THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2000**

Instructions

1. This paper consists of Section A and B.

2. Answer all questions in section A and any four questions in section B.

1. (a) Evaluate 2562.9×0.0064, expressing your answer in standard form correct to four significant figures

$2562.9 \times 0.0064 \approx 16.40256$

Standard form: 1.640×10^1 (4 significant figures)

Answer: 1.640×10^1

(b) If $m = (m^2 + m^4)$, find the value of $(3^4/1^2)^2$

$m = m^2 + m^4 \rightarrow m^4 + m^2 - m = 0 \rightarrow m(m^3 + m - 1) = 0$

$m = 0$ (discard) or solve $m^3 + m - 1 = 0$ (numerical solution ≈ 0.754)

Recheck: $(3^4/1^2)^2 = (81/1)^2 = 6561$ (problem likely independent of m)

Answer: 6561

(c) Evaluate $(1625^2 - 175^2)$

$1625^2 - 175^2 = (1625 - 175)(1625 + 175) = 1450 \times 1800 = 2{,}610{,}000$

Answer: 2,610,000

2. (a) If E = {b, c, d, e}, A = {a, b} and B = {e, d}, find $A \cap B$

$A \cap B = \{a, b\} \cap \{e, d\} = \emptyset$

Answer: $\emptyset$

(b) (i) $A \cap B'$

$B' = \{b, c\}$

$A \cap B' = \{a, b\} \cap \{b, c\} = \{b\}$

Answer: {b}

(ii) $(A \cap B')$

$(A \cap B') = \{b\}$ (as above)

Answer: {b}

(c) In a class of 42 students, 31 students study History and 26 study Physics...

Total = 42, History = 31, Physics = 26, Both = x

History only = 31 - x, Physics only = 26 - x, Neither = 42 - (31 - x + x + 26 - x) = x - 15

$x - 15 = 0 \rightarrow x = 15$

History only = 16, Physics only = 11, Neither = 0

Venn diagram confirms all participate.

Answer: History only = 16, Physics only = 11, Both = 15, Neither = 0

3. (a) The distance between two towns on a map of scale 1:500,000 is 9 cm...

Actual distance = 9 × 500,000 = 4,500,000 cm = 45 km

Answer: 45 km

(b) Three classes working 8 hours a day take 5 days to harvest maize from a shamba...

Total work = 3 × 8 × 5 = 120 class-hours

2 classes, 10 hours: Total work = $2 \times 10 \times d = 120 \rightarrow d = 6$ days

Answer: 6 days

4. The second, fifth and eleventh terms of an arithmetic progression are in geometric progression...

Arithmetic: $T_2 = a + d$, $T_5 = a + 4d$, $T_{11} = a + 10d$

Geometric: $(a + 4d)^2 = (a + d)(a + 10d)$

$(a + 4d)^2 - (a + d)(a + 10d) = 0 \rightarrow 3ad + 6d^2 = 0 \rightarrow d(a + 2d) = 0$

$d \neq 0 \rightarrow a = -2d$

$T_2 = -d$, $T_5 = -2d$, $T_{11} = 8d$

Common ratio: $(-2d) / (-d) = 2$, $8d / (-2d) = -4$ (not geometric)

Recheck: $a + 4d = 2(a + d) \rightarrow a = 2d$

$T_2 = 3d$, $T_5 = 6d$, $T_{11} = 12d$

Common ratio = $6d / 3d = 2$, $12d / 6d = 2$

(a) the common ratio of the geometric progression

Common ratio = 2

Answer: 2

(b) A function is defined by the arithmetic progression

$f(x) = x - 2$

$a = 2d, d = d \rightarrow f(x) = (2d) + (x - 1)d = dx + d$

Recheck: AP terms 3d, 6d, 12d → $f(x) = 3d\,x$

Recheck: $f(x) = x - 2$ not related to AP

Answer: Recheck problem

(c) Find

(i) the inverse f^{-1} of this function

$f(x) = x - 2 \rightarrow f^{-1}(x) = x + 2$

Answer: $f^{-1}(x) = x + 2$

(ii) the value of $f^{-1}(2)$

$f^{-1}(2) = 2 + 2 = 4$

Answer: 4

(iii) the domain of f^{-1}

Domain of f^{-1} = Range of f = $\mathbb{R}$

Answer: $\mathbb{R}$

(d) Rewrite 2x + 3 < 7 when the absolute value sign and hence sketch the graph of the resulting inequality

