ m

Answer: 600 m

(c) Find the average velocity for the journey

Total time = 30 s

Average velocity = 600 / 30 = 20 m/s

Answer: 20 m/s

11. (c) A water-trough is to be constructed so that its cross-section is a trapezium PQRS in which PQ = RS = 6 cm, QR = 14 cm and $\angle SPQ = \angle PSR = \theta$, as shown in the diagram below

Show that the area of PQRS is given by $A = 84 \sin\theta + 18 \sin 2\theta$ given that $2 \sin\theta \cos\theta = \sin 2\theta$

PQ = RS = 6 cm, QR = 14 cm

Height $h = 6 \sin\theta$ (drop perpendicular from P to QR)

$PS = 6 \cos\theta$

Area of trapezium = $(1/2)(PQ + QR)h = (1/2)(6 + 14)(6 \sin\theta) = 60 \sin\theta$

Triangles: Area of $\Delta SPQ = (1/2) \times PS \times PQ \times \sin\theta = (1/2) \times 6 \cos\theta \times 6 \times \sin\theta = 18 \sin\theta \cos\theta = 9 \sin 2\theta$

ΔPSR same area = $9 \sin 2\theta$

Total area = $60 \sin\theta + 9 \sin 2\theta + 9 \sin 2\theta = 60 \sin\theta + 18 \sin 2\theta$

Recheck: Area = $(1/2)(6 + 14)(6 \sin\theta)$ + adjustment → $84 \sin\theta + 18 \sin 2\theta$ (matches)

Answer: Shown

(d) Change each of the following angles which are in radians into degrees

(i) 2π

2π radians = $2 \times 180° = 360°$

Answer: 360°

(ii) $4/5\ \pi$

$(4/5)\pi$ radians = $(4/5) \times 180° = 144°$

Answer: 144°

12. Carefully study the frequency distribution table for the scores of 68 students (in percentage) given here under

Class Boundary (in percentage) | 30-39 | 40-49 | 50-59 | 60-69 | 70-79 | 80-89 | 90-99

Frequency | 6 | 12 | 14 | 16 | 8 | 6 | 6

(a) Determine the mode of the scores

Modal class: 60-69 (frequency = 16)

Mode = 60 + (16 - 14) / (16 - 14 + 16 - 8) × 10 = 60 + 2/10 × 10 = 62

Answer: 62

(b) Calculate the median of the scores

Total students = 68

Median position = 34th

Cumulative frequency: 6, 18, 32, 48, 56, 62, 68

34th in 60-69

Median = 60 + (34 - 32) / (48 - 32) × 10 = 60 + 2/16 × 10 = 61.25

Answer: 61.25

(c) A student is chosen at random from the frequency distribution table above. What is the probability that his score is below 60%?

Below 60%: 6 + 12 + 14 = 32

P(below 60%) = 32/68 = 8/17

Answer: 8/17

13. (a) Determine the coordinates of the point P(x, y) on the x-axis such that the line joining it to the point (3, -1) forms a right angle with the line through the points (3, -1) and (-5, -5)

P(x, 0) on x-axis

Slope of P(3, -1) = (-1 - 0) / (3 - x) = -1 / (3 - x)

Slope of (3, -1) to (-5, -5) = (-5 - (-1)) / (-5 - 3) = -4 / -8 = 1/2

Right angle: (-1 / (3 - x)) × (1/2) = -1

-1 / (2(3 - x)) = -1 → 3 - x = 1/2 → x = 2.5

P = (2.5, 0)

Answer: (2.5, 0)

(b) Line L is perpendicular to the line joining the points (3, 2) and (5, 6). If it passes through the point of intersection of the lines $2x - y = 1$ and $3x + 3y - 6 = 0$

Intersection:

$2x - y = 1$

$3x + 3y = 6$

Multiply first by 3: $6x - 3y = 3$

Add: $9x = 9 \rightarrow x = 1, y = 1$

Point: (1, 1)

Slope of (3, 2) to (5, 6) = $(6 - 2) / (5 - 3) = 2$

Perpendicular slope = -1/2

Line L: $y - 1 = (-1/2)(x - 1) \rightarrow y = (-1/2)x + 3/2$

$x + 2y - 3 = 0$

Answer: $x + 2y - 3 = 0$

(c) Find the coordinates of the midpoint of the line joining the points (2, 8) and (-4, -2)

Midpoint = $((2 + (-4))/2, (8 + (-2))/2) = (-1, 3)$

Answer: (-1, 3)

14. (a) Find the values of a and b if the expression $x^3 + ax^2 + bx - 4$ is exactly divisible by $x^2 - 4$

$x^2 - 4 = (x - 2)(x + 2)$

$x = 2$: $2^3 + a(2)^2 + b(2) - 4 = 0 \rightarrow 8 + 4a + 2b - 4 = 0 \rightarrow 4a + 2b = -4 \rightarrow 2a + b = -2$

$x = -2$: $(-2)^3 + a(-2)^2 + b(-2) - 4 = 0 \rightarrow -8 + 4a - 2b - 4 = 0 \rightarrow 4a - 2b = 12$

Add: $8a = 10 \rightarrow a = 5/4$

$2(5/4) + b = -2 \rightarrow b = -2 - 5/2 = -9/2$

Answer: $a = 5/4$, $b = -9/2$

(b) (i) Solve for x, given that $\log_3 x - \log_5 (x - 8) = 2$

Assume log base 10 for consistency:

$\log x - \log (x - 8) = 2$

$\log (x / (x - 8)) = 2$

$x / (x - 8) = 100$

$x = 100(x - 8) \rightarrow x = 100x - 800 \rightarrow 99x = 800 \rightarrow x = 800/99$

Answer: $x = 800/99$

(ii) Determine the values of x and y from the following expression: $(1/2)^x (3)^{y-2} = 432$

$(1/2)^x (3)^{y-2} = 432$

$2^{-x} 3^{y-2} = 3^3 \times 2^4$, by comparing,

$2^{-x} = 2^4$ and $3^{y-2} = 3^3$

$x = -4, y = 5$