THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2007**

Instructions

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

Prepared by: Maria Marco for TETEA

1. (a) Given $x = 4.5 \times 10^{10}$ and $z = 7.2 \times 10^{10}$, find y in standard form if z = xy

z = xy

$7.2 \times 10^{10} = (4.5 \times 10^{10})y$

$y = (7.2 \times 10^{10}) / (4.5 \times 10^{10})$

y = 7.2 / 4.5 = 1.6

$y = 1.6 \times 10^{0}$

Answer: 1.6×10^{0}

(b) Express 2/153 as a fraction

2/153 is already a fraction, but assuming simplification or typo:

2/153 cannot be simplified further (GCD of 2 and 153 is 1).

Answer: 2/153

(c) Evaluate 2/5 + 0.28 + 1/35

Convert to common denominator:

2/5 = 14/35, 0.28 = 28/100 = 7/25 = 49/125, 1/35

LCM of 35 and 125 = 875

14/35 = 350/875, 49/125 = 343/875, 1/35 = 25/875

(350 + 343 + 25) / 875 = 718 / 875

Give in fraction form: 718/875

Answer: 718/875

2. (a) There are 60 people at a meeting. 35 are businesspersons, 32 are employees and 13 are both businesspersons and employees

(i) How many are businesspersons or employees?

Businesspersons or employees = 35 + 32 - 13 = 54

Answer: 54

(ii) How many are neither businesspersons nor employees?

Total = 60

Neither = 60 - (businesspersons or employees) = 60 - 54 = 6

Answer: 6

(b) If $n(A \cap B') = 8$, $n(B \cap A') = 5$ and $n(A \cup B) = 20$

(i) Display the information in a Venn diagram

(ii) Give the values of $n(A)$ and $n(B)$

$n(A \cap B') = 8$, $n(B \cap A') = 5$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Let $n(A \cap B) = x$

$n(A) = 8 + x$, $n(B) = 5 + x$

$20 = (8 + x) + (5 + x) - x$

$20 = 13 + x$

$x = 7$

$n(A) = 8 + 7 = 15$, $n(B) = 5 + 7 = 12$

Answer: $n(A) = 15$, $n(B) = 12$

3. Let A be the acute angle of a right angled triangle ABC such that B = 90° and Cos C = 1/3. Find the value of Sin A

In triangle ABC, B = 90°, Cos C = 1/3

Cos C = adjacent/hypotenuse = BC/AC

Let BC = 1, AC = 3

$AB = \sqrt{(3^2 - 1^2)} = \sqrt{8} = 2\sqrt{2}$

Sin A = opposite/hypotenuse = BC/AC = 1/3

But A is opposite B, so Sin A = AB/AC = $2\sqrt{2} / 3$

Answer: $2\sqrt{2} / 3$

4. (a) If a = i + 2j, b = i + 2j and p = 2q = 14j, find the values of the scalars p and q

p = 2q = 14j

p = 14, q = 7 (since p and q are scalars, 14j implies magnitude 14)

Answer: $p = 14$, $q = 7$

(b) Use common logarithmic tables to find the value of $2.055 \times 20.35 / 100.5 \times 0.045$

$\log(2.055 \times 20.35 / (100.5 \times 0.045))$

$= \log 2.055 + \log 20.35 - (\log 100.5 + \log 0.045)$

$\log 2.055 \approx 0.3126$, $\log 20.35 \approx 1.3085$

$\log 100.5 \approx 2.0022$, $\log 0.045 \approx -1.3468$

$0.3126 + 1.3085 - (2.0022 - 1.3468)$

$= 1.6211 - 0.6554 = 0.9657$

$\text{Antilog}(0.9657) \approx 9.24$

Answer: 9.24

(c) Solve for x if $\sqrt{(3x^2 - 17)} = 8$

$\sqrt{(3x^2 - 17)} = 8$

$3x^2 - 17 = 64$

$3x^2 = 81$

$x^2 = 27$

$x = \pm\sqrt{27} = \pm 3\sqrt{3}$

Answer: $x = \pm 3\sqrt{3}$

5. (a) Prove that the opposite angles of any quadrilateral inscribed in a circle are supplementary

Let ABCD be a cyclic quadrilateral.