$2x + 3 < 7 \rightarrow -7 < 2x + 3 < 7 \rightarrow -10 < 2x < 4 \rightarrow -5 < x < 2$

Answer: $-5 < x < 2$

6. (a) Find the values of x and y given that $3x - y = 3$ and $9x^2 + y^2 - 45$

$3x - y = 3 \rightarrow y = 3x - 3$

$9x^2 + (3x - 3)^2 = 45$

$9x^2 + 9x^2 - 18x + 9 = 45$

$18x^2 - 18x - 36 = 0$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2 \rightarrow y = 3(2) - 3 = 3$

$x = -1 \rightarrow y = 3(-1) - 3 = -6$

Answer: (2, 3), (-1, -6)

(b) Make W the subject of the formula $T = W / \sqrt{5x}$

$T = W / \sqrt{5x} \rightarrow W = T\sqrt{5x}$

Answer: $W = T\sqrt{5x}$

7. (a) Determine the value of x in the figure below where O is the centre of the circle

$\angle AOB = 50°$ (angle at center)

$\angle ACB = (1/2) \times 50° = 25°$ (angle at circumference)

ΔAOC: OA = OC (radii)

$\angle OAC = \angle OCA = x$

$x + x + 50 = 180 \rightarrow 2x = 130 \rightarrow x = 65°$

Answer: $x = 65°$

(b) Prove that the tangents from an external point to a circle are equal

Tangents from point P to circle at points A and B:

PA = PB (tangent property, radii perpendicular to tangents, congruent triangles)

9. (a) Find the image of (7, 6) under a rotation through 180° followed by another rotation of 90°

Rotation 180° about origin: $(7, 6) \rightarrow (-7, -6)$

Rotation 90° counterclockwise: $(-7, -6) \rightarrow (6, -7)$

Answer: (6, -7)

(b) A translation T maps the point (3, 2) onto (4, 3). Find where T maps

(i) the point (0, 0)

T: $(3, 2) \rightarrow (4, 3) \rightarrow$ Translation vector = (1, 1)

$(0, 0) \rightarrow (0 + 1, 0 + 1) = (1, 1)$

Answer: (1, 1)

(ii) the point (7, 4)

$(7, 4) \rightarrow (7 + 1, 4 + 1) = (8, 5)$

Answer: (8, 5)

10. A manufacturer has 150 and 90 kilograms of wood and plastic respectively...

Let x = units of A, y = units of B

Wood: $1x + 3y \leq 150$

Plastic: $2x + 1y \leq 90$

$x \geq 0, y \geq 0$

Maximize income: $I = 4000x + 6000y$

Vertices:

(0, 0): I = 0

(0, 50): $3y = 150 \rightarrow y = 50 \rightarrow I = 6000(50) = 300{,}000$

(45, 0): $2x = 90 \rightarrow x = 45 \rightarrow I = 4000(45) = 180{,}000$

(30, 30): $1x + 3y = 150$, $2x + 1y = 90 \rightarrow x = 30, y = 30 \rightarrow I = 4000(30) + 6000(30) = 120{,}000 + 180{,}000 = 300{,}000$

Maximum income at (0, 50) and (30, 30): 300,000

Choose (30, 30) for balanced production: 30 units of A, 30 units of B

Answer: 30 units of A, 30 units of B

11. (a) If A = [9 7; 8 6] and B = [-6 1; -2 5], find

(i) AB

AB = [9(-6) + 7(-2) 9(1) + 7(5); 8(-6) + 6(-2) 8(1) + 6(5)] = [-68 44; -60 38]

Answer: [-68 44; -60 38]

(ii) BA

BA = [-6(9) + 1(8) -6(7) + 1(6); -2(9) + 5(8) -2(7) + 5(6)] = [-46 -36; 22 16]

Answer: [-46 -36; 22 16]

(b) If the matrix A = [1 3; 2 4], find $(A^{-1})^4$

Determinant of A = $1\times4 - 3\times2 = -2$

A^{-1} = (1/-2) [4 -3; -2 1] = [-2 3/2; 1 -1/2]

$(A^{-1})^4 = (A^{-1})^2 (A^{-1})^2$

$(A^{-1})^2$ = [-2 3/2; 1 -1/2] [-2 3/2; 1 -1/2] = [11/2 -9/4; -3/2 5/4]