Angle at A ($\angle DAB$) and angle at C ($\angle BCD$) are opposite.

Inscribed angle theorem: $\angle DAB = (1/2) \times$ measure of arc BCD

$\angle BCD = (1/2) \times$ measure of arc BAD

Arc BCD + arc BAD = 360° (full circle)

$\angle DAB + \angle BCD = (1/2)(\text{arc } BCD) + (1/2)(\text{arc } BAD) = (1/2)(360°) = 180°$

Similarly, $\angle ABC + \angle CDA = 180°$.

Thus, opposite angles are supplementary.

Answer: Proved

(b) In figure 1 below, O is the centre of the circle, $A\hat{O}B = 120°$ and CDB = 15°. Find the value of x

$A\hat{O}B = 120°$, CDB = 15°

$\angle ACB = (1/2) \times A\hat{O}B = (1/2) \times 120° = 60°$

In triangle CDB, $\angle CDB = 15°$, $\angle DCB = x$

$\angle CBD = 180° - 15° - x = 165° - x$

$\angle ACB = \angle DCB$ (same arc CB)

$x = 60°$

Answer: $x = 60°$

(c) For a tank given in the figure 2 above, calculate the angle between DF and the base ABCD

Dimensions: AB = 4 cm, BC = 4 cm, BF = 2 cm

DF in triangle DFB:

$DB = \sqrt{(4^2 + 4^2)} = 4\sqrt{2}$

$DF = \sqrt{((4\sqrt{2})^2 + 2^2)} = \sqrt{(32 + 4)} = 6$

Angle θ between DF and ABCD:

Height = 2 cm, Base DB = $4\sqrt{2}$

$\tan \theta = 2 / (4\sqrt{2}) = 1 / (2\sqrt{2})$

$\theta = \tan^{-1}(1 / (2\sqrt{2})) \approx 19.47°$

Answer: 19.47°

6. (a) The first four terms of an AP are (2, a - b), (2a + b), (a + 3b) respectively where a and b are constants

(i) Find the values of the constants a and b

$T_1 = 2$, $T_2 = a - b$, $T_3 = 2a + b$, $T_4 = a + 3b$

Common difference d:

$T_2 - T_1 = T_3 - T_2$

$(a - b) - 2 = (2a + b) - (a - b)$

a - b - 2 = a + 2b

-b - 2 = 2b

-3b = 2

b = -2/3

$T_3 - T_2 = T_4 - T_3$

(2a + b) - (a - b) = (a + 3b) - (2a + b)

a + 2b = -a + 2b

2a = 0

a = 0

Check: b = -2/3, a = 0 → 2, -(-2/3), 2(0) + (-2/3), 0 + 3(-2/3) → 2, 2/3, -2/3, -2 (not AP, recheck)

Correct: 2a = 2b → a = b

(a - b) - 2 = (2a + b) - (a - b)

a - b - 2 = a + 2b

-3b = 2

b = -2/3, a = -2/3

Answer: a = -2/3, b = -2/3

(ii) The sum of the first 10 terms

a = 2, d = (a - b) - 2 = 0 - 2 = -2

S_{10} = (10/2)[2a + (10-1)d] = 5[2(2) + 9(-2)] = 5[4 - 18] = 5(-14) = -70

Answer: -70

(b) How long should Tshs. 1,200,000/- be invested at simple interest rate of 5% to get an interest of Tshs. 180,000/-?

I = PRT / 100

180,000 = (1,200,000 × 5 × T) / 100

180,000 = 60,000T

T = 180,000 / 60,000 = 3 years

Answer: 3 years

7. (a) Find the equation of a straight line that is parallel to the line $3x + 4y = 1$ and cuts the x-axis where $x = -1$

Parallel to $3x + 4y = 1$ → Slope: $3x + 4y = 1 \rightarrow 4y = -3x + 1 \rightarrow y = (-3/4)x + 1/4 \rightarrow$ slope = -3/4

Line: $y = (-3/4)x + c$

Cuts x-axis at $x = -1$: $y = 0$ when $x = -1$

$0 = (-3/4)(-1) + c$

$0 = 3/4 + c$

$c = -3/4$

Equation: $y = (-3/4)x - 3/4$

Express as $ax + by + c = 0$:

$(3/4)x + y + 3/4 = 0$

Multiply by 4: $3x + 4y + 3 = 0$

Answer: $3x + 4y + 3 = 0$

(b) Express the equation of the perpendicular bisector joining the points A (3, -1) and B (5, 2), in the form $y = mx + c$ where m and c are constants

Midpoint of AB: $((3 + 5)/2, (-1 + 2)/2) = (4, 1/2)$

Slope of AB: $(2 - (-1)) / (5 - 3) = 3/2$

Perpendicular slope: -2/3

Equation: $y - (1/2) = (-2/3)(x - 4)$

$y - 1/2 = (-2/3)x + 8/3$

$y = (-2/3)x + 8/3 + 1/2$

$y = (-2/3)x + 16/6 + 3/6 = (-2/3)x + 19/6$

$m = -2/3$, $c = 19/6$

Answer: $y = (-2/3)x + 19/6$

8. (a) A cylindrical solid of radius 7 cm and height 25 cm is cut equally from the top to bottom resulting into equal half solids (see figure 3). Find the total surface area of one half solid

Radius = 7 cm, Height = 25 cm

Cut vertically: Half cylinder → Curved surface + two semicircles + flat face

Curved surface area of half = $(1/2) \times 2\pi rh = \pi \times 7 \times 25 = 175\pi$

Semicircles (top and bottom): $2 \times (1/2) \times \pi r^2 = \pi \times 7^2 = 49\pi$

Flat face: $7 \times 25 = 175$

Total surface area = $175\pi + 49\pi + 175 = 224\pi + 175$

Using $\pi = 22/7$: $224 \times 22/7 = 704$

Total = $704 + 175 = 879$ cm²

Answer: 879 cm²

(b) What is the area of a regular 45 sides polygon inscribed in a circle of radius 6 cm?

Number of sides $n = 45$, radius $r = 6$ cm

Area of regular polygon = $(1/2) \times n \times r^2 \times \sin(360°/n)$

Angle = $360°/45 = 8°$

Area = $(1/2) \times 45 \times 6^2 \times \sin 8°$

$\sin 8° \approx 0.1392$

Area = $(1/2) \times 45 \times 36 \times 0.1392 = 22.5 \times 36 \times 0.1392 \approx 112.75$ cm²

Answer: 112.75 cm²

9. (a) Mary received a certain amount of money from her father to go to school. She spent one third in her journey to school. At school she paid three fifths of the remaining amount as school fees and remained with 24,000/- as her pocket money. Showing the procedure; calculate the total amount of money she received from her father

Let total amount = x

After journey: $(2/3)x$

After school fees: $(2/5)$ of $(2/3)x = (4/15)x$

Remaining: $(2/3)x - (4/15)x = (10/15)x - (4/15)x = (6/15)x = (2/5)x$

$(2/5)x = 24{,}000$

$x = 24{,}000 \times (5/2) = 60{,}000$

Answer: 60,000

(b) If $f(x) = x^3 + kx^2 + 4x + 4$ has a remainder 16 when divided by $x - 2$. Find the value of k

$f(x) = x^3 + kx^2 + 4x + 4$, remainder when divided by $x - 2$:

$f(2) = 16$

$2^3 + k(2^2) + 4(2) + 4 = 16$

$8 + 4k + 8 + 4 = 16$

$20 + 4k = 16$

$4k = -4$

$k = -1$

Answer: $k = -1$

10. (a) The graph in figure 4 represents the journey made by a car between two sets of traffic lights. How far is it between the traffic lights?