$(A^{-1})^4$ = [11/2 -9/4; -3/2 5/4] [11/2 -9/4; -3/2 5/4] = [121/4 -33/8; -33/8 13/8]

Answer: [121/4 -33/8; -33/8 13/8]

(c) Solve the simultaneous equations below using the matrix method

4x + 2y = 40

x + 3y = 35

[4 2; 1 3] [x; y] = [40; 35]

Determinant = 4×3 - 2×1 = 10

Inverse = (1/10) [3 -2; -1 4]

[x; y] = (1/10) [3 -2; -1 4] [40; 35] = (1/10) [50; 100] = [5; 10]

Answer: x = 5, y = 10

12. (a) If u = 4i + 3j, and v = 2i + 4j, find

(i) 2u + 3v

2u = 8i + 6j, 3v = 6i + 12j

2u + 3v = 14i + 18j

Answer: 14i + 18j

(ii) |7u|

7u = 7(4i + 3j) = 28i + 21j

$|7u| = \sqrt{(28^2 + 21^2)} = \sqrt{(784 + 441)} = \sqrt{1225} = 35$

Answer: 35

(iii) If u = v

4i + 3j = 2i + 4j → 4i - 2i = 4j - 3j → 2i = j (not possible, vectors not equal)

Answer: Not possible

(b) A student walks 500 m in the direction S 45° E from the classroom to the basketball ground...

S 45° E = (-500 sin 45°, -500 cos 45°) ≈ (-353.55, -353.55)

W = (200, 0)

Displacement = (-353.55 + 200, -353.55) ≈ (-153.55, -353.55)

Magnitude = $\sqrt{(-153.55)^2 + (-353.55)^2} \approx 384.62$ m

Direction: tan θ = 353.55 / 153.55 ≈ 2.3 → θ ≈ 66.5° below negative x-axis

Answer: ≈ 384.62 m, 66.5° below negative x-axis

13. The table below shows the masses of 100 students to the nearest kilogram

Mass (kg.)	60 - 62	63 - 65	66 - 68	69 - 71	72 - 74
Frequency	5	18	42	27	8

(a) Determine the mean of the masses

Midpoints: 61, 64, 67, 70, 73

Mean = (5×61 + 18×64 + 42×67 + 27×70 + 8×73) / 100

= (305 + 1152 + 2814 + 1890 + 584) / 100 = 6745 / 100 = 67.45 kg

Answer: 67.45 kg

(b) Find the mode

Modal class: 66-68 (frequency = 42)

Mode = 66 + (42-18)/(84-18) × 3 = 66 + 24/66 × 3 ≈ 67.09 kg

Answer: 67.09 kg (approximate)

(c) Draw a cumulative frequency curve and use it to determine the median of the masses

Cumulative frequency:

60-62: 5

63-65: 23

66-68: 65

69-71: 92

72-74: 100

Median (50th percentile): In 66-68 → 66 + (50-23)/(65-23) × 3 ≈ 67.93 kg

Answer: 67.93 kg (approximate)

14. (a) Find the equation of the straight line joining the point O(0, 0) to the mid-point of the line joining A(3, 2) and B(5, -1)

Midpoint of AB = (4, 0.5)

Slope = 0.5 / 4 = 0.125

Equation: $y = 0.125x$

Answer: $y = 0.125x$

(b) Find the coordinates of the point of intersection P of the straight lines $4x + 3y = 7$ and $3x - 4y = -1$

$4x + 3y = 7$

$3x - 4y = -1$

Multiply first by 4, second by 3:

$16x + 12y = 28$

$9x - 12y = -3$

Add: $25x = 25 \rightarrow x = 1$

$4(1) + 3y = 7 \rightarrow 3y = 3 \rightarrow y = 1$

P = (1, 1)

Answer: (1, 1)

(c) Determine the equation of a line which passes through the point N(5, 0) and is parallel to the line $3x + 4y = 12$

$3x + 4y = 12 \rightarrow y = (-3/4)x + 3$

Slope = -3/4

Equation: $y - 0 = (-3/4)(x - 5) \rightarrow y = (-3/4)x + 15/4$

$3x + 4y - 15 = 0$

Answer: $3x + 4y - 15 = 0$