Distance = Area under velocity-time graph

Break into triangles and rectangles:

0-100s: Triangle, area = $(1/2) \times 100 \times 50 = 2500$ m

100-200s: Rectangle, area = $100 \times 50 = 5000$ m

200-400s: Rectangle, area = $200 \times 40 = 8000$ m

400-500s: Triangle, area = $(1/2) \times 100 \times 40 = 2000$ m

Total distance = 2500 + 5000 + 8000 + 2000 = 17,500 m = 17.5 km

Answer: 17.5 km

(b) Solve the simultaneous equations

{ $x - y^2 = 2$

{ $2x^2 + 3y^2 = 15$

From first: $x = y^2 + 2$

Substitute into second: $2(y^2 + 2)^2 + 3y^2 = 15$

$2(y^4 + 4y^2 + 4) + 3y^2 = 15$

$2y^4 + 8y^2 + 8 + 3y^2 = 15$

$2y^4 + 11y^2 - 7 = 0$

$y^4 + 11/2\ y^2 - 7/2 = 0$

$2y^4 + 11y^2 - 7 = 0$

Let $u = y^2$: $2u^2 + 11u - 7 = 0$

$u = [-11 \pm \sqrt{(121 + 56)}] / 4 = [-11 \pm \sqrt{177}] / 4$

$u \approx 0.577$ or $u \approx -6.077$ (discard negative)

$y^2 \approx 0.577 \rightarrow y \approx \pm 0.76$

$x = y^2 + 2 \approx 0.577 + 2 \approx 2.58$

Solutions: (2.58, 0.76), (2.58, -0.76)

Answer: (2.58, 0.76), (2.58, -0.76)

7. (a) Find the equation of a straight line that is parallel to the line $3x + 4y = 1$ and cuts the x-axis where $x = -1$

Parallel to $3x + 4y = 1 \rightarrow$ Slope: $3x + 4y = 1 \rightarrow 4y = -3x + 1 \rightarrow y = (-3/4)x + 1/4 \rightarrow$ slope $= -3/4$

Line: $y = (-3/4)x + c$

Cuts x-axis at $x = -1$: $y = 0$ when $x = -1$

$0 = (-3/4)(-1) + c$

$0 = 3/4 + c$

$c = -3/4$

Equation: $y = (-3/4)x - 3/4$

Express as $ax + by + c = 0$:

$(3/4)x + y + 3/4 = 0$

Multiply by 4: $3x + 4y + 3 = 0$

Answer: $3x + 4y + 3 = 0$

(b) Express the equation of the perpendicular bisector joining the points A (3, -1) and B (5, 2), in the form $y = mx + c$ where m and c are constants

Midpoint of AB: $((3 + 5)/2, (-1 + 2)/2) = (4, 1/2)$

Slope of AB: $(2 - (-1)) / (5 - 3) = 3/2$

Perpendicular slope: $-2/3$

Equation: $y - (1/2) = (-2/3)(x - 4)$

$y - 1/2 = (-2/3)x + 8/3$

$y = (-2/3)x + 8/3 + 1/2$

$y = (-2/3)x + 16/6 + 3/6 = (-2/3)x + 19/6$

$m = -2/3$, $c = 19/6$

Answer: $y = (-2/3)x + 19/6$

8. (a) A cylindrical solid of radius 7 cm and height 25 cm is cut equally from the top to bottom resulting into equal half solids (see figure 3). Find the total surface area of one half solid

Radius = 7 cm, Height = 25 cm

Cut vertically: Half cylinder → Curved surface + two semicircles + flat face

Curved surface area of half = $(1/2) \times 2\pi rh = \pi \times 7 \times 25 = 175\pi$

Semicircles (top and bottom): $2 \times (1/2) \times \pi r^2 = \pi \times 7^2 = 49\pi$

Flat face: $7 \times 25 = 175$

Total surface area = $175\pi + 49\pi + 175 = 224\pi + 175$

Using $\pi = 22/7$: $224 \times 22/7 = 704$

Total = $704 + 175 = 879\ cm^2$

Answer: $879\ cm^2$

(b) What is the area of a regular 45 sides polygon inscribed in a circle of radius 6 cm?

Number of sides $n = 45$, radius $r = 6$ cm

Area of regular polygon = $(1/2) \times n \times r^2 \times \sin(360°/n)$

Angle = $360°/45 = 8°$

Area = (1/2) × 45 × 6^2 × sin 8°

sin 8° ≈ 0.1392

Area = (1/2) × 45 × 36 × 0.1392 = 22.5 × 36 × 0.1392 ≈ 112.75 cm^2

Answer: 112.75 cm^2

9. (a) Mary received a certain amount of money from her father to go to school. She spent one third in her journey to school. At school she paid three fifths of the remaining amount as school fees and remained with 24,000/- as her pocket money. Showing the procedure; calculate the total amount of money she received from her father

Let total amount = x

After journey: (2/3)x

After school fees: (2/5) of (2/3)x = (4/15)x

Remaining: (2/3)x - (4/15)x = (10/15)x - (4/15)x = (6/15)x = (2/5)x

(2/5)x = 24,000

x = 24,000 × (5/2) = 60,000

Answer: 60,000

(b) If $f(x) = x^3 + kx^2 + 4x + 4$ has a remainder 16 when divided by x - 2. Find the value of k

$f(x) = x^3 + kx^2 + 4x + 4$, remainder when divided by x - 2:

f(2) = 16

$2^3 + k(2^2) + 4(2) + 4 = 16$

8 + 4k + 8 + 4 = 16

20 + 4k = 16

4k = -4

k = -1

Answer: k = -1

10. (a) The graph in figure 4 represents the journey made by a car between two sets of traffic lights. How far is it between the traffic lights?

Distance = Area under velocity-time graph

Break into triangles and rectangles:

0-100s: Triangle, area = $(1/2) \times 100 \times 50 = 2500$ m

100-200s: Rectangle, area = $100 \times 50 = 5000$ m

200-400s: Rectangle, area = $200 \times 40 = 8000$ m

400-500s: Triangle, area = $(1/2) \times 100 \times 40 = 2000$ m

Total distance = $2500 + 5000 + 8000 + 2000 = 17{,}500$ m = 17.5 km

Answer: 17.5 km

(b) Solve the simultaneous equations

$\{ x - y^2 = 2$

$\{ 2x^2 + 3y^2 = 15$

From first: $x = y^2 + 2$

Substitute into second: $2(y^2 + 2)^2 + 3y^2 = 15$

$2(y^4 + 4y^2 + 4) + 3y^2 = 15$

$2y^4 + 8y^2 + 8 + 3y^2 = 15$

$2y^4 + 11y^2 - 7 = 0$

$y^4 + 11/2\ y^2 - 7/2 = 0$

$2y^4 + 11y^2 - 7 = 0$

Let $u = y^2$: $2u^2 + 11u - 7 = 0$

$u = [-11 \pm \sqrt{(121 + 56)}] / 4 = [-11 \pm \sqrt{177}] / 4$

$u \approx 0.577$ or $u \approx -6.077$ (discard negative)

$y^2 \approx 0.577 \rightarrow y \approx \pm 0.76$

$x = y^2 + 2 \approx 0.577 + 2 \approx 2.58$

Solutions: (2.58, 0.76), (2.58, -0.76)

Answer: (2.58, 0.76), (2.58, -0.76)